**Aileron Design**

# Chapter 12
# Design of Control Surfaces

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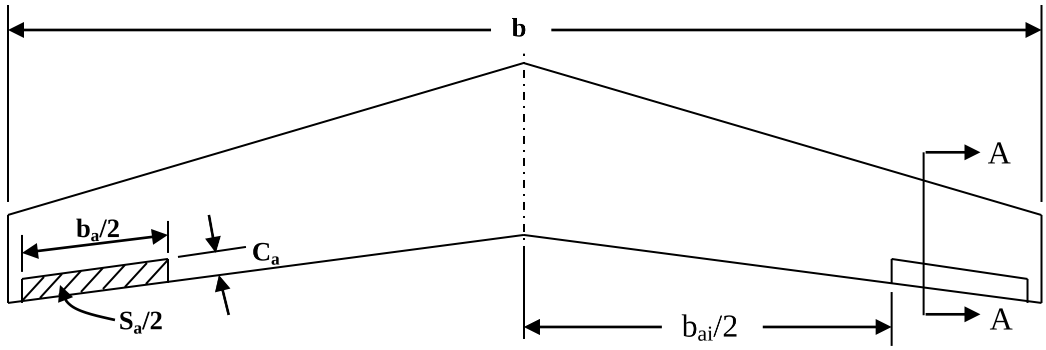
### 12.4.1. Introduction

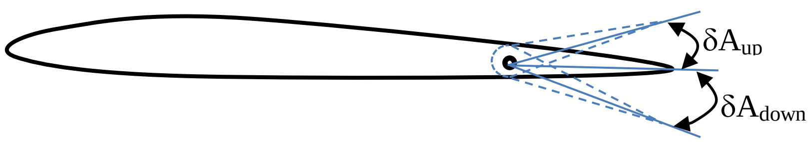
The primary function of an aileron is the lateral (i.e. roll) control of an aircraft; however, it also affects the directional control. Due to this reason, the aileron and the rudder are usually designed concurrently. Lateral control is governed primarily through a roll rate (P). Aileron is structurally part of the wing, and has two pieces; each located on the trailing edge of the outer portion of the wing left and right sections. Both ailerons are often used symmetrically, hence their geometries are identical. Aileron effectiveness is a measure of how good the deflected aileron is producing the desired rolling moment. The generated rolling moment is a function of aileron size, aileron deflection, and its distance from the aircraft fuselage centerline. Unlike rudder and elevator which are displacement control, the aileron is a rate control. Any change in the aileron geometry or deflection will change the roll rate; which subsequently varies constantly the roll angle.

The deflection of any control surface including the aileron involves a hinge moment. The hinge moments are the aerodynamic moments that must be overcome to deflect the control surfaces. The hinge moment governs the magnitude of augmented pilot force required to move the corresponding actuator to deflect the control surface. To minimize the size and thus the cost of the actuation system, the ailerons should be designed so that the control forces are as low as possible.

In the design process of an aileron, four parameters need to be determined. They are: 1. aileron planform area ($S_a$); 2. aileron chord/span ($C_a/b_a$); 3. maximum up and down aileron deflection ($\pm \delta_{Amax}$); and 4. location of inner edge of the aileron along the wing span ($b_{ai}$). Figure 12.10 shows the aileron geometry. As a general guidance, the typical values for these parameters are as follows: $S_a/S$ = 0.05 to 0.1, $b_a/b$ = 0.2-0.3, $C_a/C$ = 0.15-0.25, $ba_i/b$ = 0.6-0.8, and $\delta_{Amax} = \pm 30$ degrees. Based on this statistics, about 5 to 10 percent of the wing area is devoted to the aileron, the aileron-to-wing-chord ratio is about 15 to 25 percent, aileron-to-wing-span ratio is about 20-30 percent, and the inboard aileron span is about 60 to 80 percent of the wing span. Table 12.17 illustrates the characteristics of aileron of several aircraft.




*a. Top-view of the wing and aileron*




*b. Side-view of the wing and aileron (Section AA)*
*Figure 12.1. Geometry of aileron*

Factors affecting the design of the aileron are: 1. the required hinge moment, 2. the aileron effectiveness, 3. aerodynamic and mass balancing, 4. flap geometry, 5. the aircraft structure, and 6. cost. Aileron effectiveness is a measure of how effective the aileron deflection is in producing the desired rolling moment. Aileron effectiveness is a function of its size and its distance to aircraft center of gravity. Hinge moments are also important because they are the aerodynamic moments that must be overcome to rotate the aileron. The hinge moments governs the magnitude of force required of the pilot to move the aileron. Therefore, great care must be used in designing the aileron so that the control forces are within acceptable limits for the pilots. Finally, aerodynamic and mass balancing deals with techniques to vary the hinge moments so that the stick force stays within an acceptable range. Handling qualities discussed in the previous section govern these factors. In this section, principals of aileron design, design procedure, governing equations, constraints, and design steps as well as a fully solved example are presented.

### 12.4.2. Principles of Aileron Design

A basic item in the list of aircraft performance requirements is the maneuverability. Aircraft maneuverability is a function of engine thrust, aircraft mass moment of inertia, and control power. One of the primary control surfaces which cause the aircraft to be steered along its three-dimensional flight path (i.e. maneuver) to its specified destination is aileron. Ailerons are like plain flaps placed at outboard of the trailing edge of the wing. Right aileron and left aileron are deflected differentially and simultaneously to produce a rolling moment about x-axis. Therefore, the main role of aileron is the roll control; however it will affect yaw control as well. Roll control is the fundamental basis for the design of aileron.

| **No** | **Aircraft** | **Type** | **$m_{TO}$ (kg)** | **b (m)** | **$C_A/C$** | **Span ratio** | | **$\delta_{Amax}$ (deg)** | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | **$b_i/b/2$** | **$b_o/b/2$** | **up** | **down** |
| 1 | Cessna 182 | Light GA | 1,406 | 11 | 0.2 | 0.46 | 0.95 | 20 | 14 |
| 2 | Cessna Citation III | Business jet | 9,979 | 16.31 | 0.3 | 0.56 | 0.89 | 12.5 | 12.5 |
| 3 | Air Tractor AT-802 | Agriculture | 7,257 | 18 | 0.36 | 0.4 | 0.95 | 17 | 13 |
| 4 | Gulfstream 200 | Business jet | 16,080 | 17.7 | 0.22 | 0.6 | 0.86 | 15 | 15 |
| 5 | Fokker 100A | Airliner | 44,450 | 28.08 | 0.24 | 0.6 | 0.94 | 25 | 20 |
| 6 | Boeing 777-200 | Airliner | 247,200 | 60.9 | 0.22 | 0.32[1] | 0.76[2] | 30 | 10 |
| 7 | Airbus 340-600 | Airliner | 368,000 | 63.45 | 0.3 | 0.64 | 0.92 | 25 | 20 |
| 8 | Airbus A340-600 | Airliner | 368,000 | 63.45 | 0.25 | 0.67 | 0.92 | 25 | 25 |

*Table 12.1. Characteristics of aileron for several aircraft*

Table 12.12 (lateral directional handling qualities requirements) provides significant criteria to design the aileron. This table specifies required time to bank an aircraft at a specified bank angle. Since the effectiveness of control surfaces are the lowest in the slower speed, the roll control in a take-off or landing operations is the flight phase at which the aileron is sized. Thus, in designing the aileron one must consider only level 1 and most critical phases of flight that is usually phase B.

Based on the Newton's second law for a rotational motion, the summation of all applied moments is equal to the time rate of change of angular momentum. If the mass and the geometry of the objet (i.e. vehicle) are fixed, the law is reduced to a simpler version: The summation of all moments is equal to the mass moment of inertia time of the object about the axis or rotation multiplied by the rate of change of angular velocity. In the case of a rolling motion, the summation of all rolling moments (including the aircraft aerodynamic moment) is equal to the aircraft mass moment of inertia about x-axis multiplied by the time rate of change ($\partial/\partial t$) of roll rate (P).

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Inboard aileron [1]
Outboard aileron [2]

$$\sum L_{cg} = I_{xx}\frac{\partial P}{\partial t} \tag{12.7}$$

or

$$\dot{P} = \frac{\sum L_{cg}}{I_{xx}} \tag{12.8}$$

Generally speaking, there are two forces involved in generating the rolling moment: 1. An incremental change in wing lift due to a change in aileron angle, 2. Aircraft rolling drag force in the yz plane. Figure 12.11 illustrates the front-view of an aircraft where incremental change in the lift due to aileron deflection (ΔL) and incremental drag due to the rolling speed are shown.

The aircraft in Figure 12.11 is planning to have a positive roll, so the right aileron is deflected up and left aileron down (i.e. $+\delta_A$). The total aerodynamic rolling moment in a rolling motion is:

$$\sum M_{cg_x} = 2\Delta L \cdot y_A - \Delta D \cdot y_D \tag{12.9}$$

The factor 2 has been introduced in the moment due to lift to account for both left and right ailerons. The factor 2 is not considered for the rolling moment due to rolling drag calculation, since the average rolling drag will be computed later. The parameter $y_L$ is the average distance between each aileron and the x-axis (i.e. aircraft center of gravity). The parameter $y_D$ is the average distance between rolling drag center and the x-axis (i.e. aircraft center of gravity). A typical location for this distance is about 40% of the wing semispan from root chord.

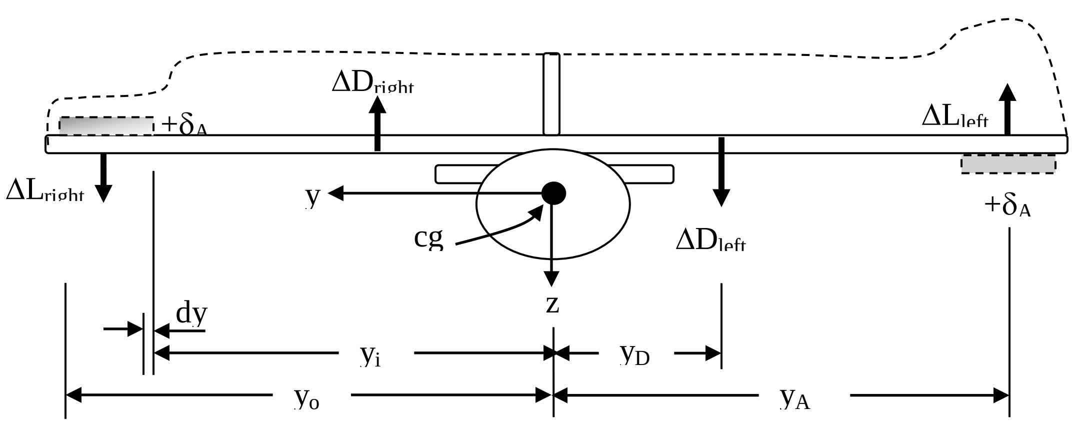



*Figure 12.2. Incremental change in lift and drag in generating a rolling motion*

In an aircraft with short wingspan and large aileron (e.g. fighter such as General Dynamics F-16 Fighting Falcon (Figure 3.12)) the drag does not considerably influence on the rolling speed. However, in an aircraft with a long wingspan and small aileron; such as bomber Boeing B-52 (Figures 8.20 and 9.4); the rolling induced drag force has a significant effect on the rolling speed. For instance, the B-52 takes about 10 seconds to have a bank angle of 45 degrees at low speeds, while for the case of a fighter such as F-16; it takes only a fraction of a second for such roll.

Owing to the fact that ailerons are located at some distance from the center of gravity of the aircraft, incremental lift force generated by ailerons deflected up/down, creates a rolling moment.

$$L_A = 2\Delta L \cdot y_A \tag{12.10}$$

However, the aerodynamic rolling moment is generally modeled as a function of wing area (S), wing span (b), dynamic pressure (q) as:

$$L_A = \bar{q} S C_l b \tag{12.11}$$

where $C_l$ is the rolling moment coefficient and the dynamic pressure is:

$$\bar{q} = \frac{1}{2}\rho V_T^2 \tag{12.12}$$

where $\rho$ is the air density and $V_T$ is the aircraft true airspeed. The parameter $C_l$ is a function of aircraft configuration, sideslip angle, rudder deflection and aileron deflection. In a symmetric aircraft with no sideslip and no rudder deflection, this coefficient is linearly modeled as:

$$C_l = C_{l_{\delta_A}} \delta_A \tag{12.13}$$

The parameter $C_{l_{\delta_A}}$ is referred to as the aircraft rolling moment-coefficient-due-to-aileron-deflection derivative and is also called the aileron roll control power. The aircraft rolling drag induced by the rolling speed may be modeled as:

$$D_R = \Delta D_{left} + \Delta D_{right} = \frac{1}{2}\rho V_R^2 S_{tot} C_{D_R} \tag{12.14}$$

where aircraft average $C_{DR}$ is the aircraft drag coefficient in rolling motion. This coefficient is about 0.7 – 1.2 which includes the drag contribution of the fuselage. The parameter $S_{tot}$ is the summation of wing planform area, horizontal tail planform area, and vertical tail planform area.

$$S_{tot} = S_w + S_{ht} + S_{vt} \tag{12.15}$$

The parameter $V_R$ is the rolling linear speed in a rolling motion and is equal to roll rate (P) multiplied by average distance between rolling drag center (See Figure 12.11) along y-axis and the aircraft center of gravity:

$$V_R = P \cdot y_D \tag{12.16}$$

Since all three lifting surfaces (wing, horizontal tail, and vertical tail) are contributing to the rolling drag, the $y_D$ is in fact, the average of three average distances. The non-dimensional control derivative $C_{l_{\delta_A}}$ is a measure of the roll control power of the aileron; it represents the change in rolling moment per unit change of aileron deflection. The larger the $C_{l_{\delta_A}}$, the more effective the aileron is at creating a rolling moment. This control derivative may be calculated using method introduced in [19]. However, an estimate of the roll control power for an aileron is presented in this Section based on a simple *strip integration method*. The aerodynamic rolling moment due to the lift distribution may be written in coefficient form as:

$$\Delta C_l = \frac{\Delta L_A}{\bar{q}Sb} = \frac{\bar{q}C_{L_A}C_a y_A dy}{\bar{q}Sb} = \frac{C_{L_A}C_a y_A dy}{Sb} \tag{12.17}$$

The section lift coefficient $C_{L_A}$ on the sections containing the aileron may be written as

$$C_{L_A} = C_{L_\alpha}\alpha = C_{L_\alpha}\frac{d\alpha}{d\delta_A}\delta_A = C_{L_\alpha}\tau_a \cdot \delta_A \tag{12.18}$$

where $\tau_a$ is the aileron effectiveness parameter and is obtained from Figure 12.12, given the ratio between aileron-chord and wing-chord. Figure 12.12 is a general representative of the control surface effectiveness; it may be applied to aileron ($\tau_a$), elevator ($\tau_e$), and rudder ($\tau_r$). Thus, in Figure 12.12, the subscript of parameter $\tau$ is dropped to indicate the generality.

Integrating over the region containing the aileron yields

$$C_l = \frac{2C_{L_{\alpha_w}}\tau\delta_A}{Sb}\int_{y_i}^{y_o} Cydy \tag{12.19}$$

where $C_{L_{\alpha_w}}$ has been corrected for three-dimensional flow and the factor 2 is added to account for the two ailerons. For the calculation in this technique, the wing sectional lift curve slope is assumed to be constant over the wing span. Therefore, the aileron sectional lift curve slope is equaled to the wing sectional lift curve slope. The parameter $y_i$ represents the inboard position of aileron with respect to the fuselage centerline, and $y_o$ the outboard position of aileron with respect to the fuselage centerline (See Figure 12.11).

The aileron roll control derivative can be obtained by taking the derivative with respect to $\delta_A$:

$$C_{l_{\delta_A}} = \frac{2C_{L_{\alpha_w}}\tau}{Sb}\int_{y_i}^{y_o} Cy\,dy \tag{12.20}$$

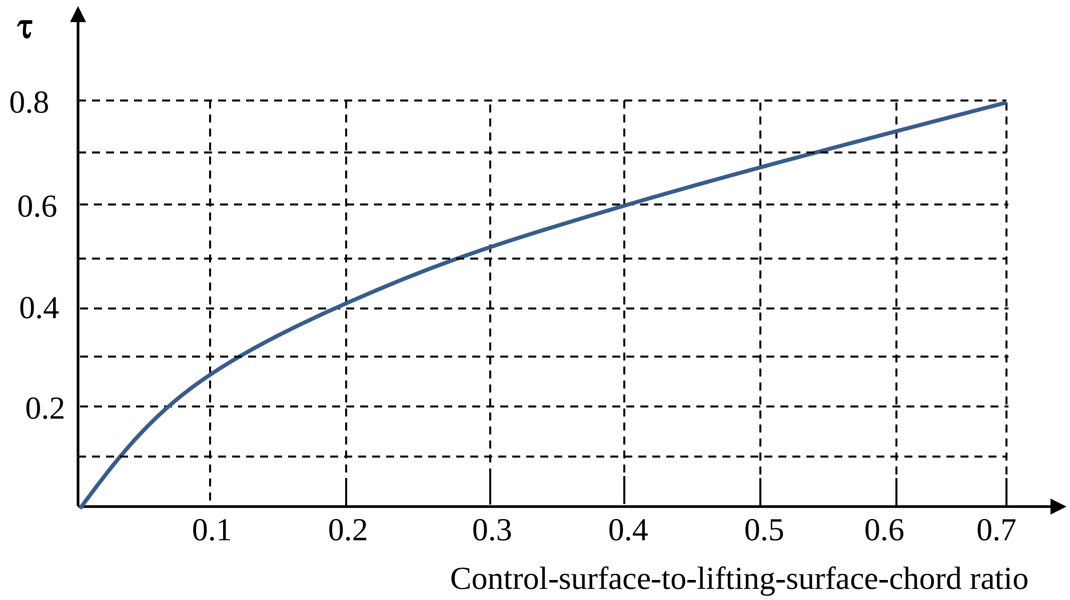



*Figure 12.3. Control surface angle of attack effectiveness parameter*

The wing chord (C) as a function of y (along span) for a tapered wing can be expressed by the following relationship:

$$C = C_r\left[1 + 2\left(\frac{\lambda - 1}{b}\right)y\right] \tag{12.21}$$

where $C_r$ denotes the wing root chord, and $\lambda$ is the wing taper ratio. Substituting this relationship back into the expression for $C_{l_{\delta_A}}$ (Equ. 12.20) yields:

$$C_{l_{\delta_A}} = \frac{2C_{L_{\alpha_w}}\tau}{Sb}\int_{y_i}^{y_o} C_r\left[1 + 2\left(\frac{\lambda - 1}{b}\right)y\right]y\,dy \tag{12.22}$$

or

$$C_{l_{\delta_A}} = \frac{2C_{L_{\alpha_w}}\tau C_r}{Sb}\left[\frac{y^2}{2} + \frac{2}{3}\left(\frac{\lambda - 1}{b}\right)y^3\right]_{y_i}^{y_o} \tag{12.23}$$

This equation can be employed to estimate roll control derivative $C_{l_{\delta_A}}$ using the aileron geometry and estimating $\tau$ from Figure 12.12. Getting back to equation 12.12, there are two pieces of ailerons; each at one left and right sections of the wing. These two pieces may have a similar magnitude of deflections or slightly different deflections, due to the adverse yaw. At any rate, only one value will enter to the calculation of rolling moment. Thus, an average value of aileron deflection will be calculated as follows:

$$\delta_A = \frac{1}{2}\left[\left|\delta_{A_{left}}\right| + \left|\delta_{A_{right}}\right|\right] \tag{12.24}$$

The sign of this $\delta_A$ will later be determined based on the convention introduced earlier; a positive $\delta_A$ will generate a positive rolling moment. Substituting equation 12.9 into equation 12.7 yields:

$$L_A + \Delta D \cdot y_D = I_{xx} \dot{P} \tag{12.25}$$

As the name implies, $\dot{P}$ is the time rate of change of roll rate:

$$\dot{P} = \frac{d}{dt} P \tag{12.26}$$

On the other hand, the angular velocity about x-axis (P) is defined as the time rate of change of bank angle:

$$P = \frac{d}{dt} \Phi \tag{12.27}$$

Combining equations 12.26 and 12.27 and removing dt from both sides, results in:

$$\dot{P}\, d\Phi = PdP \tag{12.28}$$

Assuming that the aircraft is initially at a level cruising flight (i.e. $P_o = 0$, $\phi_o = 0$), both sides may be integrated as:

$$\int_0^{\phi} \dot{P}\, d\Phi = \int_0^{P_{ss}} PdP \tag{12.29}$$

Thus, the bank angle due to a rolling motion is obtained as:

$$\Phi = \int \frac{P}{\dot{P}} dP \tag{12.30}$$

where $\dot{P}$ is obtained from equation 12.25. Thus:

$$\Phi = \int_0^{P_{ss}} \frac{I_{xx} P}{L_A + \Delta D \cdot y_D} dP \tag{12.31}$$

Both aerodynamic rolling moment and aircraft drag due to rolling motion are functions of roll rate. Plugging these two moments into equation 12.31 yields:

$$\Phi_1 = \int_0^{P_{ss}} \frac{I_{xx} P}{\bar{q} S C_l b + \frac{1}{2}\rho (P \cdot y_D)^2 (S_w + S_{ht} + S_{vt}) C_{D_R} \cdot y_D} dP \tag{12.32}$$

The aircraft rate of roll rate response to the aileron deflection has two distinct states: 1. A transient state, 2. A steady state (See Figure 12.13). The integral limit for the roll rate (P) in equation 12.32 is from an initial trim point of no roll rate (i.e. wing level and $P_o = 0$) to a steady-state value of roll rate ($P_{ss}$). Since the aileron is featured as a rate control, the deflection of aileron will eventually result in a steady-state roll rate (Figure 12.13). Thus, unless the ailerons are returned to the initial zero deflection, the aircraft will not stop at a specific bank angle. Table 12.12 defines the roll rate requirements in terms of the desired bank angle ($\Phi_2$) for the duration of t seconds. The equation 12.32 has a closed-form solution and can be solved to determine the bank angle ($\Phi_1$) when the roll rate reaches its steady-state value.

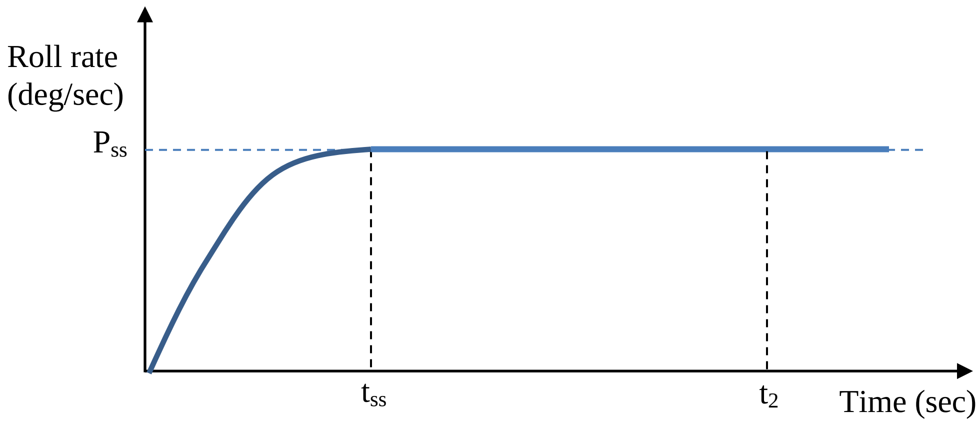



*Figure 12.4. Aircraft roll rate response to an aileron deflection*

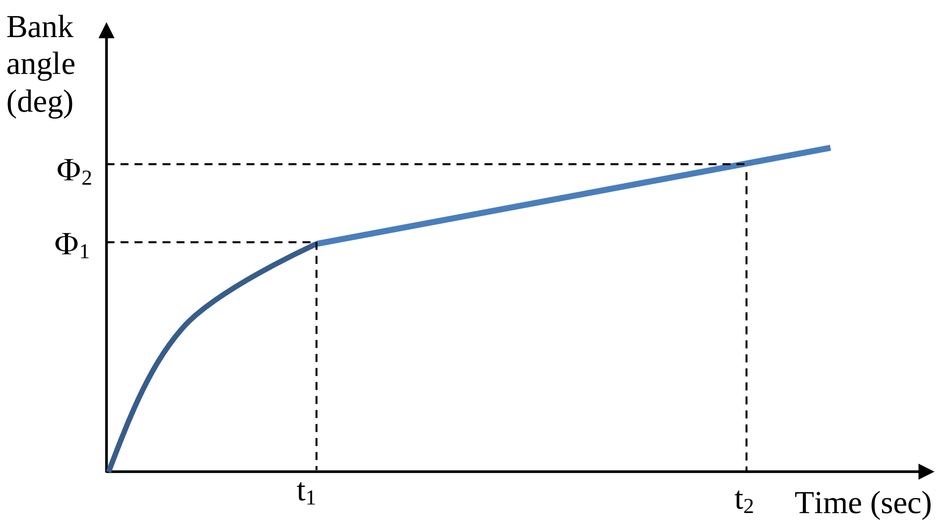



*Figure 12.5. Aircraft bank angle response to an aileron deflection*

When the aircraft has a steady-state ($P_{ss}$) roll rate, the new bank angle (Figure 12.14) after $\Delta$t seconds (i.e. $t_2$-$t_{ss}$) is readily obtained by the following linear relationship:

$$\Phi_2 = P_{ss} \cdot (t_2 - t_{ss}) + \Phi_1 \tag{12.33}$$

Due to the fact that the aircraft drag due to roll rate is not constant and is increased with an increase to the roll rate; the rolling motion is not linear. This implies that the variation of the roll rate is not linear; and there is an angular rotation about x-axis. However, until the resisting moment against the rolling motion is equal to the aileron generated aerodynamic rolling moment; the aircraft will experience an angular acceleration about x-axis. Soon after the two rolling moments are equal, the aircraft will continue to roll with a constant roll rate ($P_{ss}$). The steady-state value for roll rate ($P_{ss}$) is obtained by considering that the fact that when the aircraft is rolling with a constant roll rate, the aileron generated aerodynamic rolling moment is equal to the moment of aircraft drag in the rolling motion.

$$L_A = \Delta D_R \cdot y_D \tag{12.34}$$

Combining equations 12.14, 12.15, and 12.16, the aircraft drag due to the rolling motion is obtained as:

$$D_R = \frac{1}{2}\rho(P \cdot y_D)^2 (S_w + S_{ht} + S_{vt})C_{D_R} \tag{12.35}$$

Inserting the equation 12.35 into equation 12.34 yields:

$$L_A = \frac{1}{2}\rho(P \cdot y_D)^2 (S_w + S_{ht} + S_{vt})C_{D_R} \cdot y_D \tag{12.36}$$

Solving for the steady-state roll rate ($P_{ss}$) results in:

$$P_{ss} = \sqrt{\frac{2 \cdot L_A}{\rho(S_w + S_{ht} + S_{vt})C_{D_R} \cdot y_D^3}} \tag{12.37}$$

On the other hand, the equation 12.32 is simply a definite mathematical integration. This integration may be modeled as the following general integration problem:

$$y = k\int \frac{xdx}{x^2 + a^2} \tag{12.38}$$

According to [20], there is a closed form solution to such integration as follows:

$$y = k\frac{1}{2}\ln\left(x^2 + a^2\right) \tag{12.39}$$

The parameters *k* and *a* are obtained by comparing equation 12.38 with equation 12.32.

$$k = \frac{2I_{xx}}{\rho y_D^3 (S_w + S_{ht} + S_{vt})C_{D_R}} \tag{12.40}$$

$$a^2 = \frac{V^2 S C_l b}{(S_w + S_{ht} + S_{vt})C_{D_R} {y_D}^3} \tag{12.41}$$

Hence, the solution to the integration in equation 12.32 is determined as:

$$\Phi_1 = \left[ \frac{I_{xx}}{\rho y_D^3 (S_w + S_{ht} + S_{vt}) C_{D_R}} \ln\left( P^2 + \frac{V^2 S C_l b}{(S_w + S_{ht} + S_{vt}) C_{D_R} y_D{}^3} \right) \right]_0^{P_{ss}} \tag{12.42}$$

Applying the limits (from 0 to $P_{ss}$) to the solution results in:

$$\Phi_1 = \frac{I_{xx}}{\rho y_D^3 (S_w + S_{ht} + S_{vt}) C_{D_R}} \ln\left( P_{ss}^2 \right) \tag{12.43}$$

Recall that we are looking to determine aileron roll control power. In another word, it is desired to obtain how long it takes ($t_2$) to bank to a desired bank angle when ailerons are deflected. This duration tends to have two parts: 1. The duration ($t_{ss}$) that takes an aircraft to reach the steady-state roll rate ($P_{ss}$), 2. The time ($\Delta t_R$) to roll linearly from $\Phi_{ss}$ to $\Phi_2$ (See Figure 12.14).

$$t_2 = t_{ss} + \Delta t_R \tag{12.44}$$

where

$$\Delta t_R = \frac{\Phi_2 - \Phi_1}{P_{ss}} \tag{12.45}$$

Comparing Figures 12.13 and 12.14 indicates that $t_1 = t_{ss}$. The time ($t_{ss}$) that takes an aircraft to achieve a steady-state roll rate due to an aileron deflection is a function of angular acceleration ($\dot{P}$). Based on classical dynamics, this accelerated roll can be expressed as:

$$\Phi_1 = \Phi_o + \frac{1}{2} \dot{P} t_{ss}^2 \tag{12.46}$$

It is assumed that the aircraft is initially at a wing-level flight condition (i.e. $\Phi_o = 0$). Hence:

$$t_1 = t_{ss} = \sqrt{\frac{2\Phi_1}{\dot{P}}} \tag{12.47}$$

where $\Phi_1$ is determined from equation 12.43. In addition, in an accelerated rolling motion, the relationship between final roll rate ($P_1$) and initial roll rate ($P_o$) is a function of the rate of roll rate ($\dot{P}$) and the final bank angle ($\Phi_1$). Based on classical dynamics, an accelerated rolling motion can be expressed as:

$$P_1^2 - P_o^2 = 2 \dot{P} \Phi_1 \tag{12.48}$$

It is assumed that the aircraft is initially at a wing-level flight condition (i.e. $P_o = 0$) and the new roll rate is the steady-state roll rate (i.e. $P_1 = P_{ss}$). Thus:

$$\dot{P} = \frac{P_{ss}^2}{2\Phi_1} \tag{12.49}$$

where $P_{ss}$ is determined from equation 12.45.

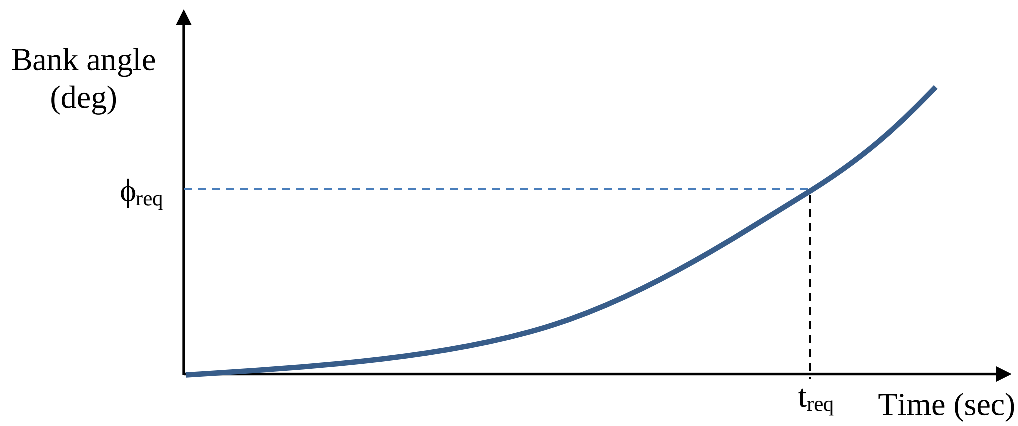



*Figure 12.6. Bank angle versus time*

For a GA and transport aircraft, the time to reach the steady state rolling motion ($t_1$) is long (more than ten seconds). Thus, the application of equations 12.48 and 12.49 is often not needed for aileron design, since the roll requirement is within a few seconds. However, for a fighter aircraft and missile, the rolling motion (See Figure 12.15) is very fast (the time $t_1$ is within a few seconds), so the application of equations 12.48 and 12.49 is usually needed for aileron design. For this reason, when the bank angle ($\Phi_1$) corresponding to steady state roll rate ($P_{ss}$) is beyond 90 degrees, the equation 12.46 serves as the relationship between the required time to reach a desired bank angle. Therefore, the duration required ($t_{req}$) to achieve a desired bank angle ($\Phi_{des}$) will be determined as follows:

$$t_2 = \sqrt{\frac{2\Phi_{des}}{\dot{P}}} \qquad (12.50)$$

The equations and relationships introduced and developed in this section provide the necessary tools to design the aileron to satisfy the roll control requirements. Table 12.12 addresses the military aircraft roll control requirements; for a civil aircraft, it is suggested to adopt a similar list of requirements. To have the greatest roll control by aileron to produce a rolling moment, consider the aileron outboard of the wing toward wing tip. Therefore flap will be considered at the inboard of the wing. This approach will result in the smallest, lightest and most economical aileron surfaces. The aileron design technique and the design procedure will be presented in Section 12.4.4.

### 12.4.3. Aileron Design Constraints

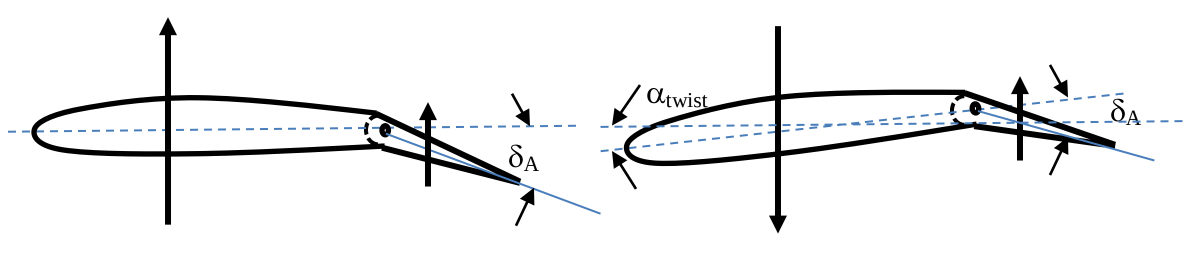
Any design problem in the engineering discipline is usually limited by various constraints and the aileron design is no exception. In this section, a number of constraints on the aileron design will be introduced.

### 12.4.3.1. Aileron Reversal

A number of aircraft when flying near their maximum speed are subject to an important aero-elastic phenomenon. No real structure is ideally rigid; and it has static and dynamic flexibility. Wings are usually produced from aerospace materials such as aluminum and composite materials and have structures which are flexible. This flexibility causes the wing not to be able to maintain its geometry and integrity especially in high speed flight operations. This phenomenon which is referred to as aileron reversal negatively influences the aileron effectiveness.

Consider the right-section of a flexible wing with a downward deflected aileron to create a negative rolling moment. At subsonic speeds, the increment aerodynamic load due to aileron deflection has a centroid somewhere near the middle of the wing chord. At supersonic speeds, the control load acts mainly on the deflected aileron itself, and hence has its centroid even farther to the rear. If this load centroid is behind the elastic axis of the wing structure, then a nose-down twist ($\alpha_{twist}$) of the main wing surface (about y axis) results. The purpose of this deflection was to raise the right-wing section. However, the wing twist reduces the wing angle of attack, and consequently a reduction of the lift on the right-section of the wing (Figure 12.16). In extreme cases, the down-lift due to aero-elastic twist will exceed the commanded up-lift, so the net effect is reversed. This change in the lift direction will consequently generate a positive rolling moment.

This undesired rolling moment implies that the aileron has lost its effectiveness and the roll control derivative; $C_{l_{\delta_A}}$ has changed its sign. Such phenomenon is referred to as aileron reversal. This phenomenon poses a significant constraint on aileron design. In addition, structural design of the wing must examine this aeroelasticity effect of aileron deflection. The aileron reversal often occurs at high speeds. Most high performance aircraft have an aileron reversal speed beyond which the ailerons lose their effectiveness. The F-14 fighter aircraft experiences aileron reversal at high speed.




a. An ideal and desired aileron　　　b. An aileron with aileron reversal

*Figure 12.7. Aileron reversal*

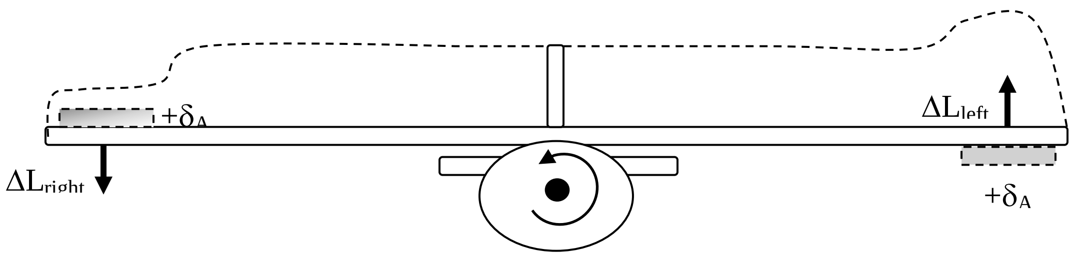
Clearly, such aileron reversal is not acceptable within the flight envelope, and must be considered during design process. A number of solutions for such problem are: 1. Making the wing stiffer, 2. Limit the range of aileron deflections at high speed, 3. Employing two sets of ailerons; one set at inboard wing section for high speed flight, and one set at outboard wing section for high speed flight, 4. Reduce the aileron chord, and 5. Using spoiler for roll control, 6. Moving the ailerons toward wing inboard section. The transport aircraft Boeing 747 has three different types of roll control device: inboard ailerons, outboard ailerons, and spoilers. The outboard ailerons are disabled except in low-speed flights when the flaps are also deflected. Spoilers are essentially flat plates of about 10-15% chord located just ahead of the flaps. When spoilers are raised, they cause a flow separation and local loss of lift. Thus, to avoid roll reversal within the operational flight envelope, the wing structure must be designed with sufficient stiffness.

#### 12.4.3.2. Adverse Yaw

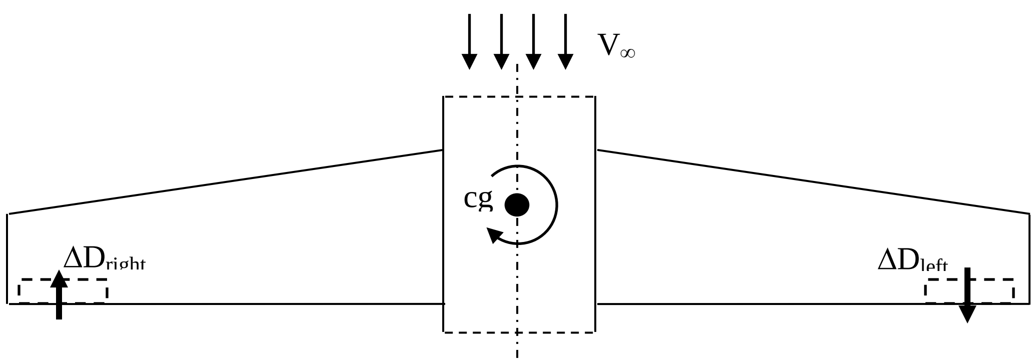
When an airplane is banked to execute a turn, it is desired that aircraft yaws and rolls simultaneously. Furthermore, it is beneficial to have the yawing and rolling moments in the same direction (i.e. both either positive or negative). For instance, when an aircraft is to turn to the right, it should be rolled (about x-axis) clockwise and yawed (about z-axis) clockwise. In such a turn, the pilot will have a happy and comfortable feeling. Such yawing moment is referred to as pro-verse yaw, and such turn is a prerequisite for a coordinated turn. This yaw keeps the aircraft pointing into the relative wind. On the other hand, if the aircraft yaws in a direction opposite to the desired turn direction (i.e. a positive roll, but a negative yaw); pilot will not have a desirable feeling and aircraft turn is not coordinated. This yawing moment is referred to as adverse yaw. When a turn is not coordinated, the aircraft will either slip or skid.

To see why and how these turns may happen, see Figure 12.17 where the pilot is planning to turn to the right. For such a goal, the pilot must apply a positive aileron deflection (i.e. left-aileron down and right-aileron up). The lift distribution over the wing in a cruising flight is symmetric; i.e. the right-wing-section lift and the left-wing-section lift are the same. When the left aileron is deflected down and right aileron is deflected up, the lift distribution varies such that the right-wing-section lift is more than left-wing-section lift. Such deflections create a clockwise rolling moment (Fig. 12.17a) as desired.

However, the aileron deflection simultaneously alters the induced drag of right and left wing differently. Recall that wing drag components of two parts: zero-lift drag ($D_o$) and induced drag ($D_i$). The wing induced drag is a function of wing lift coefficient ( $C_{D_i} = K \cdot C_L^2$). Since the right-wing-section local lift coefficient is higher than the left-wing-section local lift coefficient, the right-wing-section drag is higher than the left-wing-section drag. The drag is an aerodynamic force and has an arm relative to the aircraft center of gravity. The drag direction is rearward, so this wing-drag-couple is generating a negative (See Fig. 12.17b) yawing moment (i.e. adverse yaw). Thus, if the rudder is not deflected simultaneously with aileron deflection, the direction of the aileron-generated rolling moment and the wing-drag generated yawing moment would not be coordinated. Thus, when a pilot deflects a conventional aileron to make a turn, the aircraft will initially yaw in a direction opposite to that expected.




*a. Front view (positive roll)*




*b. Down-view (negative yaw)*
*Figure 12.8. Adverse yaw due to wing drag*

The phenomenon of adverse yaw imposes a constraint on the aileron design. To avoid such an undesirable yawing motion (i.e., adverse yaw), there are a number of solutions; four of which are as follows: 1. Employ a simultaneous aileron-rudder deflections such that to eliminate the adverse yaw. This requires an interconnection between aileron and rudder. 2. Differential ailerons; i.e. up-deflection of the aileron in one side is greater than the down-deflection of the other aileron. This causes equal induced drag in right and left wing during a turn. 3. Employ Frise aileron in which the aileron hinge line is higher than the regular location. 4. Employ spoiler. Both Frise aileron and spoiler are creating a wing drag such the both wing-sections drags are balanced. Most Cessna aircraft use Frise ailerons, but most Piper aircraft employ differentially deflected ailerons. The critical condition for an adverse yaw occurs when the airplane is flying at slow speeds (i.e. high lift coefficient). This phenomenon means that designer must consider the application of one or combination of the above-mentioned techniques to eliminate adverse yaw.

### 12.4.3.3. Flap

The wing trailing edge in a conventional aircraft is the home for two control surfaces; one primary (i.e. aileron), and one secondary (i.e. trailing edge high lift device such as flap). As the aileron and the flap are next to each other along the wing trailing edge, they impose a span limit on one another (Fig. 12.16). The balance between aileron span ($b_a$) and flap span ($b_f$) is a function of the priority of roll control over the take-off/landing performance. To improve the roll control power, the ailerons are to be placed on the outboard and the flap on the inboard part of the wing sections. The application of high lift device applies another constraint on the aileron design which must be dealt with in the aircraft design process.

The spanwise extent of aileron depends on the amount of span required for trailing edge high lift devices. In general, the outer limit of the flap is at the spanwise station where the aileron begins. The exact span needed for ailerons primarily depend of the roll control requirements. A low speed aircraft usually utilizes about 40% of the total wing semispan for ailerons. This means that flaps can start at the side of the fuselage and extend to the 60% semispan station. However, with the application of spoilers, the ailerons are generally reduced in size, and the flaps may extend to about 75% of the wing semispan. Furthermore, if a small inboard aileron is provided for gentle maneuvers, the effective span of the flaps is reduced.

If the take-off/landing performance is of higher importance in the priority list, try to devote a small span to aileron; so that a large span can be occupied by powerful flaps. This in turn means lower stall speed and more safety. On the other hand, if roll control has higher priority than the take-off/landing performance, the ailerons should be designed before the flaps are designed. Due to of the importance of the roll control in a fighter aircraft, span of the flaps must be selected as short as possible, so that the span of the aileron is long enough. Therefore, in a fighter aircraft, it is advised to design the aileron prior to flap design. On the other hand, in the case of civil GA and transport aircraft, it is recommended to design flap first, while in the case of a fighter aircraft, design aileron first.

### 12.4.3.4. Wing Rear Spar

Another aileron design constraint in a conventional aircraft is applied by the wing rear spar. Aileron needs a hinge line to rotate about and to provide the aileron with a sufficient freedom to operate. To have lighter and a less complicated wing structure, it is advised to consider the wing rear spar as the most forward limit for the aileron. This may limit the aileron chord; but at the same time, improves the wing structural integrity. In addition, it is structurally better to have the same chord for aileron and flaps. This selection results in a lighter structure and allows the rear spar to hold both flap and aileron. Therefore the aileron-to-wing attachment through the rear spar (See Fig. 12.18) is considered as both a constraint, and at the same time, an attachment point.

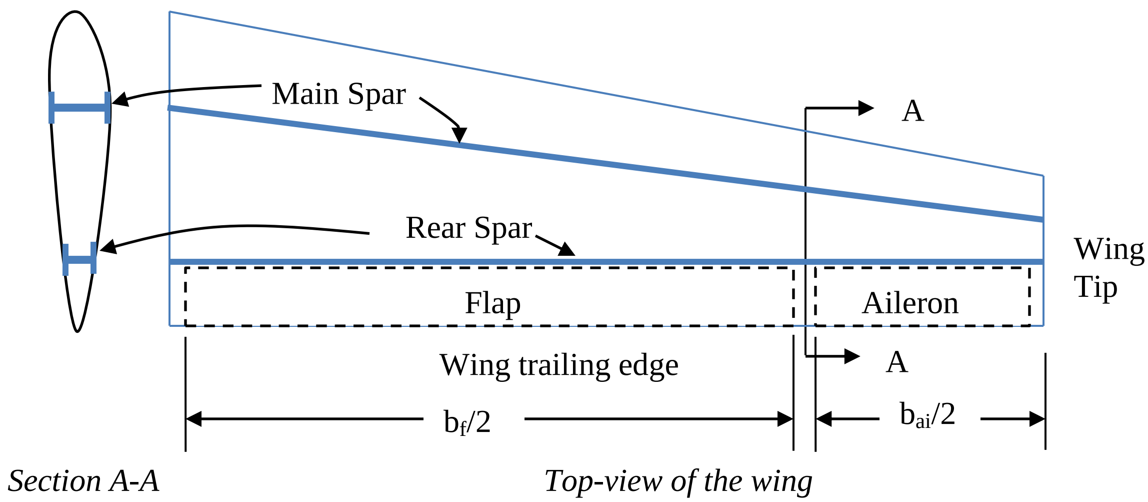



*Figure 12.9. Flap, aileron and rear spar*

**12.4.3.5. Aileron Stall**

When ailerons are deflected more than about 20-25 degrees, flow separation tends to occur. Thus, the ailerons will lose their effectiveness. Furthermore, close to wing stall, even a small downward aileron deflection can produce flow separation and loss of roll control effectiveness. To prevent roll control effectiveness, it is recommended to consider the aileron maximum deflection to be less than 25 degrees (both up and down). Hence, the maximum aileron deflection is dictated by the aileron stall requirement. Table 12.19 provides a technique to determine the stall angle of a lifting surface (e.g. wing) when its control surface (e.g. aileron) is deflected.

**12.4.3.6. Wingtip**

Due to a spanwise component of airflow along the wing span, there is a tendency for the flow to leak around the wing tips. This flow establishes a circulatory motion that trails downstream of the wing. Thus, a trailing vortex is created at each wing tip. To consider effects of vortex flow at the tip of the wing, span of the ailerons must not run toward wingtip. In another word, some distance must exist between outer edge of the aileron and tip of the wing (See Fig 12.16).

### 12.4.4. Steps of Aileron Design

In Sections 12.4.1 through 12.4.3, aileron function, design criteria, parameters, governing rules and equations, formulation, design requirements have been developed and presented. In addition, Section 12.3 introduces the roll control and lateral handling qualities requirements for various aircraft and flight phases. In this section, aileron design procedures in terms of design steps are introduced. It must be noted that there is no unique solution to satisfy the customer requirements in designing an aileron. Several aileron designs may satisfy the roll control requirements, but each will have a unique advantages and disadvantages. Based on the systems engineering approach, the aileron detail design begins with identifying and defining design requirements and ends with optimization. The following is the aileron design steps for a conventional aircraft:

1. Layout design requirements (e.g. cost, control, structure, manufacturability, and operational)
2. Select roll control surface configuration
3. Specify maneuverability and roll control requirements
4. Identify the aircraft class and critical flight phase for roll control
5. Identify the handling qualities design requirements (Section 12.3) from resources such as aviation standards (e.g. Table 12.12). The design requirements primarily include the time ($t_{req}$) that takes an aircraft to roll from an initial bank angle to a specified bank angle. The total desired bank angle is denoted as $\Phi_{des}$.
6. Specify/Select the inboard and outboard positions of the aileron as a function of wing span (i.e. $b_{a_i}/b$ and $b_{a_o}/b$). If the flaps are already designed, identify the outboard position of the flap; then consider the inboard location of the aileron to be next to the outboard position of the flap.
7. Specify/Select the ratio between aileron-chord to the wing-chord (i.e. $C_a/C$). An initial selection for the aileron leading edge may be considered as the next to the wing rear spar.
8. Determine aileron effectiveness parameter ($\tau_a$) from Figure 12.12.
9. Calculate aileron rolling moment coefficient derivative ($C_{l_{\delta_A}}$). You may use references such as [19] or estimate the derivative by employing equation 12.23.
10. Select the maximum aileron deflection ($\delta_{Amax}$). A typical value is about ±25 degrees.
11. Calculate aircraft rolling moment coefficient ($C_l$) when aileron is deflected with the maximum deflection (equation 12.13). Both positive and negative deflections will serve the same.
12. Calculate aircraft rolling moment ($L_A$) when aileron is deflected with the maximum deflection (equation 12.10)
13. Determine the steady-state roll rate ($P_{ss}$) employing equation 12.37.
14. Calculate the bank angle ($\Phi_1$) at which the aircraft achieves the steady state roll rate (equation 12.43)

15. Calculate the aircraft rate of roll rate ($\dot{P}$) that is produced by the aileron rolling moment until the aircraft reaches the steady-state roll rate ($P_{ss}$) by using equation 12.49.
16. If the bank angle ($\Phi_1$) calculated in step 14 is greater than the bank angle ($\Phi_{req}$) of step 5, determine the time (t) that takes the aircraft to achieve the desired bank angle using equation 12.50. The desired bank angle is determined in step 5.
17. If the bank angle ($\Phi_1$) calculated in step 14 is less than the bank angle ($\Phi_{req}$) of step 5, determine the time ($t_2$) that takes the aircraft to reach the desired bank angle ($\Phi_2$ or $\Phi_{req}$) using equations 12.44 and 12.45.
18. Compare the roll time obtained in step 16 or 17 with the required roll time ($t_{req}$) expressed in step 5. In order for the aileron design to be acceptable, the roll time obtained in step 16 or 17 must be equal or slightly longer than the roll time specified in step 5.
19. If the duration obtained in step 16 or 17 is equal longer than the duration ($t_{req}$) stated in step 5, the aileron design requirement has been met and move to step 26.
20. If the duration obtained in step 16 or 17 is shorter than the duration ($t_{req}$) stated in step 5, the aileron design has not met the requirement. The solution is; either to increase the aileron size (aileron span or chord); or to increase the aileron maximum deflection.
21. If the aileron geometry is changed, return to step 7. If the aileron maximum deflection is changed, return to step 10.
22. In case where an increase in the geometry of aileron does not resolve the problem; the entire wing must be redesigned; or the aircraft configuration must be changed.
23. Check aileron stall when deflected with its maximum deflection angle. If aileron stall occurs, the deflection must be reduced.
24. Check the features of adverse yaw. Select a solution to prevent it.
25. Check the aileron reversal at high speed. If it occurs; either redesign the aileron, or reinforce the wing structure.
26. Aerodynamic balance/mass balance if necessary (Section 12.7)
27. Optimize the aileron design
28. Calculate aileron span, chord, area, and draw the final design for the aileron

### 12.8.1. Aileron Design Example

**Example 12.4**

**Problem statement:** Design the roll control surface(s) for a land-based military transport aircraft to meet roll control MIL-STD requirements. The aircraft has a conventional configuration and the following geometry and weight characteristics:

$$m_{TO} = 6{,}500 \text{ kg}, S = 21 \text{ m}^2, AR = 8, \lambda = 0.7, S_h = 5.3 \text{ m}^2, S_v = 4.2 \text{ m}^2, V_s = 80 \text{ knot}, C_{L\alpha w} = 4.5 \text{ 1/rad}, I_{xx} = 28{,}000 \text{ kg.m}^2$$

Furthermore, the control surface must be of low cost and manufacturable. The high lift device has been already designed and the outboard flap location is determined to be at 60% of the wing semispan. The wing rear spar is located at 75% of the wing chord.

**Solution:**

**Step 1:**
The problem statement specified the maneuverability and roll control requirements to comply with military standards.

**Step 2:**
Due to the aircraft configuration, simplicity of design, and a desire for a low cost, a conventional roll control surface configuration (i.e. aileron) is selected.

**Step 3:**
Hence, Table 12.12 will be the reference for the aileron design which expresses the requirement as the time to achieve a specified bank angle change.

**Step 4:**
Based on Table 12.5, a land-based military transport aircraft with a mass of 6,500 kg belongs to Class II. The critical flight phase for a roll control is at the lowest speed. Thus, it is required that the aircraft must be roll controllable at approach flight condition. According to Table 12.6, the approach flight operation is considered as a phase C. To design the aileron, level of acceptability of 1 is considered. Therefore:

| **Class** | **Flight phase** | **Level of acceptability** |
|---|---|---|
| II | C | 1 |

**Step 5:**
The roll control handling qualities design requirement is identified from Table 12.12-b which states that the aircraft in Class II, flight phase C for a level of acceptability of 1 is required to be able to achieve a bank angle of $30^o$ in 1.8 seconds.

**Step 6:**
According to the problem statement, the outboard flap location is at 60% of the wing span. So the inboard and outboard positions of the aileron as a function of wing span (i.e. $b_{a_i}/b$ and $b_{a_o}/b$) are tentatively selected to be at 70% and 95% of the wing span respectively.

**Step 7:**
The wing rear spar is located at 75% of the wing chord, so the ratio between aileron-chord to the wing-chord (i.e. $C_a/C$) is tentatively selected to be 20%.

**Step 8:**

The aileron effectiveness parameter ($\tau_a$) is determined from Figure 12.12. Since the aileron-to-wing chord ratio is 0.2, so the aileron effectiveness parameter will be 0.41.

**Step 9:**
The aileron rolling moment coefficient derivative ($C_{l_{\delta_A}}$) is calculated employing equation 12.22.

$$C_{l_{\delta_A}} = \frac{2C_{L_{\alpha_w}}\tau C_r}{Sb}\left[\frac{y^2}{2}+\frac{2}{3}\left(\frac{\lambda-1}{b}\right)y^3\right]_{y_i}^{y_o} \tag{12.23}$$

We first need to determine wing span, wing mean aerodynamic chord, and wing root chord.

$$AR = \frac{b^2}{S} \Rightarrow b = \sqrt{S \cdot AR} = \sqrt{21\times 10} \Rightarrow b = 14.49 \quad m \tag{5.19}$$

$$AR = \frac{b}{\overline{C}} \Rightarrow \overline{C} = \frac{b}{AR} = \frac{14.49}{10} \Rightarrow \overline{C} = 1.449 \quad m \tag{5.18}$$

$$\overline{C} = \frac{2}{3}C_r\left(\frac{1+\lambda+\lambda^2}{1+\lambda}\right) \Rightarrow 1.449 = \frac{2}{3}C_r\left(\frac{1+0.8+0.8^2}{1+0.8}\right) \Rightarrow C_r = 1.604 \quad m \tag{5.26}$$

The inboard and outboard positions of the aileron as a function of wing span are selected to be at 70% and 90% of the wing span respectively. Therefore:

$$y_i = 0.7\frac{b}{2} = 0.7\times\frac{14.49}{2} = 5.072 \quad m$$

$$y_o = 0.95\frac{b}{2} = 0.95\times\frac{14.49}{2} = 6.883 \quad m$$

Plugging the values for the parameters in equation 12.23 is as follows:

$$C_{l_{\delta_A}} = \frac{2\times 4.5\times 0.41\times 1.604}{21\times 14.49}\left\{\left[\frac{6.883^2}{2}+\frac{2}{3}\left(\frac{0.8-1}{14.49}\right)\times 6.883^3\right]-\left[\frac{5.072^2}{2}+\frac{2}{3}\left(\frac{0.8-1}{14.49}\right)\times 5.072^3\right]\right\}$$

which yields:

$$C_{l_{\delta_A}} = 0.176\frac{1}{rad}$$

**Step 10:**
A maximum aileron deflection ($\delta_{Amax}$) of ±20 degrees is selected.

**Step 11:**
The aircraft rolling moment coefficient ($C_l$) when aileron is deflected with the maximum deflection is:

$$C_l = C_{l_{\delta_A}} \delta_A = 0.176 \times \frac{20}{57.3} = 0.061 \qquad (12.13)$$

**Step 12:**

The aircraft rolling moment ($L_A$) when aileron is deflected with the maximum deflection is calculated. The typical approach velocity is 1.1 to 1.3 times the stall speed, thus the aircraft is considered to approach with a speed of 1.3 $V_s$. In addition, the sea level altitude is considered for the approach flight operation.

$$V_{app} = 1.3V_s = 1.3 \times 80 = 104 \quad knot = 53.5 \quad \frac{m}{s}$$

$$L_A = \frac{1}{2}\rho V_{app}^2 S C_l b = \frac{1}{2} \times 1.225 \times 53.5^2 \times 21 \times 0.061 \times 14.49 = 32{,}692.6 \quad Nm \qquad (12.11)$$

**Step 13:**

The steady-state roll rate ($P_{ss}$) is determined.

$$P_{ss} = \sqrt{\frac{2 \cdot L_A}{\rho(S_w + S_{ht} + S_{vt})C_{D_R} \cdot y_D^3}} \qquad (12.37)$$

An average value of 0.9 is selected for wing-horizontal tail-vertical tail rolling drag coefficient is selected. The drag moment arm is assumed to at 405 of the wing span, so:

$$y_D = 0.4\frac{b}{2} = 0.4 \times \frac{14.49}{2} = 2.898 \quad \frac{m}{s}$$

$$P_{ss} = \sqrt{\frac{32{,}692.6}{1.225(21 + 5.3 + 4.2) \times 0.9 \cdot (2.898)^3}} = 8.937\frac{rad}{\sec} \qquad (12.37)$$

**Step 14:**

Calculate the bank angle ($\Phi_1$) at which the aircraft achieves the steady state roll rate:

$$\Phi_1 = \frac{I_{xx}}{\rho y_D^3 (S_w + S_{ht} + S_{vt})C_{D_R}} \ln\left(P_{ss}^2\right) = \frac{28{,}000}{1.225 \times (2.898)^3 (21 + 5.3 + 4.2) \times 0.9} \ln\left(8.937^2\right) \qquad (12.43)$$

$$\Phi_1 = 149.82 \quad rad = 8584.14 \quad \deg$$

**Step 15:**

Calculate the aircraft rate of roll rate ( $\dot{P}$ ) that is produced by the aileron rolling moment until the aircraft reaches the steady-state roll rate ($P_{ss}$).

$$\dot{P} = \frac{P_{ss}^2}{2\Phi_1} = \frac{8.937^2}{2 \times 149.82} = 0.267 \quad \frac{rad}{\sec^2} \qquad (12.49)$$

**Step 16 and 17:**

The bank angle ($\Phi_1$) calculated in step 14 is compared with the bank angle ($\Phi_{req}$) of step 5. Since the bank angle calculated in step 14 (i.e. 8,584 degrees) is shorter than the bank angle ($\Phi_{req}$) of step 5 (i.e. 30 degrees), the time that takes the aircraft to achieve the bank angle of 30 degrees is determined.

$$t_2 = \sqrt{\frac{2\Phi_{des}}{\dot{P}}} = \sqrt{\frac{2\times 30}{0.267}} = 1.982 \quad \text{sec} \tag{12.47}$$

**Step 18:**

The roll time obtained in step 16 or 17 is compared with the required roll time ($t_{req}$) expressed in step 5. The roll time obtained in step 16 or 17 to achieve the bank angle of 30 degrees (i.e. 1.982 sec) is longer than the roll time expressed in step 5 (i.e. 1.8 sec). Hence the current aileron design does not satisfy the requirements and must be redesigned.

**Step 19 and 20:**

The duration obtained in step 16 or 17 is shorter than the duration ($t_{req}$) stated in step 5, so the aileron design has not met the requirement. The solution is; either to increase the aileron size (aileron span or chord); or to increase the aileron maximum deflection. Due to the aileron stall concern, and the location of rear spar, the maximum aileron deflection and the aileron-chord-to-wing-chord ratio are not altered. The flap outboard location is located at 60% of the wing span, thus the safest solution is to increase the aileron span.

**Step 21:**

By trial and error, it is determined that the aileron inboard span at 61% of the wing span will satisfy the roll control requirements. The calculation is as follows:

$$y_i = 0.61\frac{b}{2} = 0.61\times\frac{14.49}{2} = 4.42 \quad m$$

$$y_o = 0.95\frac{b}{2} = 0.95\times\frac{14.49}{2} = 6.883 \quad m$$

$$C_{l_{\delta_A}} = \frac{2\times 4.5\times 0.41\times 1.604}{21\times 14.49}\left\{\left[\frac{6.883^2}{2} + \frac{2}{3}\left(\frac{0.8-1}{14.49}\right)\times 6.883^3\right] - \left[\frac{4.42^2}{2} + \frac{2}{3}\left(\frac{0.8-1}{14.49}\right)\times 4.42^3\right]\right\}$$

$$C_{l_{\delta_A}} = 0.228\frac{1}{rad}$$

$$C_l = C_{l_{\delta_A}}\delta_A = 0.228\times\frac{20}{57.3} = 0.08 \tag{12.13}$$

$$L_A = \frac{1}{2}\rho V_{app}^2 S C_l b = \frac{1}{2}\times 1.225\times 53.5^2 \times 21\times 0.08\times 14.49 = 42{,}429.6 \quad Nm \tag{12.11}$$

$$P_{ss} = \sqrt{\frac{2\cdot L_A}{\rho\left(S_w + S_{ht} + S_{vt}\right)C_{D_R}\cdot y_D^3}} \tag{12.37}$$

$$P_{ss} = \sqrt{\frac{42{,}429.6}{1.225(21 + 5.3 + 4.2) \times 0.9 \cdot (2.898)^3}} = 10.181 \frac{rad}{\sec} \qquad (12.37)$$

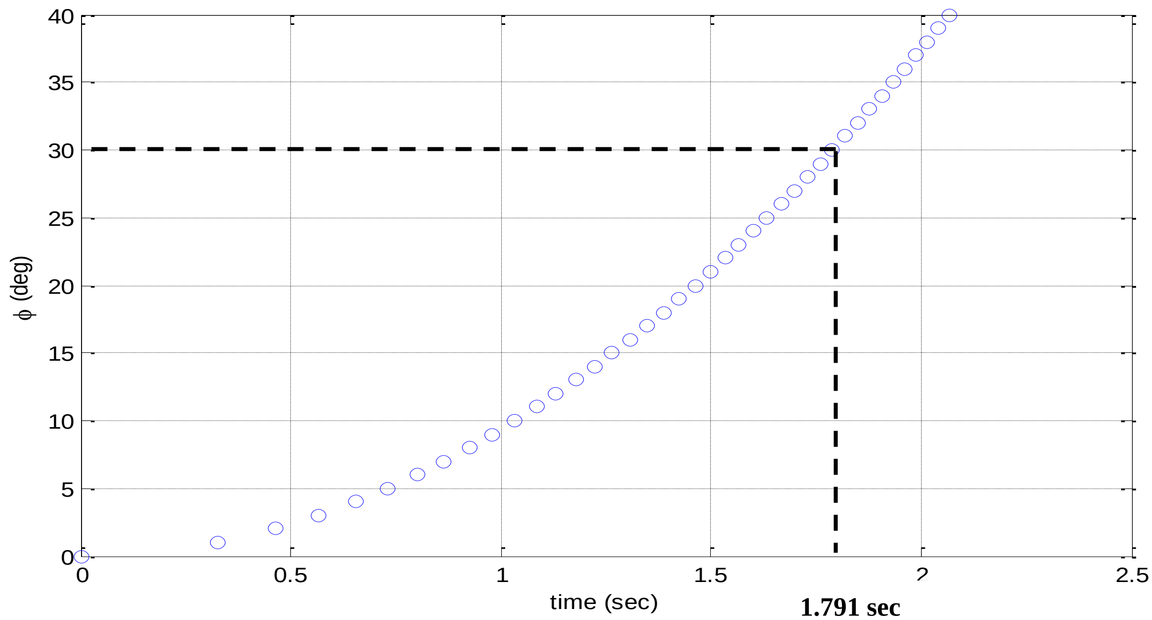



*Figure 12.10. Variations of bank angle versus time for aircraft in Example 12.4*

$$\Phi_1 = \frac{I_{xx}}{\rho y_D^3 (S_w + S_{ht} + S_{vt}) C_{D_R}} \ln(P_{ss}^2) = \frac{28{,}000}{1.225 \times (2.898)^3 (21 + 5.3 + 4.2) \times 0.9} \ln(10.181^2)$$

$$\Phi_1 = 158.74 \quad rad = 9095 \quad \deg$$

$$\dot{P} = \frac{P_{ss}^2}{2\Phi_1} = \frac{10.181^2}{2 \times 158.74} = 0.327 \quad \frac{rad}{\sec^2} \qquad (12.49)$$

$$t_2 = \sqrt{\frac{2\Phi_{des}}{\dot{P}}} = \sqrt{\frac{2 \times 30}{0.327}} = 1.791 \quad \sec \qquad (12.50)$$

The variations of bank angle versus time are plotted in Figure 12.44.

**Steps 26:** Aerodynamic balance/mass balance (out of scope of this example)

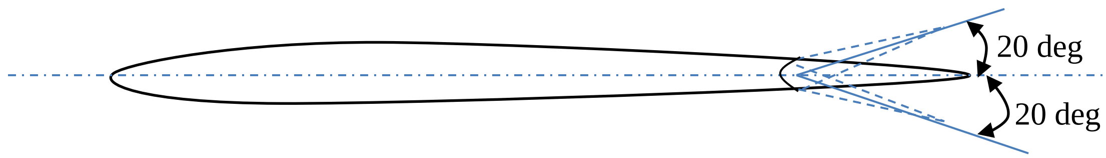
**Steps 27:** Optimization (out of scope of this example)

**Steps 28:** Geometry

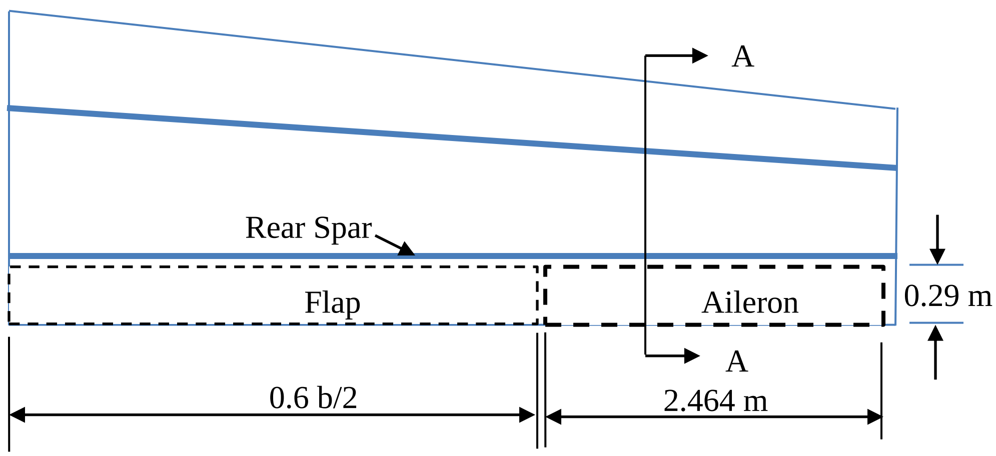
The geometry of each aileron is as follows:

$$b_A = y_{oA} - y_{iA} = 6.883 - 4.42 = 2.464 \quad m$$

$$C_A = 0.2 C_w = 0.2 \times 1.449 = 0.29 \quad cm$$




*Section A-A*




*Top-view of the right wing section*

*Figure 12.11. Wing and aileron of Example 12.4*

The overall planform area of both left and right ailerons is:

$$A_A = 2b_A C_A = 2 \times 2.464 \times 0.29 = 1.428 \quad m^2 \tag{12.43}$$

The top and side views of the right wing section including the aileron are shown in Figure 12.45. It must be noted that the aileron chord was assumed to be 20% of the wing chord throughout the aileron span. However, the wing is tapered, so in fact, the aileron needs to be tapered too. To make the application and the fabrication of the aileron easier, a constant chord aileron is selected; hence, a change in the aileron chord must be applied in the later design stage to correct for the assumption.