Elevator Design

Chapter 12
Design of Control Surfaces

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12.5.1. Introduction

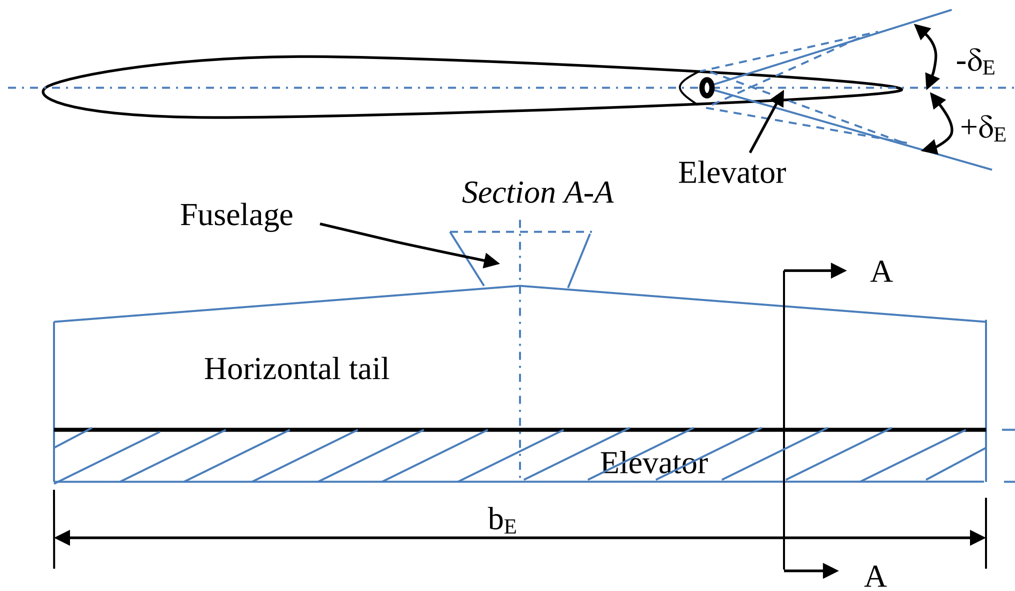
A very fundamental requirement of a safe flight is longitudinal control; which is assumed to be the primary function of an elevator. An aircraft must be longitudinally controllable, as well as maneuverable within the flight envelope (Figure 12.7). In a conventional aircraft, the longitudinal control is primarily applied though the deflection of elevator (δ_E), and engine throttle setting (δ_T). Longitudinal control is governed through pitch rate (Q) and consequently angular acceleration ($\ddot{\theta}$) about y-axis (or rate of pitch rate). Longitudinal control of an aircraft is achieved by providing an incremental lift force on horizontal tail. Thus, elevator which is classified as a primary control surface is considered as a pitch control device.

The incremental tail lift can be generated by deflecting the entire tail or by deflecting elevator which is located at the tail trailing edge. Since the horizontal tail is located at some distance from the aircraft center of gravity, the incremental lift force creates a pitching moment about the aircraft cg. Pitch control can be achieved by changing the lift on either aft horizontal tail or canard.

There are two groups of requirements in the aircraft longitudinal controllability: 1. Pilot force, 2. Aircraft response to the pilot input. In order to deflect the elevator, the pilot must apply a force to stick/yoke/wheel and hold it (in the case of an aircraft with a stick-fixed control system). In an aircraft with a stick-free control system, the pilot force is amplified through such devices as tab and spring. The pilot force analysis is out of scope of this text; the interested reader is referred to study references such as [11] and [12].

In a conventional symmetric aircraft, the longitudinal control is not coupled with the lateral-directional control. Thus, the design of the elevator is almost entirely independent of the design of the aileron and the rudder. This issue simplifies the design of the elevator. In the design of the elevator, four parameters should be determined. They are: 1) elevator planform area (S_E), 2) elevator chord (C_E), 3) elevator span (b_E), and 4) maximum elevator deflection ($\pm\delta_{Emax}$). As a general guidance, the typical values for these parameters are as follows: S_E/S_h = 0.15 to 0.4, b_E/b_h = 0.8-1, C_E/C_h = 0.2-0.4, and

δ_{Emax_up} = -25 degrees, δ_{Emax_down} = +20 degrees. Figure 12.19 shows the geometry of the horizontal tail and elevator. As a convention, the up deflection of elevator is denoted negative, and down deflection as positive. Thus a, negative elevator deflection is creating a negative horizontal tail list while generating a positive (nose up) pitching moment.



Top-view of the horizontal tail and elevator
Figure 12.1. Horizontal tail and elevator geometry

Prior to the design of elevator, the wing and horizontal tail must be designed, as well as the most aft and most forward locations of aircraft center of gravity must be known. In this section, principals of elevator design, design procedure, governing equations, constraints, and design steps as well as a fully solved example are presented.

12.5.2. Principles of Elevator Design

Elevator is a primary control surface placed on the trailing edge of the horizontal tail or canard. Longitudinal control and longitudinal trim are two main functions of the elevator; and it has minor influence on the longitudinal stability. Elevator is flap-like and is deflected up and down. With this deflection, the camber of the airfoil of the tail is changed, and consequently tail lift coefficient (C_{Lh}) is changed. The main objective of elevator deflection is to increase or decrease tailplane lift and hence tailplane pitching moment.

Factors affecting the design of an elevator are elevator effectiveness, elevator hinge moment, and elevator aerodynamic and mass balancing. The elevator effectiveness is a measure of how effective the elevator deflection is in producing the desired pitching moment. The elevator effectiveness is a function of elevator size and tail moment arm. Hinge moment is also important because it is the aerodynamic moment that must be overcome to rotate the elevator. The hinge moment governs the magnitude of force required of the pilot to move the stick/yoke/wheel. Therefore, great care must be used in designing an elevator so that the stick force is within acceptable limits for the pilots. Aerodynamic and mass balancing (See Section 12.7) deal with technique to vary the hinge moment so that the stick force stays within an acceptable range; and no aero-elastic phenomenon occurs.

The longitudinal control handling qualities requirements during take-off operation is stated as follows: in an aircraft with a tricycle landing gear, the pitch rate should have a value such that the take-off rotation does not take longer than a specified length of time. Since the take-off rotation dynamics is governed by Newton's second law, the take-off rotation time may be readily expressed in terms of the aircraft angular acceleration ($\ddot{\theta}$) about the main gear rotation point. For instance, in a transport aircraft, the acceptable value for the take-off rotation time is 3-5 seconds. The equivalent value for the angular rotation rate to achieve such requirement is 4-6 deg/sec^2. This requirement must be satisfied when the aircraft center of gravity is located at the most forward location. Table 12.9 provides take-off angular acceleration requirements for various types of aircraft. These specifications are employed in the design of elevator.

In the elevator detail design process, the following parameters must be determined:

1. Elevator-chord-to-tail-chord ratio (C_E / C_h)
2. Elevator-span-to-tail-span ratio (b_E / b_h)
3. Maximum up elevator deflection ($-\delta_{E_{max}}$)
4. Maximum down elevator deflection ($+\delta_{E_{max}}$)
5. Aerodynamic balance of the elevator
6. Mass balance of the elevator

The first four elevator parameters (chord, span, and deflections) are interrelated. When the value of one elevator parameter is increased, the value of other parameters could be decreased. On the other hand, each parameter has unique constraint. For instance, the elevator maximum deflection should be less than the value that causes flow separation or causes the horizontal tail to stall. In addition, the ease of fabrication suggests to having an elevator chord of span that is more convenient. Thus, for simplicity in the design and manufacture, the elevator span is often selected to be equal to the horizontal tail span (i.e. $b_E/b_h = 1$).

When elevator is deflected more than about 20-25 degrees, flow separation over the tail tends to occur. Thus, the elevator will lose its effectiveness. Furthermore, close to horizontal tail stall, even a small downward elevator deflection can produce flow separation and loss of pitch control effectiveness. To prevent pitch control effectiveness, it is recommended to consider the elevator maximum deflection to be less than 25 degrees (both up and down). Hence, the maximum elevator deflection is dictated by the elevator/tail stall requirement.

No	**Aircraft**	**Type**	**m_{TO} (kg)**	**S_E/S_h**	**C_E/C_h**	**δ_{Emax} (deg)**	
						down	**up**
1	Cessna 182	Light GA	1,406	0.38	0.44	22	25
2	Cessna Citation III	Business jet	9,979	0.37	0.37	15	15.5
3	Gulfstream 200	Business jet	16,080	0.28	0.31	20	27.5
4	AT-802	Agriculture	7,257	0.36	0.38	15	29
5	ATR 42-320	Regional airliner	18,600	0.35	0.33	16	26
6	Lockheed C-130 Hercules	Military cargo	70,305	0.232	0.35	15	40
7	Fokker F-28-4000	Transport	33,000	0.197	0.22	15	25
8	Fokker F-100B	Airliner	44,450	0.223	0.32	22	25
9	McDonnell Douglas DC-8	Transport	140,600	0.225	0.25	10	25
10	McDonnell Douglas DC-9-40	Transport	51,700	0.28	0.30	15	25
11	McDonnell Douglas DC-10-40	Transport	251,700	0.225	0.25	16.5	27
12	McDonnell Douglas MD-11	Transport	273,300	0.31	0.35	20	37.5
13	Boeing 727-100	Transport	76,820	0.23	0.25	16	26
14	Boeing 737-100	Transport	50,300	0.224	0.25	20	20
15	Boeing 777-200	Transport	247,200	0.30	0.32	25	30
16	Boeing 747-200	Transport	377,842	0.185	0.23	17	22
17	Airbus A-300B	Transport	165,000	0.295	0.30	-	-
18	Airbus 320	Transport	78,000	0.31	0.32	17	30
19	Airbus A340-600	Airliner	368,000			15	30
20	Lockheed L-1011 Tristar	Transport	231,000	0.215	0.23	0	25
21	Lockheed C-5A	Cargo	381,000	0.268	0.35	10	20

Table 12.1. Specifications of elevators for several aircraft

Provided that the elevator is designed to have the full span (i.e. $b_E = b_h$), and the deflection is at its maximum allowable value, the elevator chord must be long enough to generate the desired change in the tail lift. However, as the elevator chord is increased, the tail becomes more prone to the flow separation. If the required elevator chord is more than 50% of horizontal tail chord (i.e. $C_E/C_h > 0.5$), an all moving tail (i.e. $C_E = C_h$) is recommended. Fighter aircraft are often equipped with all moving horizontal tail to create the maximum amount of pitching moment in order to improve the pitch maneuverability. Most fighter aircraft have such tail, since they are required to be highly maneuverable. Table 12.18 shows specifications of elevators for several aircraft.

The most critical flight condition for pitch control is when the aircraft is flying at a low speed due to the fact that elevator is less effective. Two flight operations which feature a very low speed are take-off and landing. Take-off control is much harder than the landing control due to the safety considerations. A take-off operation is usually divided into three sections: 1. Ground section, 2. Rotation or transition, 3. Climb. The longitudinal control in a take-off is mainly applied during the rotation section which the nose is pitched up by rotating the aircraft about main gear.

A fundamental criterion for elevator design is the elevator effectiveness. The elevator effectiveness is a representative of longitudinal control power and is frequently measured by three non-dimensional derivatives ($C_{m_{\delta_E}}, C_{L_{\delta_E}}, C_{L_{h_{\delta E}}}$) as follows:

1. The primary production of the elevator is an aircraft pitching moment to control the pitch rate. The non-dimensional derivative which represents the longitudinal control power derivative is the rate of change of aircraft pitching moment coefficient with respect to elevator deflection ($C_{m_{\delta_E}}$). This is determined as:

$$C_{m_{\delta_E}} = \frac{\partial C_m}{\partial \delta_E} = -C_{L_{\alpha_h}} \eta_h \cdot \overline{V}_H \cdot \frac{b_E}{b_h} \tau_e \tag{12.51}$$

   where $C_{L_{\alpha_h}}$ is the horizontal tail lift curve slope, $\overline{V}_H$ denotes the horizontal tail volume coefficient, η_h is the horizontal tail dynamic pressure ratio. The parameter τ_e is the angle of attack effectiveness of the elevator which is primarily a function of elevator-to-tail-chord ratio (C_E/C_h). The latter variable (τ_e) is determined from Figure 12.12. The typical value for derivative $C_{m_{\delta_E}}$ is about -0.2 to -4 1/rad.

2. Another measure of elevator effectiveness is a parameter which represents the contribution of elevator to the aircraft lift ($C_{L_{\delta_E}}$). This non-dimensional derivative is the rate of change of aircraft lift coefficient with respect to elevator deflection and is defined as follows:

$$C_{L_{\delta_E}} = \frac{\partial C_L}{\partial \delta_E} = C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} \frac{b_E}{b_h} \tau_e \tag{12.52}$$

   where η_h is the horizontal tail dynamic pressure ratio, and S_h is the horizontal tail planform area.

3. The third measure of elevator effectiveness is a non-dimensional derivative which represents the contribution of elevator to the tail lift ($C_{L_{h_{\delta E}}}$). This derivative is the rate of change of tail lift coefficient with respect to elevator deflection and is defined as follows:

$$C_{L_{h_{\delta E}}} = \frac{\partial C_{L_h}}{\partial \delta_E} = \frac{\partial C_{L_h}}{\partial \alpha_h}\frac{\partial \alpha_h}{\partial \delta_E} = C_{L_{\alpha_h}} \tau_e \tag{12.53}$$

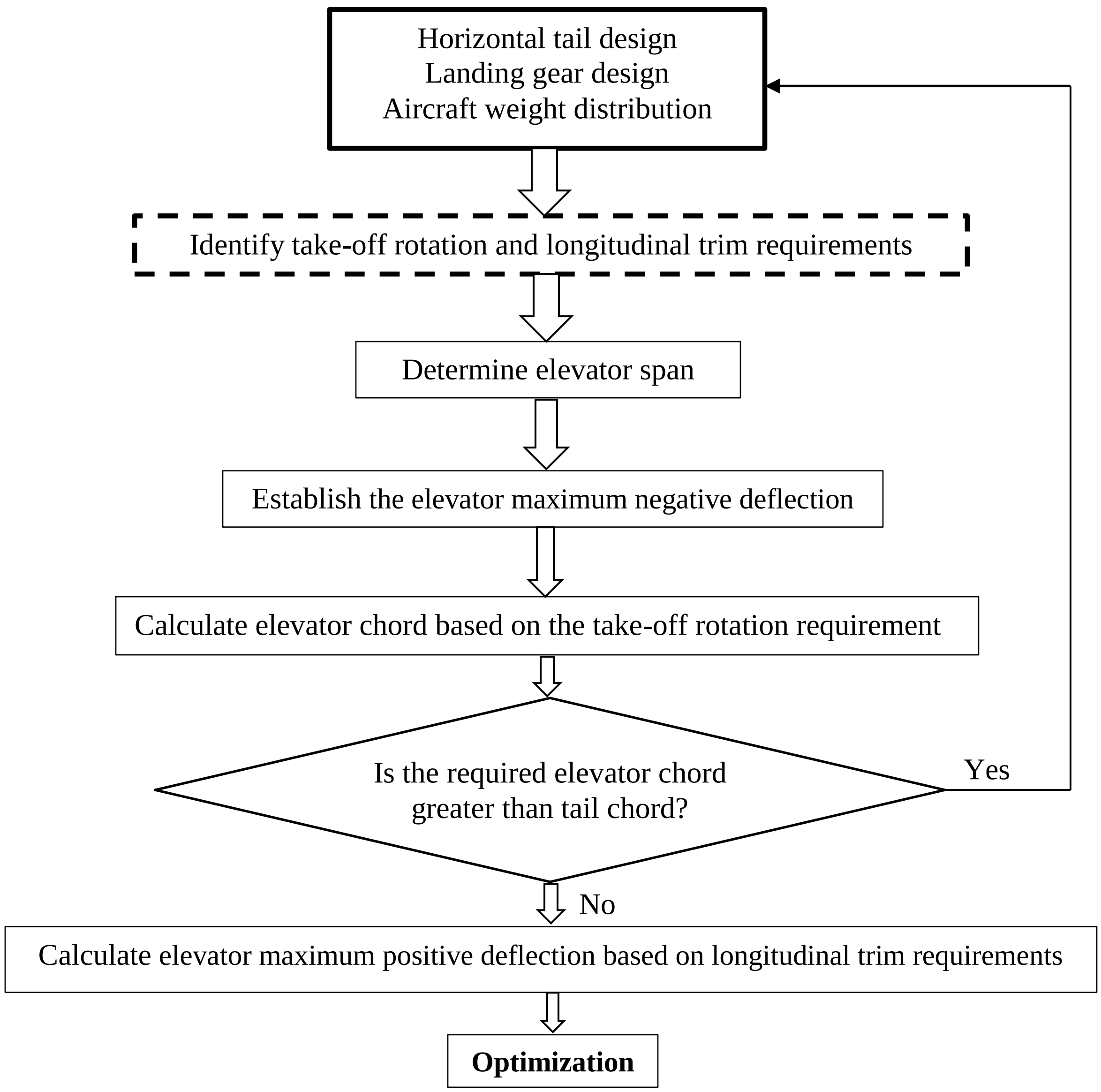


Figure 12.2. Elevator design flowchart

The most significant elevator design requirement is the take-off rotation requirement. This design requirement is a function of aircraft mission and the landing gear configuration. Two more popular configurations are: 1. Nose gear or tricycle, 2. Tail gear. These two landing gear configurations tend to desire different take-off rotation requirement as follows:

1. In aircraft with a tricycle landing gear, the elevator must be powerful enough to rotate the aircraft about the main gear and lift the nose with specified angular pitch acceleration. This requirement shall be satisfied when the aircraft has 80 percent of take-off speed (0.8 V_{TO}) and the aircraft center of gravity is at its most allowable forward position. This requirement is equivalent to a rotation at stall speed with a specified angular acceleration.

2. In aircraft with a tail gear configuration, the elevator must be such that to rotate the aircraft about main gear and lift the tail with specified angular pitch acceleration. This requirement shall be satisfied when the aircraft has 50 percent of take-off speed ($0.5 V_{TO}$) and the aircraft center of gravity is at its most allowable aft position.

The angular pitch acceleration requirement for various aircraft is given in Table 12.9. In a conventional aircraft, the take-off rotation when aircraft cg is at its most forward location; frequently requires the most negative elevator deflection (up). On the other hand, the longitudinal trim when aircraft cg is at its most aft location and the aircraft has the lowest allowable speed; usually requires the most positive elevator deflection (down). The governing equations for take-off rotation operation and technique to calculate pitch rate acceleration during rotation is developed in Section 12.5.3. The governing equations for longitudinal trim and technique to calculate the desired elevator deflection is developed in Section 12.5.4.

There are generally crucial interrelations between elevator design, landing gear design, and aircraft weight distribution (i.e. aircraft cg positions) process. Any of these three components/parameters will impose a limit/constraint on other two components/ parameters. For instance, as the aircraft most allowable cg is pushed forward, the elevator is required to be larger. Furthermore, as the main gear in a tricycle configuration is moved rearward, the elevator needs to be more powerful. Hence, it is necessary that the elevator design group has a compromising attitude and a close relationship with landing gear design team, and also with aircraft weight distribution group. Sometimes, a slight change in the landing gear design may lead to a considerable improvement in the elevator design. In the interest of minimizing the total cost/aircraft weight, changes must be incorporated as required, leading to a preferred design configuration. Therefore, the elevator and landing gear must be designed/evaluated/optimized simultaneously, and aircraft cg must be positioned such that to provide the best design environment for both landing gear and elevator. Elevator design flowchart is presented in Figure 12.20.

12.5.3. Take-off Rotation Requirement

For an aircraft with a landing gear configuration which the main gear is behind aircraft cg (e.g. tricycle landing gear), the take-off rotation requirement is employed to design the elevator. Most aircraft, to become airborne, must be rotated about the main gear to achieve the angle of attack required for lift-off. Exceptions to this are aircraft like military bomber B-52. The take-off rotation requirement requires the elevator design be such that the pitch angular acceleration ($\ddot{\theta}$) is greater than a desired value. In Chapter 9, the requirement is mathematically developed by specifically focusing on the relationship with landing gear design. In this section, the elevator design technique is established on the technique that is developed in Chapter 9.

The angular acceleration about the main gear rotation point; $\ddot{\theta}$ is a function of number of parameters including horizontal tail area, horizontal tail arm, aircraft weight, rotation speed, the distance between main gear and the aircraft cg, and finally elevator control power. Typical rotational acceleration is given in Table 12.9 for various types of aircraft. The rotation acceleration is the aircraft acceleration at the time the aircraft begins to rotate about the main gear. This speed must be slightly more that the stall speed (V_s):

$$V_R = 1.1\ V_s - 1.3\ V_s \tag{12.54}$$

However, for the safety factor, the elevator is designed to rotate the aircraft with the desired acceleration at the stall speed (V_s).

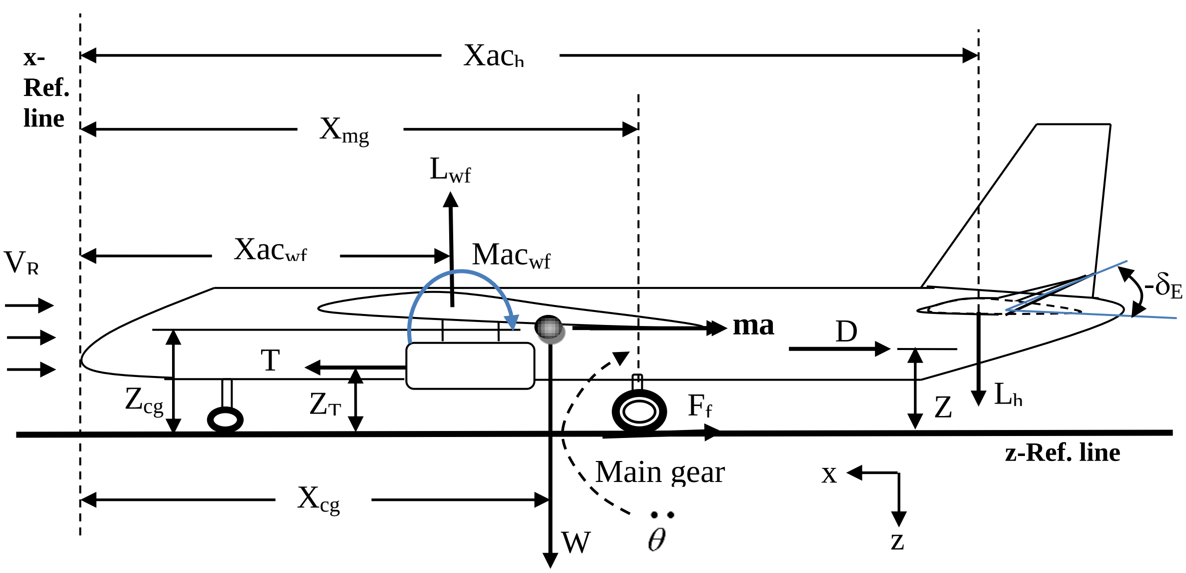


Figure 12.3. Forces and moments during take-off rotation

In this Section, an analysis of the elevator design to generate a given level of pitch angular acceleration about the main gear contact point is presented. Consider the aircraft with a tricycle landing gear in figure 12.21 which is at the onset of a rotation about the main gear in a take-off operation. The figure illustrates all forces and moments contributing to this moment of the take-off operation. Contributing forces include wing-fuselage lift (L_{wf}), horizontal tail lift (L_h), aircraft drag (D), friction force between tires and the ground (F_f), aircraft weight (W), engine thrust (T), and acceleration force (m.a). Please note that latter force (m.a) is acting backward due to the Newton's third law as a reaction to the acceleration. Furthermore, the contributing moments are the wing-fuselage aerodynamic pitching moment (Mo_{wf}) plus the moments of preceding forces about the rotation point. The distance between these forces are measured with respect to both x-reference line (i.e. fuselage nose), and z-reference line (i.e. ground) as shown in figure 12.21.

For a conventional aircraft with a tricycle landing gear, the horizontal tail lift is negative during rotation. It is recommended to consider the ground effect in calculating lift and drag to achieve more accurate results. The ground friction coefficient, μ, depends on the type of terrain. Table 9.7 introduces the friction coefficients for different terrains.

There are three governing equations of motion that govern the aircraft equilibrium at the instant of rotation; two force equations and one moment equation:

$$\sum F_x = m\frac{dV}{dt} \Rightarrow \ T - D - F_f = ma \Rightarrow \ T - D - \mu N = ma \tag{12.55}$$

$$\sum F_z = 0 \Rightarrow \ L + N = W \Rightarrow L_{wf} - L_h + N = W \Rightarrow N = W - \left(L_{wf} - L_h\right) \tag{12.56}$$

$$\sum M_{cg} = I_{yy_{mg}} \ddot{\theta} \Rightarrow - M_W + M_D - M_T + M_{L_{wf}} + M_{ac_{wf}} + M_{L_h} + M_a = I_{yy_{mg}} \ddot{\theta} \tag{12.57}$$

The equation 12.57 indicates that the aircraft negative pitching moment must be overcome by an opposite moment created by deflecting the elevator. All contributing forces and moments in equations 12.55 through 12.57 are introduced in Chapter 9; they are repeated and renumbered here for convenience. The normal force (N), the friction force (F_f), and the aircraft lift at take-off are:

$$N = W - L_{TO} \tag{12.58}$$

$$F_f = \mu N = \mu\left(W - L_{TO}\right) \tag{12.59}$$

$$L_{TO} = L_{wf} + L_h \tag{12.60}$$

The wing-fuselage lift (L_{wf}), horizontal tail lift (L_h), aerodynamic drag (D) forces and wing-fuselage pitching moment about wing-fuselage aerodynamic center are as follows. Recall that the horizontal tail lift is negative.

$$L_h = \frac{1}{2}\rho V_R^2 C_{L_h} S_h \tag{12.61}$$

$$L_{wf} \cong \frac{1}{2}\rho V_R^2 C_{L_{TO}} S_{ref} \tag{12.62}$$

$$D_{TO} = \frac{1}{2}\rho V_R^2 C_{D_{TO}} S_{ref} \tag{12.63}$$

$$M_{ac_{wf}} = \frac{1}{2}\rho V_R^2 C_{m_{ac_{wf}}} S_{ref} \overline{C} \tag{12.64}$$

where V_R denote the aircraft linear forward speed at the instant or rotation, S_{ref} represents the wing planform area, S_h is the horizontal tail planform area, ρ is the air density, and $\overline{C}$ is the wing mean aerodynamic chord. Furthermore, four coefficients of C_D, C_{Lwf}, C_{Lh}, and Cm_{ac_wf} denote drag, wing-fuselage lift, horizontal lift, and wing-fuselage pitching moment coefficients respectively. In equation 12.57, the clockwise rotation about y-axis is assumed to be as positive rotation.

The contributing pitching moments in take-off rotation control are aircraft weight moment (M_W), aircraft drag moment (M_D), engine thrust moment (M_T), wing-fuselage lift moment (M_{Lwf}), wing-fuselage aerodynamic pitching moment ($M_{ac_{wf}}$), horizontal tail lift moment (M_{Lh}), and linear acceleration moment (M_a). These moments are obtained as follows:

$$M_W = W\left(x_{mg} - x_{cg}\right) \tag{12.65}$$

$$M_D = D\left(z_D - z_{mg}\right) \tag{12.66}$$

$$M_T = T\left(z_T - z_{mg}\right) \tag{12.67}$$

$$M_{L_{wf}} = L_{wf}\left(x_{mg} - x_{ac_{wf}}\right) \tag{12.68}$$

$$M_{L_h} = L_h\left(x_{ac_h} - x_{mg}\right) \tag{12.69}$$

$$M_a = ma\left(z_{cg} - z_{mg}\right) \tag{12.70}$$

In equations 12.65 through 12.70, the subscript "mg" denotes main gear, since the distances are measured from the main gear. The inclusion of the moment generated by the aircraft acceleration (equation 12.70) is due to the fact that based on the Newton's third law; any action creates a reaction (ma). This reaction force is producing a moment when its corresponding arm is taken into account. Substituting these moments into equation 12.57 yields:

$$\begin{aligned}\sum M_{cg} = I_{yy}\ddot{\theta} \Rightarrow -W\left(x_{mg} - x_{cg}\right) + D\left(z_D - z_{mg}\right) - T\left(z_T - z_{mg}\right) + L_{wf}\left(x_{mg} - x_{ac_{wf}}\right) + \\ M_{ac_{wf}} - L_h\left(x_{ac_h} - x_{mg}\right) + ma\left(z_{cg} - z_{mg}\right) = I_{yy_{mg}}\ddot{\theta}\end{aligned} \tag{12.71}$$

where Iyy_{mg} represents the aircraft mass moment of inertia about y-axis at the main gear. In an aircraft with a tricycle landing gear, the tail lift moment, wing-fuselage moment, drag moment, and acceleration moment are all clockwise, while the weight moment, thrust moment, and wing-fuselage aerodynamic pitching moment are counterclockwise. These directions must be considered when assigning a sign to each one. The role of elevator in equation 12.71 is to create a sufficient horizontal tail lift (L_h). The result is as follows:

$$L_h = \frac{L_{wf}\left(x_{mg} - x_{ac_{wf}}\right) + M_{ac_{wf}} + ma\left(z_{cg} - z_{mg}\right) - W\left(x_{mg} - x_{cg}\right) + D\left(z_D - z_{mg}\right) - T\left(z_T - z_{mg}\right) - I_{yy_{mg}}\ddot{\theta}}{x_{ac_h} - x_{mg}} \tag{12.72}$$

Then, this horizontal tail lift must be such that to satisfy the take-off rotation requirement. The elevator contribution to this lift is through tail lift coefficient which can be obtained by using equation 12.61.

$$C_{L_h} = \frac{2L_h}{\rho V_R^2 S_h} \tag{12.73}$$

This tail lift coefficient is generally negative (about -1 to -1.5) and is a function of tail angle of attack (α_h), tail airfoil section [21] features, and tail planform parameters such as aspect ratio, sweep angle and taper ratio. The horizontal tail lift coefficient is modeled as:

$$C_{L_h} = C_{L_{ho}} + C_{L_{\alpha_h}}\alpha_h + C_{L_{h_{\delta E}}}\delta_E \tag{12.74}$$

where $C_{L_{\alpha_h}}$ is the tail lift curve slope and $C_{L_{ho}}$ is the zero angle of attack tail lift coefficient. Most horizontal tails tend to use a symmetric airfoil section, so the parameter $C_{L_{ho}}$ is normally zero. Inclusion of this statement, and plugging equation 12.53 into equation 12.74 results in:

$$C_{L_h} = C_{L_{\alpha_h}}\alpha_h + C_{L_{\alpha_h}}\tau_e\delta_E = C_{L_{\alpha_h}}\left(\alpha_h + \tau_e\delta_E\right) \tag{12.75}$$

Recall that the tail angle of attack is already (See Chapter 6) defined as:

$$\alpha_h = \alpha + i_h - \varepsilon \tag{12.76}$$

where α is the aircraft angle of attack at the onset of rotation, i_h denotes the tail incidence angle, and ε represents the downwash angle which is determined through equation 6.54. The aircraft angle of attack, when the aircraft is on the ground (i.e. onset of rotation) is usually zero.

The elevator designer can control the magnitude of the elevator control power by proper selection of the elevator geometry. Equation 12.75 enables the elevator designer to determine elevator characteristics to satisfy take-off rotation requirement. Knowing τ_e, one can use Figure 12.12 to estimate the elevator chord to tail chord ratio. This represents the minimum elevator area to satisfy the most crucial aircraft longitudinal control requirement. Please note that the take-off rotation requirement dictates the maximum up deflection of the elevator ($-\delta_{Emax}$). The maximum positive (down) deflection of the elevator ($+\delta_{Emax}$) is dictated by the longitudinal trim requirement which will be examined in the next Section. The elevator will normally generate its maximum negative pitching moment to maintain longitudinal trim when the aircraft is flying with the lowest velocity and the aircraft cg is at its most aft allowable location. However, the elevator will generally create its maximum positive pitching moment during take-off rotation when the aircraft cg is at its most forward allowable location and aircraft has the maximum take-off weight.

The satisfaction of the take-off rotation requirements is frequently generates a conflict between design groups such as landing gear design group, tail design group, weight and balance group, fuselage design group, propulsion system design group, and elevator design group. Each design group may focus on other design requirements and consider the rotation requirement at the end of the list. If that is the case, one design group will create a challenge for other design groups which may not be resolved easily. The solution is to have all design groups to debate about various solutions and adopt the least challenging one.

12.5.4. Longitudinal Trim Requirement

When all longitudinal moments and forces are in equilibrium, it is said that the aircraft is in longitudinal trim. In this section, we shall be concerned with longitudinal trim. The elevator plays a significant role in the aircraft longitudinal trim to fly at various trim conditions. To carry out the longitudinal trim analysis and to derive a relationship that represents the function of elevator in longitudinal trim, consider the aircraft in Figure 12.22 which is cruising with a constant speed. The engine is located under the wing and engine thrust (T) has an offset (z_T) with the aircraft center of gravity. The engine is creating a positive pitching moment. It is assumed that engine setting angle is zero.

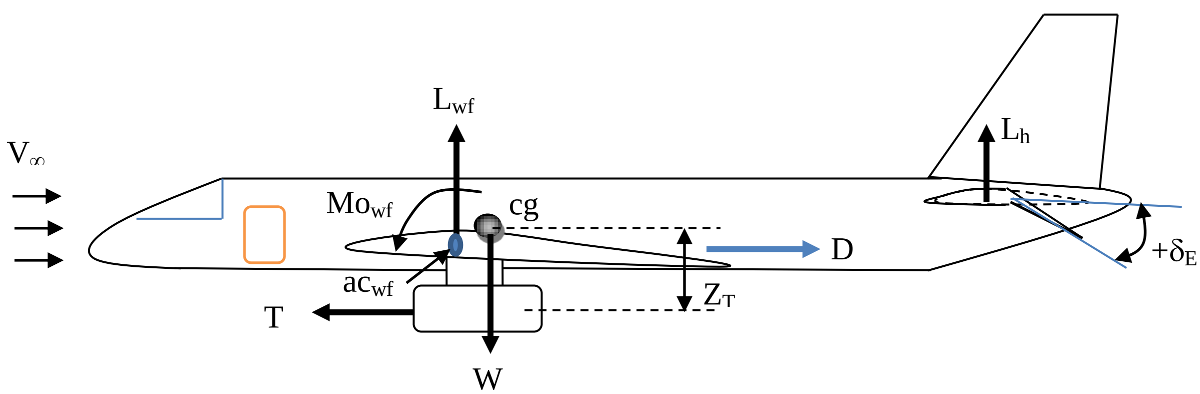


Figure 12.4. Longitudinal trim

The governing longitudinal trim equations are:

$$\sum F_z = 0 \Rightarrow L = W \tag{12.77}$$

$$\sum F_x = 0 \Rightarrow D = T \tag{12.78}$$

$$\sum M_{cg} = 0 \Rightarrow M_A + T \cdot z_T = 0 \tag{12.79}$$

It is also assumed that there is always sufficient thrust to balance the drag force, thus only two equations need to be expanded.

$$\bar{q} \cdot S \cdot C_L = W \tag{12.80}$$

$$\bar{q} \cdot S \cdot \bar{C} \cdot C_m + T \cdot z_T = 0 \tag{12.81}$$

The aerodynamic forces and moments are functions of non-dimensional derivatives, so equations 12.80 and 12.81 may be written as follows:

$$\bar{q} \cdot S \cdot \left(C_{L_o} + C_{L_\alpha}\alpha + C_{L_{\delta_E}}\delta_E\right) = W \tag{12.82}$$

$$\bar{q} \cdot S \cdot \bar{C} \cdot \left(C_{m_o} + C_{m_\alpha}\alpha + C_{m_{\delta_E}}\delta_E\right) + T \cdot z_T = 0 \tag{12.83}$$

The equations may be reformatted as:

$$C_{L_o} + C_{L_\alpha}\alpha + C_{L_{\delta_E}}\delta_E = \frac{W}{\bar{q} \cdot S} = C_{L_1} \tag{12.84}$$

$$C_{m_o} + C_{m_\alpha}\alpha + C_{m_{\delta_E}}\delta_E = -\frac{T \cdot z_T}{\bar{q} \cdot S \cdot \bar{C}} \tag{12.85}$$

where C_{L1} is the steady-state aircraft lift coefficient at this cruising flight. It is useful to recast the equations in a matrix format:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta_E}} \\ C_{m_\alpha} & C_{m_{\delta_E}} \end{bmatrix} \begin{bmatrix} \alpha \\ \delta_E \end{bmatrix} = \begin{bmatrix} C_{L_1} - C_{L_o} \\ -\dfrac{T \cdot z_T}{\bar{q} \cdot S \cdot \bar{C}} - C_{m_o} \end{bmatrix} \tag{12.86}$$

This set of equations has two unknowns; aircraft angle of attack (α), and elevator deflection (δ_E). Solutions to this set of equation employing Cramer's rule are the following:

$$\alpha = \frac{\begin{vmatrix} C_{L_1} - C_{L_o} & C_{L_{\delta_E}} \\ -\dfrac{T \cdot z_T}{\bar{q} \cdot S \cdot \bar{C}} - C_{m_o} & C_{m_{\delta_E}} \end{vmatrix}}{\begin{vmatrix} C_{L_\alpha} & C_{L_{\delta_E}} \\ C_{m_\alpha} & C_{m_{\delta_E}} \end{vmatrix}} \tag{12.87}$$

$$\delta_E = \frac{\begin{vmatrix} C_{L_\alpha} & C_{L_1} - C_{L_o} \\ C_{m_\alpha} & -\dfrac{T \cdot z_T}{\overline{q} \cdot S \cdot \overline{C}} - C_{m_o} \end{vmatrix}}{\begin{vmatrix} C_{L_\alpha} & C_{L_{\delta_E}} \\ C_{m_\alpha} & C_{m_{\delta_E}} \end{vmatrix}} \tag{12.88}$$

or

$$\alpha = \frac{\left(C_{L_1} - C_{L_o}\right)C_{m_{\delta_E}} + \left(\dfrac{T \cdot z_T}{\overline{q} \cdot S \cdot \overline{C}} + C_{m_o}\right)C_{L_{\delta_E}}}{C_{L_\alpha} C_{m_{\delta_E}} - C_{m_\alpha} C_{L_{\delta_E}}} \tag{12.89}$$

$$\delta_E = -\frac{\left(\dfrac{T \cdot z_T}{\overline{q} \cdot S \cdot \overline{C}} + C_{m_o}\right)C_{L_\alpha} + \left(C_{L_1} - C_{L_o}\right)C_{m_\alpha}}{C_{L_\alpha} C_{m_{\delta_E}} - C_{m_\alpha} C_{L_{\delta_E}}} \tag{12.90}$$

where the aircraft static longitudinal stability derivative (Cm_α) is determined by Equation 6.67 in Chapter 6. The elevator deflection to maintain the aircraft longitudinal trim can be directly obtained from equation 12.90. Please note that if the thrust line is above the aircraft cg, the parameter z_T would be negative. The elevator angle must be large enough to maintain longitudinal trim at all flight conditions; particularly when the aircraft center of gravity is located at the most allowable aft position.

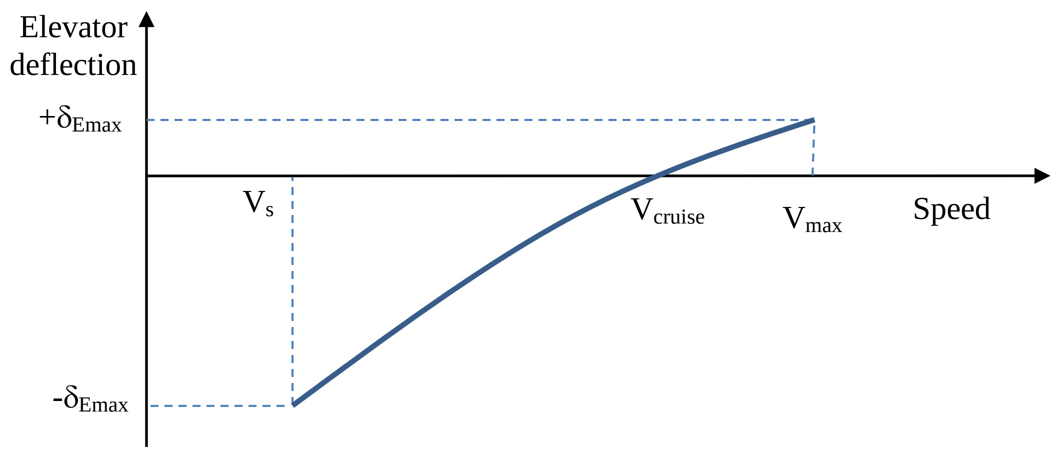


Figure 12.5. Typical variations of elevator deflection versus aircraft speed

The elevator designer must synthesize an elevator in such a way that longitudinal trim is not a limiting factor anywhere in the intended flight envelope and throughout aircraft mission. In case where the required elevator angle is more than about 30 degrees, the designer needs to increase the elevator size or even the tail arm. This is to ensure that the elevator does not cause any flow separation over the tail during its application. Figure

12.23 shows the typical variations of elevator deflection versus aircraft speed to maintain aircraft longitudinal trim. As the figure illustrates, one of the objectives in the horizontal tail design is to require a zero elevator deflection during a cruising flight. Please note that as a convention, the up deflection of the elevator is considered to be negative.

There is a constraint on the elevator design which must be considered and checked. The elevator deflection must not cause the horizontal tail to stall. The elevator deflection will decrease the tail stall angle. At the end of take-off rotation, the aircraft, wing, and tail angles of attack are all increased. The elevator may be thought of as a plain flap attached to the tail, so tail stall angle is depending upon elevator chord and deflection. So, the elevator designer should check whether the tail stall occurs when the maximum elevator deflection is employed and fuselage is lifted up. The recommendation is to keep the tail within 2 degrees of its stall angle of attack. Using equation 12.76, the relationship between horizontal tail angle at take-off and fuselage take-off angle of attack (α_{TO}) is obtained:

$$\alpha_{h_{TO}} = \alpha_{TO}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + i_h - \varepsilon_o \qquad (12.91)$$

The fuselage take-off angle of attack may be assumed to be equal to aircraft take-off angle of attack. This equation yields the maximum positive tail angle of attack that must be less than tail stall angle. On the other hand, during a cruising flight with a maximum speed, when the elevator is employed to maintain longitudinal trim, the maximum positive elevator deflection must be checked. The tail stall angle of attack during take-off rotation (α_{h_s}) is a function of a number of parameters including tail airfoil section, elevator deflection and elevator chord and is determined by:

$$\alpha_{h_s} = \pm\left(\alpha_{h_{s_{\delta E=0}}} - \Delta\alpha_{h_E}\right) \qquad (12.92)$$

δ_E (deg)	**Tail-to-elevator-chord ratio; C_E/C_h**										
	0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0	0	0	0	0	0	0	0	0	0	0	0
±5	0	0.3	0.5	1.1	1.6	2.2	2.7	3.3	3.9	4.4	5
±10	0	0.6	1	2.1	3.2	4.4	5.5	6.6	7.7	8.9	10
±15	0	0.9	1.5	3.2	4.9	6.5	8.2	9.9	11.6	13.3	15
±20	0	1.2	2	4.2	6.5	8.7	11	13.2	15.5	17.7	20
±25	0	1.6	2.5	5.3	8.1	11	13.7	16.5	19.4	22.2	25
±30	0	1.9	3	6.4	9.7	13.1	16.5	19.9	23.2	26.6	30

Table 12.2. Reduction in tail stall angle ($\Delta\alpha_{h_E}$) in degrees when elevator is deflected

where the $\alpha_{h_{s_{\delta E=0}}}$ is the tail stall angle (typically about 14 degrees) when elevator is not employed. The parameters $\Delta\alpha_{h_E}$ is the magnitude of reduction in tail stall angle of attack due to elevator deflection; and must be determined using a wind tunnel testing or referring to Aerodynamics references. Table 12.19 illustrates the empirical values (in degrees) for the parameter $\Delta\alpha_{h_E}$ as a function of elevator deflection and tail-to-elevator-chord ratio.

When the elevator is designed, the generated horizontal tail lift coefficient needs to be calculated and compared with the desired tail lift coefficient. Tools such as computational fluid dynamics technique or lifting line theory (Chapter 5, Section 5.14) may be utilized for such calculation. One of the parameters in this evaluation process is the change in the tail zero-lift angle of attack due to elevator deflection ($\Delta\alpha_{o_E}$).is The horizontal tail is a lifting surface and may be treated in the same way as the wing. Thus, the empirical equation 5.39 is reformatted to approximate the parameter $\Delta\alpha_{o_E}$ as follows:

$$\Delta\alpha_{o_E} \approx -1.15 \cdot \frac{C_E}{C_h}\delta_E \qquad (12.93)$$

The generated horizontal tail lift coefficient must be equal to the desired generated horizontal tail lift coefficient. The parameter $\Delta\alpha_{o_E}$ is employed in the application of lifting line theory to approximate the tail lift coefficient as well as the tail lift distribution.

12.5.5. Elevator Design Procedure

In Sections 12.5.1 through 12.5.4; elevator primary function, parameters, governing rules and equations, objectives, design criteria, formulation, as well as design requirements have been presented in details. In addition, Figure 12.20 illustrates the design flow chart of the elevator. In this section, elevator design procedure in terms of design steps is introduced. It must be noted that there is no unique solution to satisfy the customer requirements in designing an elevator. Several elevator designs may satisfy the requirements, but each will have a unique advantages and disadvantages. It must be noted, that there is a possibility that no elevator can satisfy the requirements due to the limits/constraints imposed by tail design and landing gear design. In such a situation, the designer must return to tail design and/or landing gear design; and redesign those components.

Based on the systems engineering approach, the elevator detail design begins with identifying and defining design requirements and ends with optimization. The following is the elevator design steps for a conventional aircraft:

1. Layout the elevator design requirements (See Section 12.5.2)

2. Identify take-off rotation acceleration requirement from Table 12.9.
3. Select elevator span (See Table 12.3)
4. Establish maximum elevator deflection to prevent flow separation (See Table 12.3)
5. Calculate the wing-fuselage lift (L_{wf}), aircraft drag (D) and wing-fuselage pitching moment about wing-fuselage aerodynamic center using equations 12.62 through 12.64.
6. Calculate aircraft linear acceleration (a) during take-off rotation using equation 12.55.
7. Calculate the contributing pitching moments during take-off rotation (i.e. aircraft weight moment (M_W), aircraft drag moment (M_D), engine thrust moment (M_T), wing-fuselage lift moment (M_{Lwf}), wing-fuselage aerodynamic pitching moment ($M_{ac_{wf}}$), and linear acceleration moment (M_a) using equations 12.65 through 12.70. For this calculation, consider the most forward aircraft center of gravity.
8. Calculate desired horizontal tail lift (L_h) during take-off rotation employing equation 12.72. For this calculation, consider the most forward aircraft center of gravity.
9. Calculate desired horizontal tail lift coefficient (C_{L_h}) employing equation 12.73.
10. Calculate the angle of attack effectiveness of the elevator (τ_e) employing equation 12.75. In this calculation, the maximum negative elevator deflection (from step 4) is considered.
11. Determine the corresponding elevator-to-tail-chord ratio (C_E/C_h) from Figure 12.12.
12. If the elevator-to-tail-chord ratio (C_E/C_h) is more than 0.5, it is suggested to select an all moving tail (i.e. $C_E/C_h = 1$).
13. If the angle of attack effectiveness of the elevator (τ_e) is greater than 1, there is no elevator which can satisfy the take-off rotation requirement by the current tail/landing gear specifications. In such a case, horizontal tail and/or landing gear must be redesigned. Then, return to step 5.
14. Using an aerodynamic technique such as Computational Fluid Dynamics technique or lifting line theory (See Chapter 5, Section 5.14); determine the horizontal tail lift distribution and horizontal tail lift coefficient when the elevator is deflected with its maximum negative angle (i.e. $-\delta_{Emax}$).
15. Compare the produced horizontal tail lift coefficient of step 13 with the desired horizontal tail lift coefficient of step 9. These two numbers must be the same. If not, adjust elevator chord or elevator span to vary the produced horizontal tail lift coefficient.
16. Calculate elevator effectiveness derivatives ($C_{m_{\delta_E}}, C_{L_{\delta_E}}, C_{L_{h_{\delta E}}}$) from equation 12.51 through 12.53. For these calculations, examine both the most aft and the most forward aircraft center of gravity.
17. Calculate the elevator deflection (δ_E) required to maintain longitudinal trim at various flight conditions using equation 12.90. For these calculations, examine the most aft and the most forward aircraft center of gravity, as well as various aircraft speeds.

18. Plot the variations of the elevator deflection versus airspeed and also versus altitude. For these calculations, consider both the most aft and the most forward aircraft center of gravity.
19. Compare the maximum required down elevator deflection ($+\delta_{Emax}$) with the maximum deflection established in step 4. If the maximum required down elevator deflection of step 15 is greater than the maximum deflection established in step 4, there is no elevator which can satisfy the longitudinal trim requirements by the current tail/landing gear specification. In such a case, horizontal tail and/or landing gear must be redesigned. Then, return to step 5.
20. Check whether or not the elevator deflection causes the horizontal tail to stall during take-off rotation by using equation 12.92.
21. If tail stall will be occurred during take-off rotation, the elevator must be redesigned by reducing elevator deflection and/or elevator chord. Return to step 3.
22. If tail stall will be occurred during take-off rotation, and none of the two elevator parameters (i.e. elevator deflection and chord) may be reduced to prevent tail stall; other aircraft components such as horizontal tail, landing gear, or aircraft center of gravity must be redesigned/relocated.
23. Aerodynamic balance/mass balance if necessary (Section 12.7)
24. Optimize the elevator.
25. Calculate elevator span, elevator chord and elevator area; and then draw the top-view and side-view of the horizontal tail (including elevator) with dimensions.

12.8.2. Elevator Design Example

Example 12.5

Problem statement: Figure 12.46 illustrates the geometry of a high-wing twin jet engine light utility aircraft which is equipped with a tricycle landing gear. Design an elevator for this aircraft which has the following characteristics.

m_{TO} = 20,000 kg, V_s = 85 KEAS, I_{yy} = 150,000 kg.m^2, T_{max} = 2×28 kN, L_f = 23 m, V_C = 360 KTAS (at 25,000 ft), C_{Lo} = 0.24, C_{DoC} = 0.024, C_{DoTO} = 0.038, $C_{L\alpha}$ = 5.7 1/rad

Wing:
S = 70 m^2, AR = 8, $C_{L\alpha wf} = C_{L\alpha w}$ = 5.7 1/rad, e = 0.88, λ = 1, $\Delta C_{LflapTO}$ = 0.5, Cm_{acwf} = 0.05, i_w = 2 deg, h_o = 0.25, α_{sTO} = 12 deg

Horizontal tail:
S_h = 16 m^2, b_h = 9 m, $C_{L\alpha h}$ = 4.3 1/rad, i_h = -1 deg, λ_h = 1, η_h = 0.96, α_{h_s} = 14 deg, Airfoil section: NACA 0009, α_{twist} = 0,

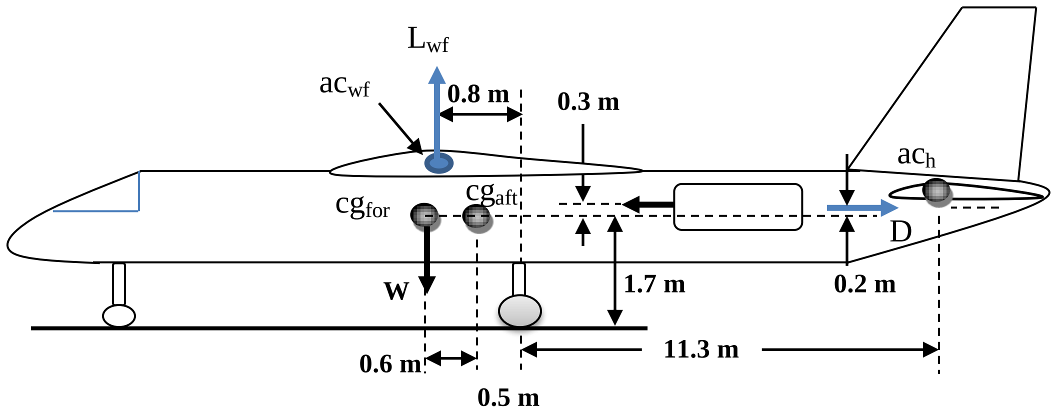


Figure 12.6. Aircraft geometry of Example 12.5

Solution:

Step 1:
The elevator design requirements are identified as follows:

a. Take-off rotation (longitudinal control) requirement. It is assumed that the airport is located at sea level altitude.
b. Longitudinal trim requirements (within flight envelope)
c. Low cost
d. Manufacturable

Step 2:
Based on Table 12.9, take-off pitch angular acceleration for this type of aircraft must be between 10 to 15 deg/sec^2. A value of 12 deg/sec^2 for the take-off pitch angular acceleration is tentatively selected.

Step 3:
Table 12.3 suggests a value of 0.8 to 1 for the elevator-span-to-tail-span ratio. A value of 1 is tentatively selected.

Step 4:
Table 12.3 suggests a value of -25 degrees for the elevator maximum deflection. A value of -25 degrees is tentatively selected.

Step 5:
Calculation of the wing-fuselage lift (L_{wf}), aircraft drag (D) and wing-fuselage pitching moment about wing-fuselage aerodynamic center. The air density at sea level is 1.225 kg/m^3, and at 25,000 ft is 0.549 kg/m^3. To obtain the wing mean aerodynamic chord, the following calculations are made:

$$b = \sqrt{S.AR} = \sqrt{70 \times 8} = 23.66 \quad m \tag{5.19}$$

$$\overline{C} = \frac{S}{b} = \frac{70}{23.66} = 2.96 \quad m \tag{5.18}$$

To find aircraft drag, we have:

$$K = \frac{1}{\pi.e.AR} = \frac{1}{3.14 \times 0.88 \times 8} = 0.045 \tag{5.22}$$

$$C_{L_C} = \frac{2W}{\rho V_C^2 S} = \frac{2 \times 20{,}000 \times 9.81}{0.549 \times (360 \times 0.5144)^2 \times 70} = 0.297 \tag{5.1}$$

$$C_{L_{TO}} = C_{L_C} + \Delta C_{L_{flap}} = 0.297 + 0.5 = 0.797 \tag{4.69c}$$

$$C_{D_{TO}} = C_{D_{oTO}} + KC_{L_{TO}}^2 = 0.038 + 0.045 \times 0.797^2 = 0.067 \tag{4.68}$$

$$V_R = V_S = 85 \quad knot = 43.73 \quad \frac{m}{s} \tag{12.54}$$

Thus, longitudinal aerodynamic forces and moment are:

$$D_{TO} = \frac{1}{2}\rho_o V_R^2 S C_{D_{TO}} = \frac{1}{2} \times 1.225 \times (43.73)^2 \times 70 \times 0.067 = 5{,}472 \quad N \tag{12.63}$$

$$L_{TO} \approx L_{wf} = \frac{1}{2}\rho_o V_R^2 S_{ref} C_{L_{TOf}} = \frac{1}{2} \times 1.225 \times (43.73)^2 \times 70 \times 0.797 = 65{,}371 \quad N \tag{12.62}$$

$$M_{ac_{wf}} = \frac{1}{2}\rho_o V_R^2 C_{m_{ac_{wf}}} S_{ref} \overline{C} = \frac{1}{2} \times 1.225 \times (43.73)^2 \times (0.05) \times 70 \times 2.96 = 12{,}125 \quad Nm \tag{12.64}$$

Step 6:

Calculation of aircraft linear acceleration (a) during take-off rotation using equation 12.55. The runway is assumed to be concrete, so from Table 9.7 (Chapter 9), a ground friction of 0.04 is selected.

$$F_f = \mu(W - L_{TO}) = 0.04(20{,}000 \times 9.81 - 65{,}371) = 5{,}230.5 \quad N$$

(12.59)

Aircraft linear acceleration at the time of take-off rotation

$$a = \frac{T - D_{TO} - F_R}{m} = \frac{2 \times 28{,}000 - 5{,}472 - 5{,}230.5}{20{,}000} \Rightarrow a = 2.265 \quad \frac{m}{s^2} \tag{12.55}$$

Step 7:

Calculation of the contributing pitching moments in the take-off rotation. The clockwise rotation about y axis is considered to be as positive direction. The most forward aircraft center of gravity is considered.

$$M_W = W(x_{mg} - x_{cg}) = -20{,}000 \times 9.81 \times 1.1 = -215{,}746 \quad Nm \tag{12.65}$$

$$M_D = D(z_D - z_{mg}) = 5{,}472 \times 1.9 = 10{,}397 \quad Nm \tag{12.66}$$

$$M_T = T(z_T - z_{mg}) = -2 \times 28{,}000 \times (1.7 + 0.3) = -112{,}000 \quad Nm \tag{12.67}$$

$$M_{L_{wf}} = L_{wf}\left(x_{mg} - x_{ac_{wf}}\right) = 65{,}371 \times 0.8 = 52{,}297 \quad Nm \tag{12.68}$$

$$M_a = ma(z_{cg} - z_{mg}) = 20{,}000 \times 2.265 \times 1.7 = 77{,}005.5 \; Nm \tag{12.70}$$

Step 8:
Calculate desired horizontal tail lift (L_h) during take-off rotation employing equation 12.72. For this calculation, consider the most forward aircraft center of gravity.

$$L_h = \frac{L_{wf}\left(x_{mg} - x_{ac_{wf}}\right) + M_{ac_{wf}} + ma(z_{cg} - z_{mg}) + W(x_{mg} - x_{cg}) + D(z_D - z_{mg}) + T(z_T - z_{mg}) - I_{yy_{mg}}\ddot{\theta}}{x_{ac_h} - x_{mg}}$$

$$L_h = \frac{52{,}297 + 12{,}125 + 77{,}005.5 - 215{,}746 + 10{,}397 - 112{,}000 - \left(150{,}000 \times \frac{12}{57.3}\right)}{11.3} \tag{12.72}$$

or

$$L_h = -18{,}348 \quad N$$

Step 9:
Calculation of the desired horizontal tail lift coefficient (C_{L_h}):

$$C_{L_h} = \frac{2L_h}{\rho_o V_R^2 S_h} = \frac{2 \times (-18{,}348)}{1.225 \times 43.73^2 \times 16} \Rightarrow C_{L_h} = -0.979 \tag{12.73}$$

Step 10:
Calculation of the angle of attack effectiveness of the elevator (τ_e). In this calculation, the maximum elevator deflection is considered.

$$C_{L_h} = C_{L_{\alpha_h}}\left(\alpha_h + \tau_e \delta_E\right) \tag{12.75}$$

The tail angle of attack is already defined as:

$$\alpha_h = \alpha + i_h - \varepsilon \tag{12.76}$$

where the downwash effect is determined as follows (6.54):

$$\varepsilon_o = \frac{2C_{L_w}}{\pi \cdot AR} = \frac{2C_{L_{TO}}}{\pi \cdot AR} = \frac{2 \times 0.797}{3.14 \times 8} = 0.063 \quad rad = 3.63 \quad \deg \tag{6.55}$$

$$\frac{\partial \varepsilon}{\partial \alpha} = \frac{2C_{L_{\alpha_w}}}{\pi \cdot AR} = \frac{2 \times 5.7}{\pi \cdot 8} = 0.454 \quad \deg/\deg \tag{6.56}$$

The wing angle of attack (α_w) at the take-off may be assumed to be equal to the wing incidence (i_w). Thus:

$$\varepsilon = \varepsilon_o + \frac{\partial \varepsilon}{\partial \alpha}\alpha_w = 0.063 + 0.454 \times \frac{2}{57.3} = 0.079 \quad rad = 4.54 \quad \deg \tag{6.54}$$

Hence, the horizontal tail angle of attack at instance of the take-off rotation is:

$$\alpha_h = \alpha + i_h - \varepsilon = 2 - 1 - 4.54 = -3.54 \quad \deg \tag{12.76}$$

The angle of attack effectiveness of the elevator from Equation 12.75 is:

$$\tau_e = \frac{\alpha_h + \left(C_{L_h}/C_{L_{\alpha_h}}\right)}{\delta_{E_{\max}}} = \frac{\frac{-3.54}{57.3} + \frac{-0.979}{4.3}}{-25/57.3} \Rightarrow \tau_e == 0.664 \tag{12.75}$$

Step 11:
The corresponding elevator-to-tail-chord ratio (C_E/C_h) to the τ_e of 0.664 (from Figure 12.12) is determined to be 0.49. Thus:

$$\frac{C_E}{C_h} = 0.49$$

In another word, the elevator-to-tail-chord ratio is determined to be 49%.

Steps 12, 13: checked.

Step 14:
Calculation of horizontal tail lift coefficient using lift line theory; when the elevator is deflected with its maximum negative angle (i.e. – δ_{Emax}). The change in the tail lift coefficient when the elevator is deflected is:

$$\Delta\alpha_{o_E} \approx -1.15 \cdot \frac{C_E}{C_h}\delta_E = -1.15 \times 0.49 \times (-25) = 14.088 \quad \deg \tag{12.93}$$

To apply of the lifting line theory, the matlab program in Chapter 5 is used and a few parameters such as tail area, tail span, and $\Delta\alpha_{o_E}$ are changed. When the program is executed the following tail lift distribution (Figure 12.47) is produced.

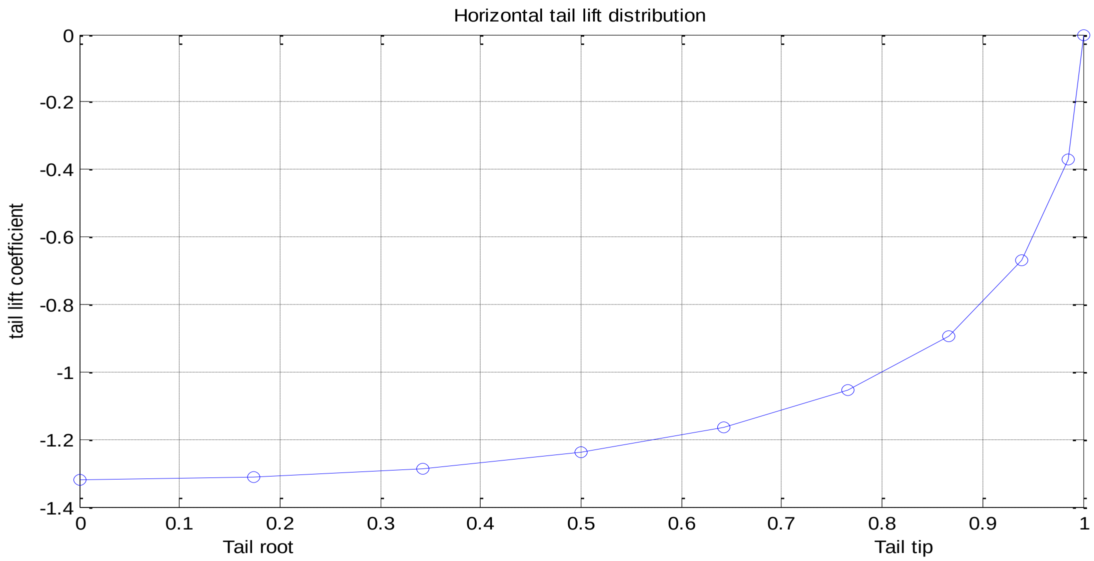


Figure 12.7. Tail lift distribution when elevator is deflected -25 degrees

The corresponding tail lift coefficient is -1.14; which is slightly greater than the desired tail lift coefficient; -0.979. Thus the elevator is acceptable.

Step 15: Checked. OK.

Step 16:
Calculation of elevator effectiveness derivatives ($C_{m_{\delta_E}}, C_{L_{\delta_E}}, C_{L_{h_{\delta E}}}$). For these calculations, the "*most aft*" aircraft center of gravity and the "*most forward*" aircraft center of gravity are considered. For the case of the most aft cg, we have the following:

$$\bar{V}_H = \frac{l_h . S_h}{S\bar{C}} = \frac{(11.3 + 0.5) \times 16}{70 \times 2.96} = 0.912 \quad (6.24)$$

$$C_{m_{\delta_E}} = -C_{L_{\alpha_h}} \eta_h \cdot \bar{V}_H \cdot \frac{b_E}{b_h} \tau_e = -4.3 \times 0.96 \times 0.912 \times 1 \times 0.664 = -2.5 \ \frac{1}{rad} \quad (12.51)$$

$$C_{L_{\delta_E}} = C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} \cdot \frac{b_E}{b_h} \tau_e = -4.3 \times 0.96 \times \frac{16}{70} \times 1 \times 0.664 = -0.626 \ \frac{1}{rad} \quad (12.52)$$

$$C_{L_{h_{\delta E}}} = C_{L_{\alpha_h}} \tau_e = 4.3 \times 0.664 = 2.85 \ \frac{1}{rad} \quad (12.53)$$

The case of the most forward aircraft cg is addressed in Step 16.

Step 17:
Calculation of the elevator deflection (δ_E) required to maintain longitudinal trim at various flight conditions. The elevator deflection when the aircraft cg is located at its most aft position and the aircraft is flying with its maximum speed is calculated as

follows. For this case, the distance between tail ac to aircraft cg is equal to 11.8 m (i.e. 11.3 +0.5).

$$\delta_E = -\frac{\left(\frac{T \cdot z_T}{\bar{q} \cdot S \cdot \bar{C}} + C_{m_o}\right) C_{L_\alpha} + \left(C_{L_1} - C_{L_o}\right) C_{m_\alpha}}{C_{L_\alpha} C_{m_{\delta_E}} - C_{m_\alpha} C_{L_{\delta_E}}} \qquad (12.90)$$

where the aircraft static longitudinal stability derivative (Cm_α) is determined as follows:

$$C_{m_\alpha} = C_{L_{\alpha_{wf}}}\left(h - h_o\right) - C_{L_{\alpha_h}} \eta_h \frac{S_h}{S}\left(\frac{l_h}{\bar{C}}\right)\left(1 - \frac{d\varepsilon}{d\alpha}\right) \qquad (6.67)$$

$$C_{m_\alpha} = 5.7\left(\frac{0.8 - 0.5}{2.96}\right) - 6.3 \times 0.96 \times \frac{16}{70}\left(\frac{11.3 + 0.5}{2.96} - 0.24\right)(1 - 0.454) = -1.479 \quad \frac{1}{rad} \qquad (6.67)$$

Thus:

$$\bar{q} = \frac{1}{2}\rho V^2 = \frac{1}{2} \times 1.225 \times (360 \times 0.514)^2 = 21008 \ \ Pa$$

$$C_{L_1} = \frac{2W}{\rho V_C^2 S} = \frac{2 \times 20{,}000 \times 9.81}{1.225 \times (360 \times 0.5144)^2 \times 70} = 0.133 \qquad (5.1)$$

$$\delta_E = -\frac{\left(\frac{56{,}000 \times (-0.3)}{21{,}008 \times 70 \times \cdot 2.96} + 0.05\right) \times 5.7 + (0.133 - 0.24) \times (-1.479)}{5.7 \times (-2.5) - (-1.479) \times (-0.626)} \qquad (12.90)$$

or

$$\delta_E = +0.029 \ \ rad = +1.637 \ \ \deg$$

Step 18:

Plot the variations of the elevator deflection versus airspeed and also versus altitude. For these calculations, the "*most aft*" aircraft center of gravity and the "*most forward*" aircraft center of gravity are considered. The following matlab program is written to calculate and plot the variations of elevator deflection to maintain longitudinal trim at various flight conditions.

```
clc
clear all

Vmax = 185; % m/s
Sw=70; % m^2
Sh = 16; % m^2
Cbar= 2.96; % m
Vs = 44; %m/sec
Tmax= 56000; %N
rho = 1.225; % kg/m^3
```

```
Cmo = 0.05;
zT = -0.3; %m
CLa = 5.2; %1/rad
CLah = 4.3; % 1/rad
CLa_wf = CLa;
g = 9.81; %m/s^2
m = 20000; % kg
CLo = 0.24;
taw = 0.664;
etha_h = 0.96;
lh = 11.3; % m from main landing gear
de_da = 0.454;
CLdE=-CLah*etha_h*Sh*taw/Sw;

% Most aft cg
xcg = 0.5; % m from main landing gear
h_to_ho = 0.3/Cbar; % m
l_h1 = lh+xcg; %m
VH1 = (l_h1*Sh)/(Sw*Cbar);
CmdE1 = -CLah*etha_h*VH1*taw;
Cma1 = CLa_wf*h_to_ho-CLah*etha_h*Sh*(l_h1/Cbar)*(1-de_da)/Sw;

% Most forward cg
xcg = 1.1; % m from main landing gear
h_to_ho = -0.3/Cbar; % m
l_h2 = lh+xcg; % m
VH2 = (l_h2*Sh)/(Sw*Cbar);
CmdE2 = -CLah*etha_h*VH2*taw;
Cma2 = CLa_wf*h_to_ho-CLah*etha_h*Sh*(l_h2/Cbar)*(1-de_da)/Sw;
i =1;

for U1=Vs:Vmax;
qbar=0.5*rho*U1^2;
CL1= (m*g)/(qbar*Sw);
f1=((Tmax*zT)/(qbar*Sw*Cbar))+Cmo;
dE1(i)=-((f1*CLa)+(CL1-CLo)*Cma1)/(CLa*CmdE1-Cma1*CLdE);
dE2(i)=-((f1*CLa)+(CL1-CLo)*Cma2)/(CLa*CmdE2-Cma2*CLdE);
V(i)=U1;
i=i+1;
end

plot(V/0.5144,dE1*57.3,'o',V/0.5144,dE2*57.3,'*')
grid
xlabel ('Speed (knot)')
ylabel ('\delta_E (deg)')
legend('Most aft cg','Most forward cg')
```

The results are plotted in Figures 12.48 and 12.49. Figure 12.48 shows the variations of elevator deflection with respect to aircraft speed at sea level to maintain longitudinal trim in a cruising flight. However, Figure 12.49 illustrates the variations of elevator deflection with respect to aircraft speed at cruise altitude (i.e. 25,000 ft.). Please note that the above *matlab* program was updated to include the cruise altitude air density in order to produce Figure 12.49. A figure such as Figure 12.48 which demonstrates the variations of elevator deflection with respect to aircraft speed in order to maintain longitudinal trim in a cruising flight is often referred to as *trim curve*.

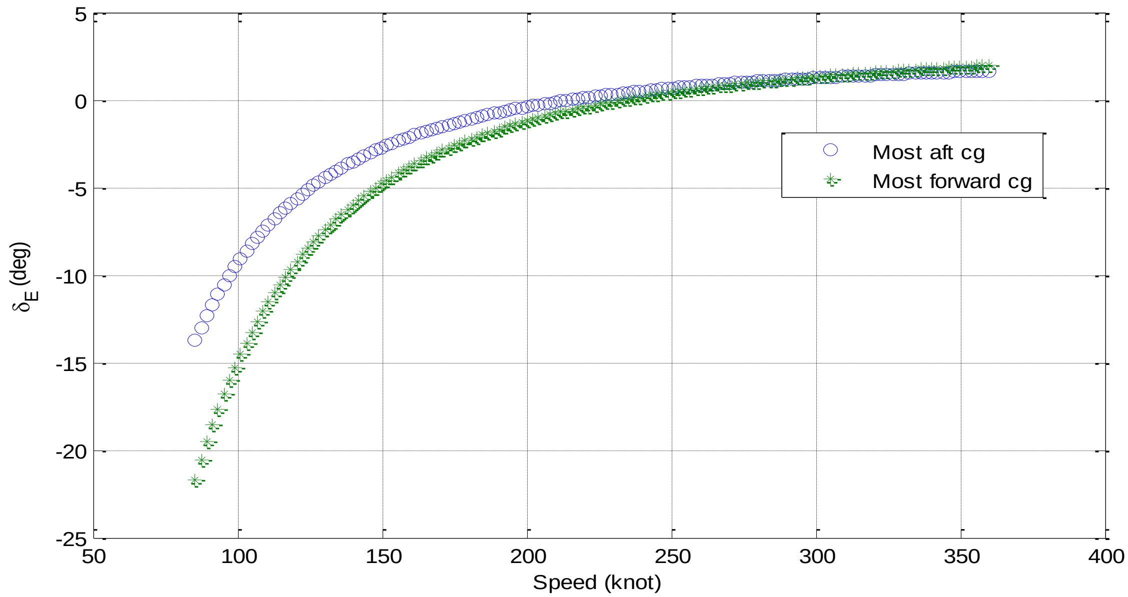


Figure 12.8. Variations of elevator deflection with respect to aircraft speed at sea level

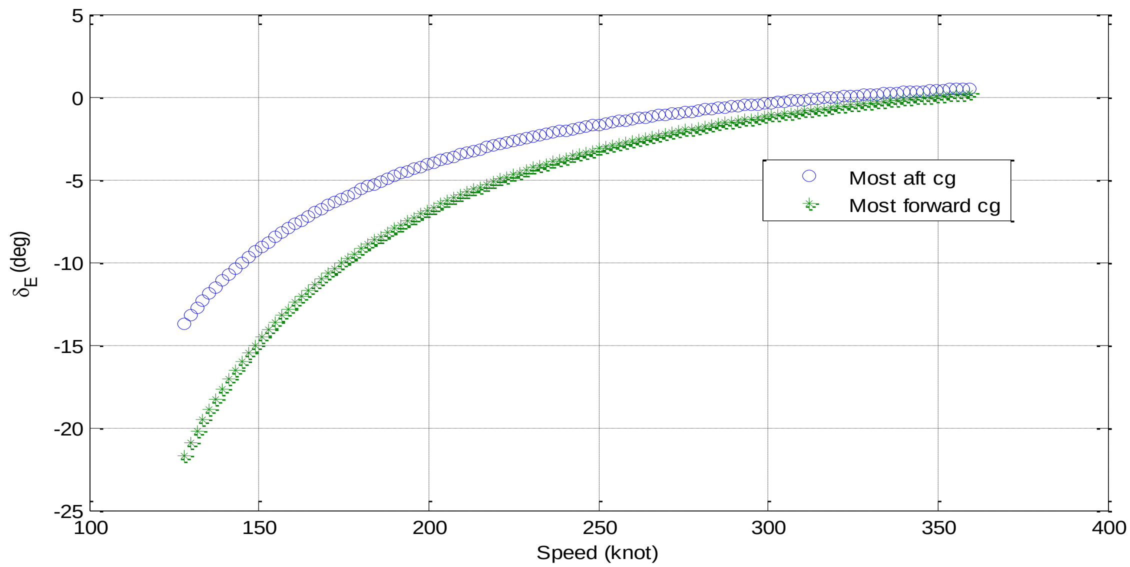


Figure 12.9. Variations of elevator deflection with respect to aircraft speed at cruise altitude

The results of steps 4 through 11 indicate that the maximum negative (up) elevator deflection is -25 degrees; while the results of step 16 (Figures 12.48 and 12.49) demonstrates that the maximum positive (down) elevator deflection is +1.96 degrees. Therefore:

$$\delta_{E_{\max_{up}}} = -25 \text{ deg}$$

$$\delta_{E_{\max_{down}}} = +1.96 \text{ deg}$$

Step 19: Checked

Step 20:
We need to check to make certain that the elevator deflection is not causing the horizontal tail to stall during take-off rotation. It is assumed that the fuselage during take-off rotation is lifted up to 2 degrees below wing stall angle:

$$\alpha_{TO} = \alpha_{s_{TO}} - 2 = 12 - 2 = 10 \text{ deg}$$

Hence, the horizontal tail take-off angle is:

$$\alpha_{h_{TO}} = \alpha_{TO}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + i_h - \varepsilon_o = 10 \times (1 - 0.454) - 1 - 3.636 = 0.828 \text{ deg} \quad (12.91)$$

The tail stall angle of attack during take-off rotation (α_{h_s}) is:

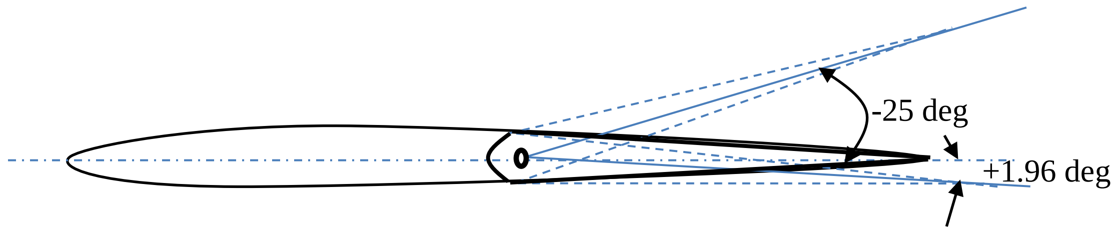
$$\alpha_{h_s} = \pm\left(\alpha_{h_{s_{\delta E=0}}} - \Delta\alpha_{h_E}\right) \quad (12.92)$$

where the $\alpha_{h_{s_{\delta E=0}}}$ is the tail stall angle when elevator is not employed and is given to be 14 degrees. The parameters $\Delta\alpha_{h_E}$ is the magnitude of reduction in tail stall angle of attack due to elevator deflection; and is determined using Table 12.19.

With an elevator deflection of 25 degrees and elevator chord ratio of 0.49; Table 12.19 shows a value of 10.71 degrees the parameter $\Delta\alpha_{h_E}$. Thus:

$$\alpha_{h_s} = \alpha_{h_{s_{\delta E=0}}} - \Delta\alpha_{h_E} = 14 - 10.71 = 3.29 \text{ deg} \quad (12.92)$$

Since the tail angle of attack at the end of rotation (i.e. 0.828 deg) is less than tail stall angle when elevator is deflected (i.e. 3.29 deg), the horizontal tail is not stalling during take-off rotation. Therefore that elevator is acceptable and passed all tests.



Section A-A

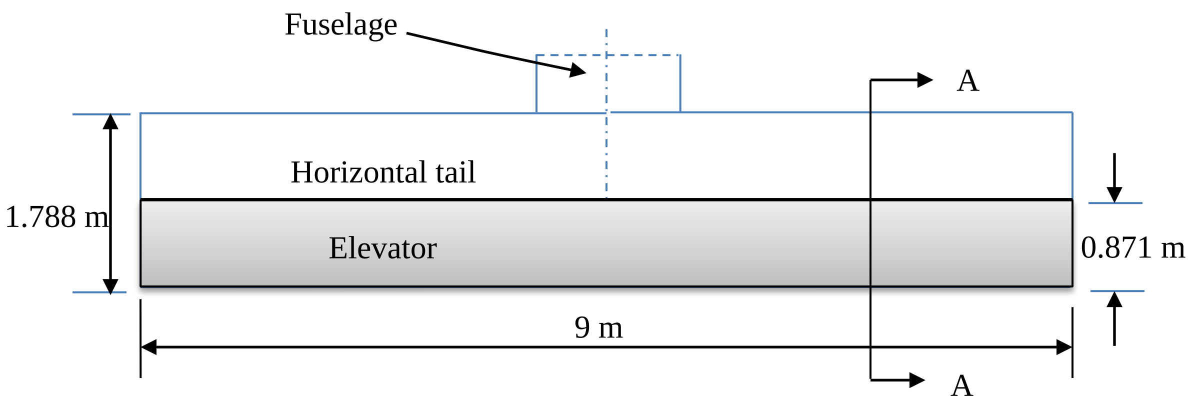


Figure 12.10. Geometry of the elevator of Example 12.5

Steps 21 and 22: Checked. OK.
Steps 23: Aerodynamic balance/mass balance (out of scope of this example)

Steps 24: Optimization (out of scope of this example).

Step 25:
Finally, the elevator geometry is as follows:

$$\frac{b_E}{b_E} = 1 \Rightarrow b_E = b_h = 9 \ \ m$$

$$S_h = b_h \overline{C}_h \Rightarrow \overline{C}_h = \frac{S_h}{b_h} = \frac{16}{9} = 1.788 \ \ m \qquad (6.66)$$

$$\frac{C_E}{C_h} = 0.49 \Rightarrow C_E = 0.49 C_h = 0.49 \times 1.788 = 0.871 \ \ m$$

$$S_E = b_E C_E = 9 \times 0.871 = 7.84 \ \ m^2$$

Figure 12.50 depicts the horizontal tail and elevator geometry.
