Optimization and control theory applications in space missions

Rafael Vázquez

Dpto. de Ingeniería Aeroespacial & Instituto de Matemáticas de la Universidad de Sevilla (IMUS)


thanks to Federico Perea (UPV), Emilio Carrizosa (US), Felipe Martín Crespo (Taitus), Christophe Louembet (Univ. Paul Sabatier, Toulouse), Manuel Sanjurjo (Univ. Carlos III Madrid)

Space Symposium,
Sevilla, March 9, 2018
Aerospace Engineering at University of Seville

• Aerospace Engineering started in 2002, in the Univ. Of Seville School of Engineering (strong tradition in Mechanical, Electrical and Chemical Engineering since the 60s)

• Teaches Undergraduate Degree, Master’s Degree and also PhD

• Mainly focused in aeronautics (a traditional industry in Seville) but there are courses in Orbital Mechanics, Spacecraft Dynamics and Spacecraft Propulsion

• The Department of Aerospace Engineering was created in 2006

• Some lines of research related to space have emerged, in collaboration with other departments

Rafael Vázquez - rvazquez1@us.es
Past and Ongoing Projects

• Three main areas:
  • Planning and scheduling for space missions (R. Vazquez, J. Galan, JM Melendez and others). Problems related to optimization. Collaboration between engineers, mathematicians and industry (Taitus software).
  • Inertia-free and adaptive attitude control laws (S. Esteban, M. Camblor in collaboration with U. Michigan). Analysis and synthesis of automatic attitude control laws that do not require knowledge of spacecraft mass distribution.

• All these lines have already produced journal and conference papers.

Rafael Vázquez - rvazquez1@us.es
Past and Ongoing projects

• Besides these, other seminal lines yet to produce publications:
  • Application of statistical tools to study uncertainty in orbital mechanics problems, such as orbit decay (R. Vazquez, FJ Camacho)
  • Analysis of groundtrak self-intersections for geocentric satellites (R. Vazquez, E. Teson, J. Galan)
  • Applications of the three-body problem dynamics to mission design (J. Galan, R. Vazquez)
Optimization in Space

- Project 1: Design and integration of a real time optimal planner for swath acquisitions for multiple EOS satellites.


- Project 2: Minimize the number of cancellations of satellite-antenna requests due to conflicts in real time.

Project 1: swath acquisitions to cover an area

How do we select the images?
How close is a feasible solution to the optimal one?
Mathematical Modeling I:

- Fixed Sensors

Satellite Sensors

Landsat-7 ETM+
Mathematical Modeling I:

Let $\mathcal{R}$ be a region, $n$ acquisitions $a_i$ of cost $c_i$.
Solve the Integer Linear Programming (ILP) problem where $x_i$ is 1 if the acquisition $a_i$ is used and 0 if not used.

$$\min \sum_{i=1}^{n} c_i x_i \quad \text{cost function}$$

s.t. $\bigcup_{i:x_i=1} a_i \supseteq \mathcal{R} \quad \text{geometrical condition}$

$$x_i \in \{0, 1\}, \quad \forall \ i = 1, 2, \ldots, n.$$ 

Note that the problem may be infeasible.
Translating geometry into equations

Find all the intersection between the acquisitions in $\mathcal{R}$. Let us define an $m \times n$ swath-subregion matrix $Q$ whose entry $q_{ij}$ takes the value 1 if subregion $SR_j$ is covered by acquisition $a_i$, and 0 otherwise.

The ILP is now:

\[
\min \sum_{i=1}^{n} c_i x_i \\
\text{s.t.} \sum_{i=1}^{n} x_i q_{ij} \geq 1, \quad \forall j = 1, \ldots, m \\
x_i \in \{0, 1\}, \quad \forall i = 1, 2, \ldots, n,
\]

The solution is the optimal covering of region $\mathcal{R}$.
Mathematical Modeling II: toy example

$R$
Mathematical Modeling II: toy example
Mathematical Modeling II: toy example

\[ R \]

\[ a_1 \]

\[ a_2 \]
Mathematical Modeling II: toy example

\[ R \]

\( a_1 \)

\( a_2 \)

\( a_3 \)
Mathematical Modeling II: toy example

\[ R \]

\( a_1 \)

\( a_2 \)

\( a_3 \)

\( a_4 \)
Human operator in the loop solution
\[ Q = \begin{pmatrix}
  0 & 1 & 0 & 0 \\
  0 & 1 & 0 & 1 \\
  0 & 1 & 1 & 1 \\
  0 & 1 & 1 & 0 \\
  0 & 1 & 0 & 0 \\
  1 & 1 & 0 & 0 \\
  1 & 1 & 0 & 1 \\
  1 & 1 & 1 & 1 \\
  1 & 1 & 0 & 1 \\
  1 & 1 & 0 & 0 \\
  1 & 0 & 0 & 0 \\
  1 & 0 & 0 & 1 \\
  1 & 0 & 1 & 1 \\
  1 & 0 & 0 & 1 \\
  1 & 0 & 0 & 0 \\
  1 & 0 & 1 & 0 \\
  0 & 0 & 1 & 1 \\
  0 & 0 & 0 & 1 
\end{pmatrix} \]
Optimal solution

Given the region $\mathcal{R}$ and the set of acquisitions $\{a_1, a_2, a_3, a_4\}$

the optimal solutions is $\{1, 1, 0, 1\} = \{a_1, a_2, a_4\}$
Generalizations and Limitations

- It is straightforward to include multiple modes for each satellite (additional restrictions).
- Optimization in time.
- Optimization in number and cost of images.

For planning purposes we need real time solutions.

- The bottleneck of the present calculations is the computation of the $Q$ matrix.
- We have implemented greedy, GRASP (Greedy randomized Adaptive Search Procedure) and two steps optimizations algorithms.
More complicated regions and acquisitions: Extremadura, Andalucia
Project 2: Satellite Range Problem

Ksat Global Network
Some orders of magnitude

- # antennas $\sim$ 50 (9 ground stations, pole to pole)
- # satellites $\sim$ 80.
- # revisits $\rightarrow$ up to 14 a day (polar orbiting satellites).
- Optimization horizon $\rightarrow$ one week $\rightarrow$ 3000 p/w ($\sim$ 45%).
- Typical pass duration $\rightarrow$ 10-20 minutes.
- operating restrictions (bands, up/down links, priorities)
- KSAT already had a deconflicting procedure (KNOS+WM).
KSAT Facility at Tromso, Norway
Some definitions

Keywords: timeline, passes, conflicts, ASAP=SRS, deconflicting tool, real time.
Deconflicting I: moving passes

If pass 1 is also compatible with antenna 2 → move the pass!
Deconflicting II: **multiple moving passes**

moving may involve more than one pass → domino effect.
Deconflicting III: shortening of passes

If pass 1 is shortable → allocated time slot > minimum duration
Deconflicting III: shortening of passes

If pass 2 is shortable → allocated time slot > minimum duration
Deconflicting IV: moving and shortening of passes

If pass 2 is movable and pass 3 shortable
Can we build an automatic Deconflicting tool?

- Clearly define the hierarchy of allowed deconflicting mechanisms.
- Define a decision variable (if possible binary).
- Define a cost function.
- Implement an optimization procedure.
- Test against experimental data.
- Use the tool to deconflict passes and (hopefully) make design decisions.
Mathematical modeling of deconflicting

compute the time intervals along the time line
for each pass and its alternatives generate all possible compatible subpasses in all the alternative antennas.

KEY INGREDIENT
Mathematical modeling of deconflicting: solutions

- Assign a **binary** value for each pass
- Add **feasibility** conditions
- Define an appropriate **linear cost function**
- Optimize (ILP).
Mathematical modeling: equations

Define the binary variable $y_{ik}$ that is 1 if pass $P_i$ is assigned to antenna $A_k$ and 0 otherwise.

Constraints:

1. Every pass has to be assigned at most to one antenna.

$$ \sum_{k \in C_i} y_{ik} \leq 1, \forall i \in F. $$

2. For a given antenna $A_k$ and a time interval $I_{jk}$ available for passes, there should be no conflict among the passes.

$$ \sum_{i \in F: j \in S_{ik}, k \in C_i} y_{ik} \leq 1, \forall k, j : \bigcup_{i \in F} S_{ik} \neq \emptyset. $$

Maximize the linear cost function:

$$ J = \sum_{i \in F} \sum_{k \in C_i} (p^* - p_{ik} + 1) \xi_{ik} y_{ik}, $$

where $p^* = \max_i p_i$ and $\xi_{ik}$ is a weighting function.
Does it work?

<table>
<thead>
<tr>
<th>Passes</th>
<th>Ant.</th>
<th>Sat.</th>
<th>Shortable passes (%)</th>
<th>Conflicts</th>
<th>Cancell.</th>
<th>Short.</th>
<th>Move. total (other site)</th>
<th>Vars.</th>
<th>Constr.</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3356</td>
<td>22</td>
<td>49</td>
<td>22</td>
<td>537</td>
<td>116</td>
<td>1</td>
<td>839 (13)</td>
<td>8090</td>
<td>9322</td>
<td>64</td>
</tr>
<tr>
<td>3006</td>
<td>22</td>
<td>47</td>
<td>22</td>
<td>196</td>
<td>75</td>
<td>3</td>
<td>703 (5)</td>
<td>11219</td>
<td>12157</td>
<td>61</td>
</tr>
<tr>
<td>3356</td>
<td>22</td>
<td>49</td>
<td>22</td>
<td>231</td>
<td>94</td>
<td>0</td>
<td>517 (6)</td>
<td>12465</td>
<td>12245</td>
<td>73</td>
</tr>
<tr>
<td>3566</td>
<td>22</td>
<td>50</td>
<td>22</td>
<td>306</td>
<td>114</td>
<td>1</td>
<td>561 (11)</td>
<td>12360</td>
<td>12714</td>
<td>73</td>
</tr>
<tr>
<td>3470</td>
<td>22</td>
<td>52</td>
<td>22</td>
<td>253</td>
<td>100</td>
<td>0</td>
<td>557 (17)</td>
<td>12016</td>
<td>12788</td>
<td>80</td>
</tr>
<tr>
<td>3408</td>
<td>22</td>
<td>52</td>
<td>22</td>
<td>196</td>
<td>91</td>
<td>0</td>
<td>478 (5)</td>
<td>12289</td>
<td>12025</td>
<td>71</td>
</tr>
<tr>
<td>1573</td>
<td>14</td>
<td>47</td>
<td>24</td>
<td>68</td>
<td>0</td>
<td>0</td>
<td>487 (7)</td>
<td>7143</td>
<td>4467</td>
<td>64</td>
</tr>
<tr>
<td>384</td>
<td>20</td>
<td>46</td>
<td>22</td>
<td>33</td>
<td>21</td>
<td>0</td>
<td>101 (0)</td>
<td>1247</td>
<td>1075</td>
<td>22</td>
</tr>
<tr>
<td>1586</td>
<td>16</td>
<td>45</td>
<td>24</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>479 (2)</td>
<td>7892</td>
<td>4964</td>
<td>63</td>
</tr>
<tr>
<td>2287</td>
<td>18</td>
<td>55</td>
<td>26</td>
<td>1053</td>
<td>250</td>
<td>4</td>
<td>312 (35)</td>
<td>6820</td>
<td>8380</td>
<td>77</td>
</tr>
</tbody>
</table>

- Implemented in C++ within a Savoir tool
- The **bottleneck** is not the dimension or the ILP solver (LPSOLVE) but the preprocessing and the **complexity** of the conflicts (Gurobi).
- We perform in real time at least as good (slightly better) as the trained deconflicting department.
Other projects... (a bit more academic)
Planning of acquisitions for agile satellites in snapshot mode.

- Agile satellites: allow rotation in all directions to acquire images
- Snapshot mode: individual images for selected coordinates
Planning of acquisitions for agile satellites in snapshot mode.
Analysis of groundtrack self-intersections for geocentric satellites

- Satellites with repeating groundtrack may or may not have points of self-intersection depending on orbital elements.
- Self-intersection are of interest because they represent points of opportunity for repeated communication and/or observation.
Analysis of groundtrack self-intersections for geocentric satellites

- In the circular case, the altitude (which determine repeating properties) and inclination are the key values.
- In the circular case, we have developed algorithms that give the exact number of self-intersection and their locations. These algorithms may help in orbit design.
Now some control theory: Rendezvous and formation flying of space vehicles

![3D chaser path](image)

- LOS constraints
- Safe zone
- Docking
- Starting point
- Forbidden area

- $\delta_1$: $\bar{\delta} = \begin{bmatrix} 0.2592 & 0.8065 & -0.0533 \end{bmatrix} \times 10^{-4}$
- $\delta_\theta$: $\bar{\delta}_i = 0.0436$, $\sqrt{\sum_{i=1}^{Np}} = 0.0436$.

Controller parameters are:
- $N_p = 60$
- $\gamma = 1000$
- $k_a = 60$
- $p = 0.95$
- $\lambda = 0.23$. 

Note: The image shows a 3D path followed by the chaser spacecraft with constraints and zones labeled accordingly.
Examples: Space Shuttle

- Different phases according to proximity.
- Close approach is manual.
Examples: Soyuz

- Russian approach: Kurs system.
- Used for MIR.
- Automatic system, does not require target actions (but requires system installed on-board target).
- Weight of 85 kg., 270 Watts of consumption (both on target and chaser).

Fig. 6  Phasing and rendezvous sequence for Soyuz/Progress vehicles.

Altitude

Downrange

Launch

Start Fly-Around

1 km

2 km

4 km

Weight of 85 kg., 270 Watts of consumption (both on target and chaser).

Fig. 7  Final approach sequence for Soyuz/Progress vehicles.
Examples: ATV

- ATV (Automated Transfer Vehicle) from ESA carries supplies to ISS and at the end of its lifetime is ejected elevating the orbit (and carrying waste away).
- Uses relative GPS and follows a pre-programmed maneuver.
- Additional pre-programmed maneuvers for all possible scenarios.
- 5 ATV vehicles had been launched (Jules Verne, Johannes Kepler, Edoardo Amaldi, Albert Einstein, Georges Lemaître).
PROBA-3 MISSION (ESA)

PRECISE FORMATION FLYING
- The relative lateral and longitudinal positions are controlled
- The absolute attitude is controlled
- The «line of sight» of the formation is controlled
- A virtual large and solid structure is built and oriented

MANEUVERS OR FUTURE ASTRONOMY MISSIONS:
- Formation re-size
- Formation re-targeting
- Combination of Station keeping, Re-size and re-targeting

Target vector oriented towards sun
Required Position control
Lateral: 5 mm (3σ @ 150 m ISD)
Longitudinal: 1.5 mm (3σ @ 150 m ISD)
The problem of designing an automatic rendezvous system of low weight and consumption is still open and an active field of research.

This would be specially attractive for small satellites that could rendezvous with larger spacecraft, such as space stations.
Rendezvous and formation flying of space vehicles

• This was one of the lines of research in F. Gavilan PhD Thesis (2012).
• We have considered two main problems:
  • Safe & optimal rendezvous subject to model uncertainties
  • Fast algorithms for rendezvous with on-off thrusters
• The problem can be modeled by linear equations when vehicles in close proximity but is time-varying (Tschauner-Hempel model)
• On-off thrusters make the problem nonlinear
Safe & optimal rendezvous subject to model uncertainties

- Problem classically solved assuming everything is known.
- If there are uncertainties, to get a robust controller, one has to solve a min-max problem: minimize fuel consumption and verify system constraints even for worst possible uncertainties.
- Uncertainties have to be estimated on-line.
- Use of Model Predictive Control.

Figure 6.2: Robust MPC with thruster disturbance ($\delta_1 \sim N(5 \cdot 10^{-5}, 0.5 \cdot 10^{-5})$, $\delta_\theta \sim N(0.0436, 0.0436)$). Controller parameters are set to: $N_p = 60$, $\gamma = 1000$, $k_a = 60$, $p = 0.95$, $\lambda = 0.23$. Mission cost $0.4382$ m/s.$^2$
Model Predictive Control

Errors minimized over a finite horizon

Constraints taken into account

Only the first control move is applied

Model of process used for predicting
Model Predictive Control

Only the first control move is applied again

Rafael Vázquez - rvazquez1@us.es
MPC vs PID

MPC vs. PID

PID:  \( u(t) = u(t-1) + g_0 e(t) + g_1 e(t-1) + g_2 e(t-2) \)
Rendezvous with on-off thrusters

• Development of an algorithm (PWM-MPC) which consider realistic thrusters or any other type of time-varying actuation (e.g. Rotating spacecraft)

• Two main ingredients:
  • Hotstart for optimization computed from impulsive actuation
  • Improvement through successive linearizations until convergence or computation time is up.
Rendezvous with on-off thrusters

Figure 4: System trajectories in the target orbital plane: open-loop PWM inputs computed from impulsive solution (dashed), closed-loop Model Predictive Control with PWM inputs using impulsive model (dot-dashed), and closed-loop Model Predictive Control with PWM inputs using the PWM planning algorithm (solid).

where

\[ R_k = h\left(\kappa_k \alpha\right)I_6 \times 6 \]  

(87)

where, \( h \) is the step function and \( \kappa_k \alpha \) is the desired arrival time for rendezvous.

Notice that the dimensions of each block matrices are shown for disambiguation. The reason for choosing (87) is that it is desired to arrive at the origin at time \( \kappa_k \alpha \) (and remain there) and at the same time minimize the control effort.

In the sequel \( \kappa_k \alpha = N_p \), so the spacecraft are expected to rendezvous after 50 minutes.

In the simulations three algorithms were considered: first, an impulsive open-loop trajectory planner, as described in Section 4.1. Next, closed-loop simulations using MPC but considering impulsive instead of PWM actuation in the model (this algorithm is denoted as impulsive MPC). Finally, closed-loop simulations using PWM actuation in the model (this algorithm is denoted as PWM MPC).

The diagram shows the trajectories of the spacecraft in the target orbital plane. The trajectories are color-coded as follows:

- Starting Point
- PWM-MPC
- Impulsive-MPC
- Open loop
- LOS constraint

The trajectories are labeled with markers indicating important points such as the starting point and constraint violation. The graph also highlights areas labeled as 'Safe area' and 'Restricted area' to illustrate the feasible and restricted regions of the spacecraft's trajectory.
Rendezvous of spacecraft

• Other prospective lines:
  • Rendezvous for a small chaser spacecraft with only 1 or 2 thrusters, and attitude control (J Sanchez Master’s Thesis). Challenging if the attitude actuator saturates. Mixes relative dynamics and attitude dynamics.
  • Rendezvous with an asteroid. Considers a triaxial asteroid, major axis spinner. To avoid impact, a tangent plane (rotating with the asteroid) can be used to establish a constraint. (JM Montilla & J Sanchez)
Rendezvous with spacecraft in NRHO (Deep Space Gateway)
EXPANDING HUMAN PRESENCE IN PARTNERSHIP
CREATING ECONOMIC OPPORTUNITIES, ADVANCING TECHNOLOGIES, AND ENABLING DISCOVERY

Now
Using the International Space Station

2020s
Operating in the Lunar Vicinity (proving ground)

Phase 0
Continue research and testing on ISS to solve exploration challenges. Evaluate potential for lunar resources. Develop standards.

Phase 1

Phase 2
Complete Deep Space Transport and conduct yearlong Mars simulation mission.

Phase 3 and 4
Begin sustained crew expeditions to Martiaина system and surface of Mars.

After 2030
Leaving the Earth-Moon System and Reaching Mars Orbit
Conclusions from these projects

- The problems are complex and challenging and there exists a **nonzero overlap** between the academic and the industry interests.

- Academia needs industry to know what problems are of practical interest and get out of the “ivory tower”...

- Industry can tap into the knowledge of academia and learn about other perspectives and ideas with minimal costs.
Thanks!

Questions?