Space Surveillance and Awareness at University of Sevilla / GIA (Grupo de Ingenieria Aeroespacial)

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Director of Indra Space Surveillance Chair

Aerospace Engineering at University of Seville

University of Seville: One of Spain oldest (>500 years) and largest (3rd after UNED and Complutense) with more than 70K students and 4K professors covering all areas of knowledge.

- ETSI (Escuela Tecnica Superior de Ingenieria) created in the 60s, strong tradition in physical/mathematical background and in technology transfer with industry through AICIA (Asociacion de Investigación y Cooperación Industrial de Andalucía)

- Aerospace Engineering started in 2002 (2nd oldest in Spain after UPM)

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Aerospace Engineering at University of Seville

- Teaches Undergraduate Degree (120, specialized in: Aerospace Vehicles, Aerial Navigation and Airports), Master's Degree (80) and also PhD, high mark required to enter -> high-level students

- Mainly focused in aeronautics (a traditional industry in Seville) but interest in Space growing and expected to grow even more (Spanish Space Agency created in Seville!), with a specialized Master in Operations of Space Systems soon to be created

- The Department of Aerospace Engineering was created in 2006 of which GIA (Grupo de Ingenieria Aeroespacial) is part of. We teach Orbital Mechanics (Basic and Applied), Spacecraft Attitude Dynamics, Spacecraft Systems.

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Space Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

Three main topics of research:

- **Space Surveillance and Awareness (SSA)**
- **Guidance, Navigation and Control (GNC) for spacecraft**
- **Optimization applied to EOSs (Earth Observation Satellites)**

Team: small group of 5 professors + 1 PhDs + several MSc students **fully devoted to space activities** (with the collaboration of the rest of the GIA group, and a professor from Applied Math, as well as several national/international collaborations)

Rafael Vazquez: full professor of orbital mechanics. Background in control, applied mathematics.

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SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

In this seminar:

- **Overview of our recent activities in maneuver detection (JM Montilla PhD).**
- In more detail: Single track IOD for low Earth orbit with approximated J2 dynamics (JM Montilla, Jan Siminski, R. Vazquez).
- Overview of Space Occupancy Filter (A. S. Rivero PhD)
- In more detail: Space Ocuppancy Path Conjunction Filter (A. S. Rivero, C. Bombardelli, G. Bau, R. Vazquez)
- Overview of other work

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Manoeuvre Detection:

- Started as CDTI-funded project in collaboration with ESA and Indra (2020-2021)
- Using S3TSR (radar operated by Indra in the Morón Air Force Base for the Spanish space tracking network S3T) to analyse the behaviour of LEO (Low Earth Orbit) satellites.
- GOAL: detect whether spacecraft have manoeuvred or not using past ephemerides and ONLY S3TSR data
- Main difficulty: as spacecraft may take quite long (24-72 hours) to be observed twice with the radar, sometimes long propagations are required -> uncertainty can explode.

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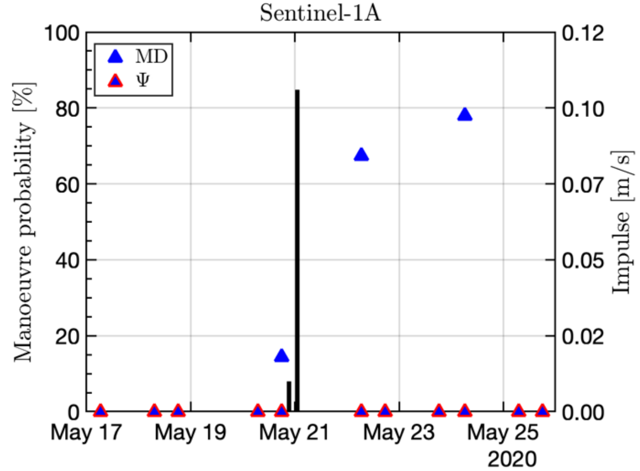
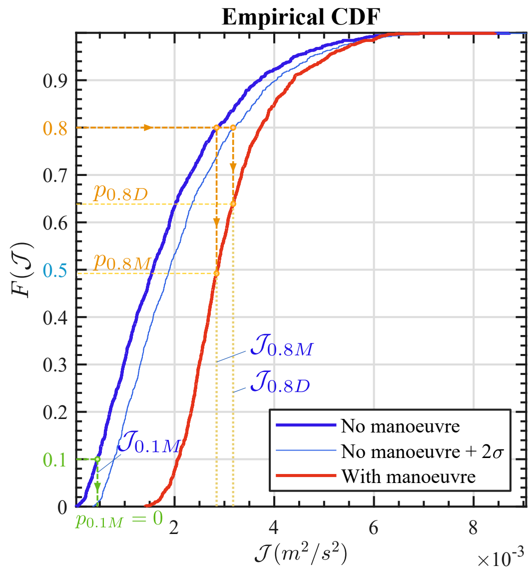
Manoeuvre Detection (INDRA PROJECT)

- Use of Mahalanobis distance, optimal control distance, and manoeuvre detection filter based on UKF.
- The obtained results were good in simulation, but the real data results are in need of improvement.
- The main identified difficulty was the scarcity of measurements (low number of tracks and/or low number of plots) which was to be expected as we had a single data source.
- Future expansion of the capabilities of S3TSR may improve the quality of the metrics, as well as considering other sources of data (laser).





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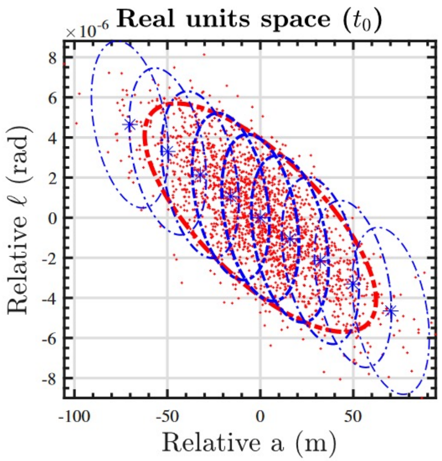
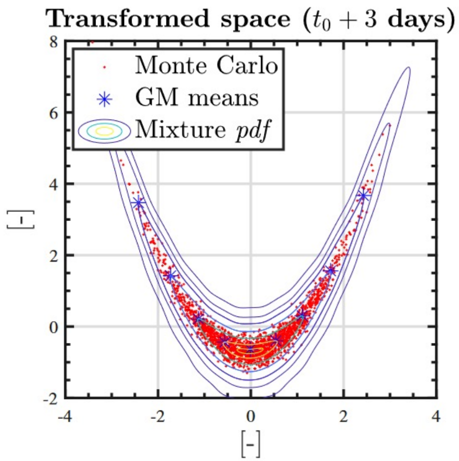
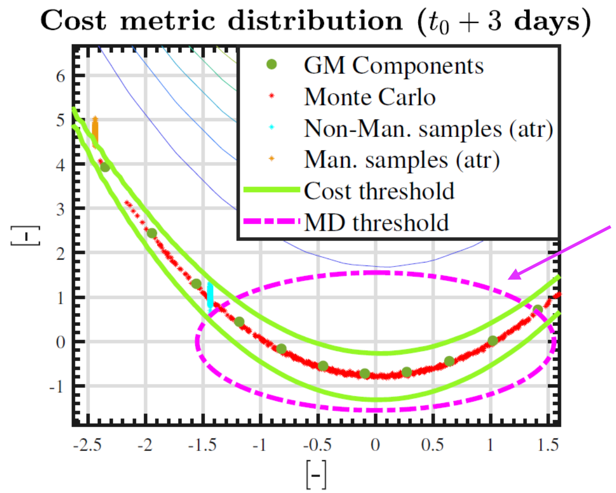
Manoeuvre Detection (JM Montilla PhD)

- Collaboration at Polimi (Pierluigi de Lizia): use of **Gaussian Mixtures** to improve covariance realism. Projecting prediction on measurement space and compare with **attributable** of radar track: **cost metric**.







More realistic than MD

$$\mathcal{PE} = \int p_1(\boldsymbol{\mu})p_2(\boldsymbol{\mu})d\boldsymbol{\mu} = \sum_{i=1}^{N} \omega_i \mathcal{N}(\hat{\boldsymbol{\mu}}_i; \boldsymbol{\mu}_{\mathcal{A}}, \hat{\Sigma}_i + \Sigma_{\mathcal{A}})$$

$$c = -\ln \mathcal{PE}$$

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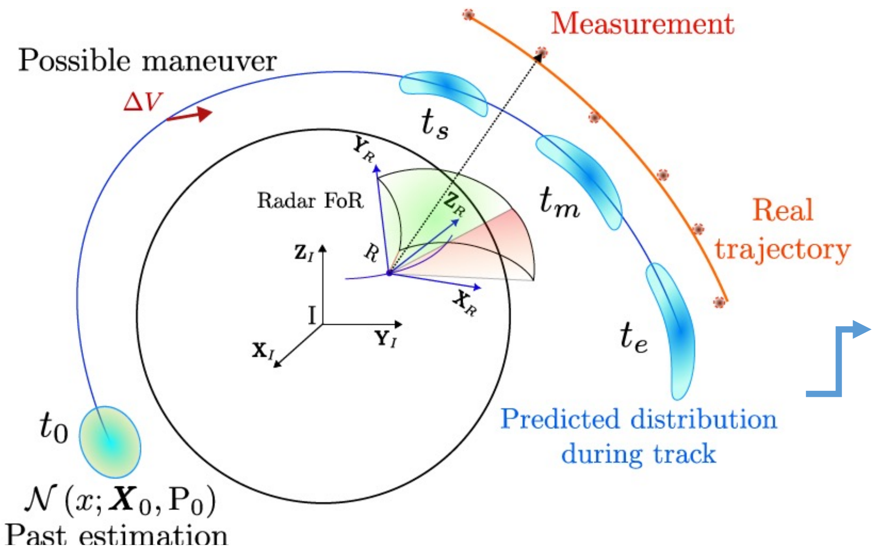
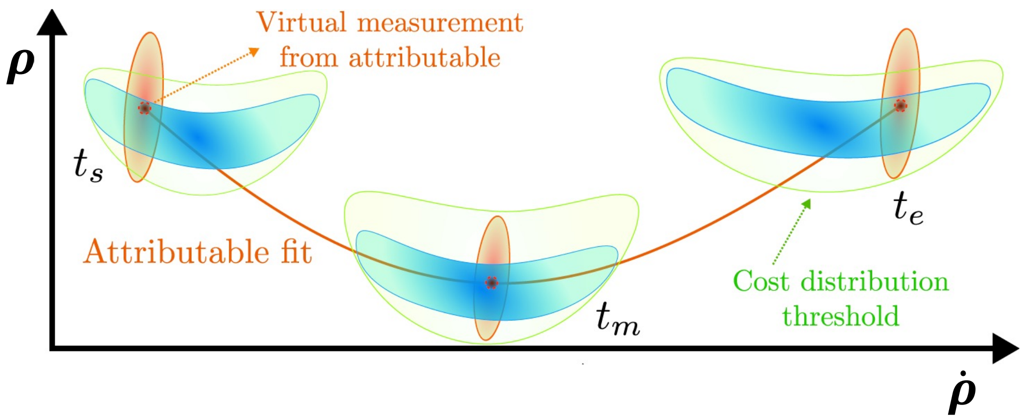
Manoeuvre Detection (JM Montilla PhD)

- Collaboration at Polimi (Pierluigi de Lizia): the **evolution of the cost** metric is analysed to overcome dimensionality loss.





$$\Delta c_{rel}(t_a, t_b) = \frac{c(t_b)}{c_{gm}(t_b)} - \frac{c(t_a)}{c_{gm}(t_a)}$$

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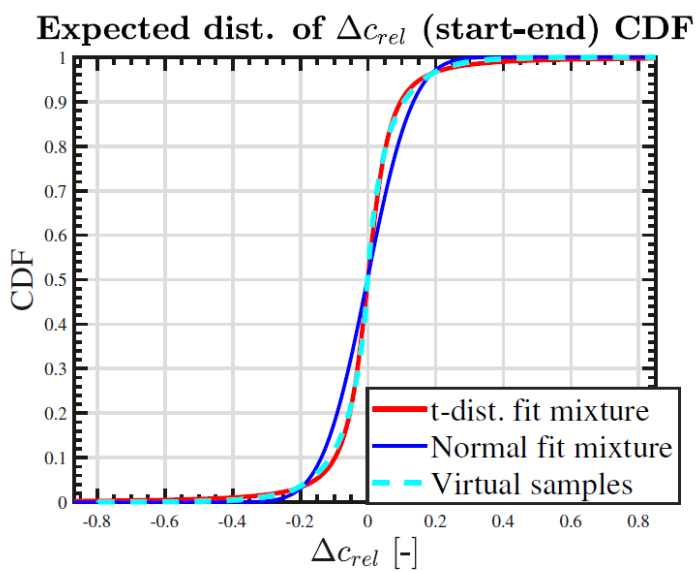

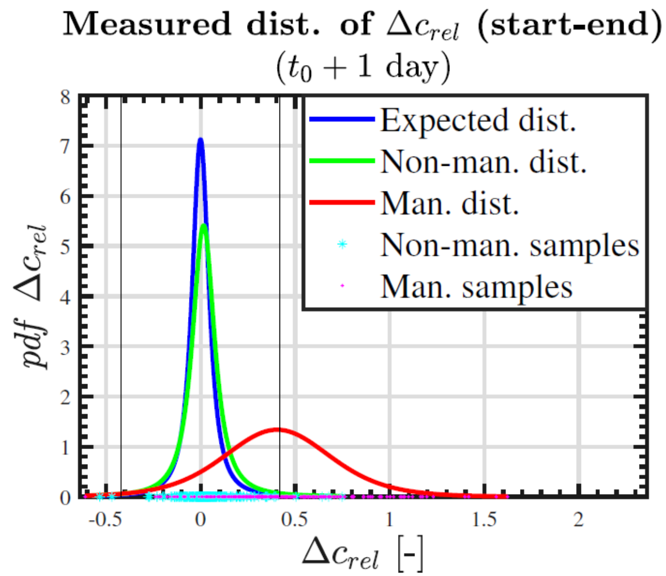

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Manoeuvre Detection (JM Montilla PhD)

- Collaboration at Polimi (Pierluigi de Lizia): High-fidelity synthetic **simulations** of Sentinel-1A like orbit

Combining cost and ΔC

[%]	$MD_{0.7}$	$c_{0.7}$	$\Delta c^{se}_{0.98}$	$\Delta c^{sm}_{0.98}$	$\Delta c^{me}_{0.98}$	$\Delta c^{all}_{0.98}$	$c + \Delta c$
TRUE POSITIVES	65.72	60.4	53.68	46.31	48.39	60.38	65.94
FALSE POSITIVES	20.42	1.5	0.73	1.02	0.69	1.65	2.79

[%]	$c_{0.4}$	$c_{0.45}$	$c_{0.5}$	$c_{0.55}$	$c_{0.6}$	$c_{0.65}$	$c_{0.7}$
TRUE POSITIVES	70.32	68.33	66.45	64.81	63.21	61.76	60.4
FALSE POSITIVES	8.64	6.65	4.93	3.967	2.84	2.05	1.5

Cost threshold change gets **worse FP rate**

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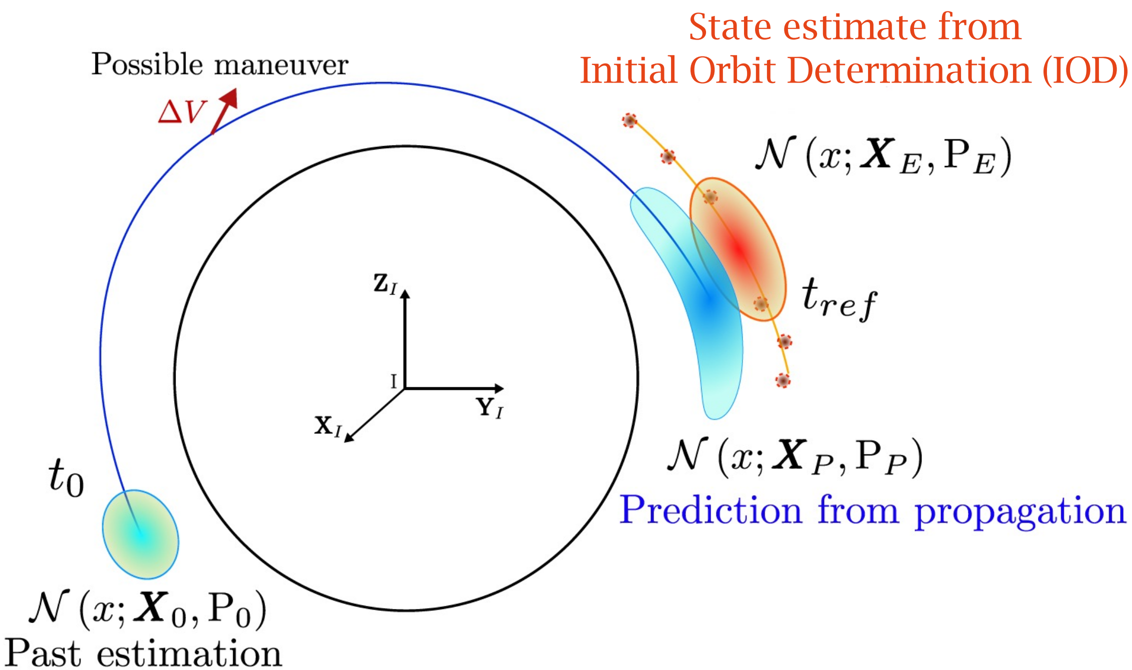
Motivation for orbit determination

- **Changes** in the cost metric (**range and range-rate space**) contain information that is usable for anomaly detection
  - The track data contains **temporal information** that was not being leveraged before with the cost metric alone (static bi-dimensional metric)
- Maybe there is **more useful information** in the track data not being used yet
  - Azimuth and elevation are being disregarded
  - The (partial) **knowledge of the dynamics** is being ignored as well

Could the **full state** (Position+Velocity) be determined with **enough certainty** (and *efficiently*) to improve anomaly detection further?

Metric definition

- ***Prediction*** and ***estimation*** comparison in a **full state** representation
  - **Mahalanobis Distance** (k^2) in an ***appropriate state space***



Curvilinear (EQCM) state representation based on the orbital path given by $\boldsymbol{X}_P$

Vallado & Alfano, 2014

$$\mathbf{X}_{P,E} = \mathbf{X}_P - \mathbf{X}_E,$$
$$k_F^2 = \mathbf{X}_{P,E}^T [\mathrm{P}_P + \mathrm{P}_E]^{-1} \mathbf{X}_{P,E}$$

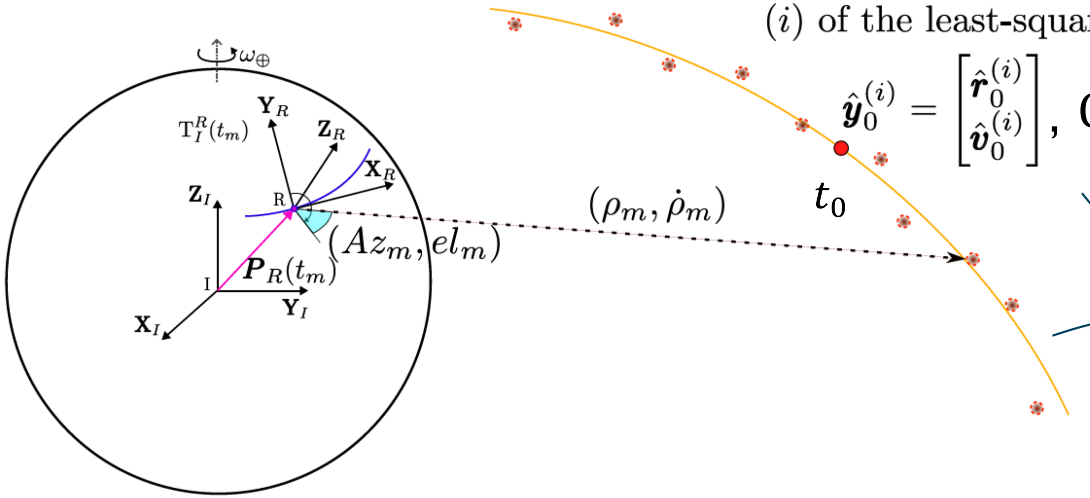
Estimation from a single track

Inputs

n **observations**

$(\rho_m, Az_m, el_m, \dot{\rho}_m)$

Radar position and orientation in inertial space



State estimate at iteration (i) of the least-squares

$$\hat{\boldsymbol{y}}_0^{(i)} = \begin{bmatrix} \hat{\boldsymbol{r}}_0^{(i)} \\ \hat{\boldsymbol{v}}_0^{(i)} \end{bmatrix}, \; \mathrm{C}_y$$

Goal

Trajectory defined by **dynamics**

Model for the dynamics

$$\hat{\boldsymbol{y}}_m = [\hat{\boldsymbol{r}}_m \, \hat{\boldsymbol{v}}_m] = \mathcal{P}\left(\hat{\boldsymbol{y}}_0, \Delta t_m\right)$$

$$\hat{z}_m = h\left(\hat{\boldsymbol{y}}_m, t_m\right)$$

Observation function (could be any of the observables)

Observed residual at t_m

$$z_m = \hat{z}_m + \xi_m$$

$$J_z = \min_{\hat{\boldsymbol{y}}_0} \sum_{m=1}^{n} (w_m \xi_m)^2$$

Non-linear minimization problem

$$w_m = 1/\sigma_m$$

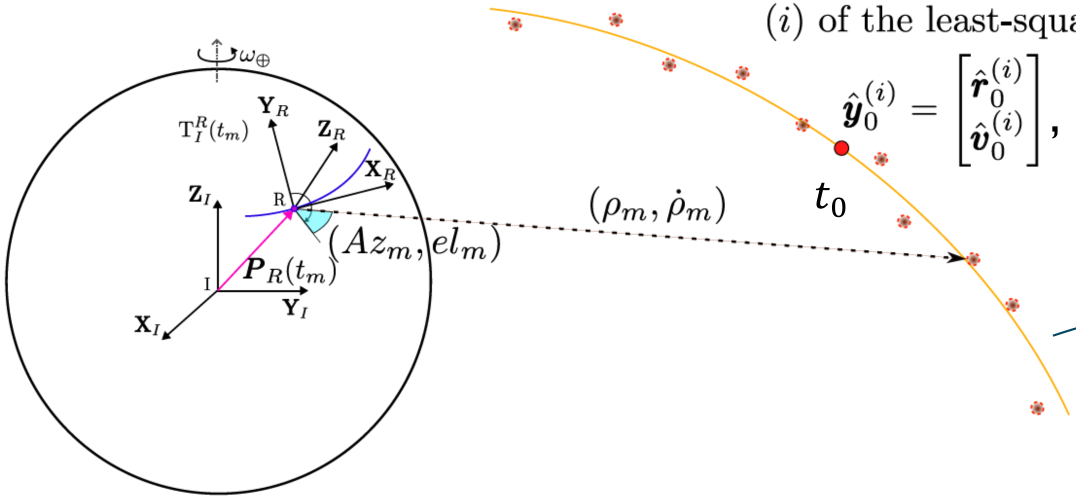
Estimation from a single track

Inputs

n **observations**

$(\rho_m, Az_m, el_m, \dot{\rho}_m)$

Radar position and orientation in inertial space



State estimate at iteration (i) of the least-squares

$$\hat{\boldsymbol{y}}_0^{(i)} = \begin{bmatrix} \hat{\boldsymbol{r}}_0^{(i)} \\ \hat{\boldsymbol{v}}_0^{(i)} \end{bmatrix}, \; C_y$$

→ **Goal**

→ Trajectory defined by **dynamics**

Model for the dynamics

$$\hat{\boldsymbol{y}}_m = [\hat{\boldsymbol{r}}_m \; \hat{\boldsymbol{v}}_m] = \mathcal{P}(\hat{\boldsymbol{y}}_0, \Delta t_m)$$

$$\hat{z}_m = h(\hat{\boldsymbol{y}}_m, t_m)$$ → *Linearize!*

Observation function (could be any of the observables)

Observed residual at t_m

$$z_m = \hat{z}_m + \xi_m$$

$$\Delta \boldsymbol{z}^{(i)} = \boldsymbol{z} - \hat{\boldsymbol{z}}^{(i)}$$

$$\Delta \boldsymbol{z}^{(i)} = A\, \Delta \boldsymbol{y}_0^{(i)} + \boldsymbol{\xi}$$

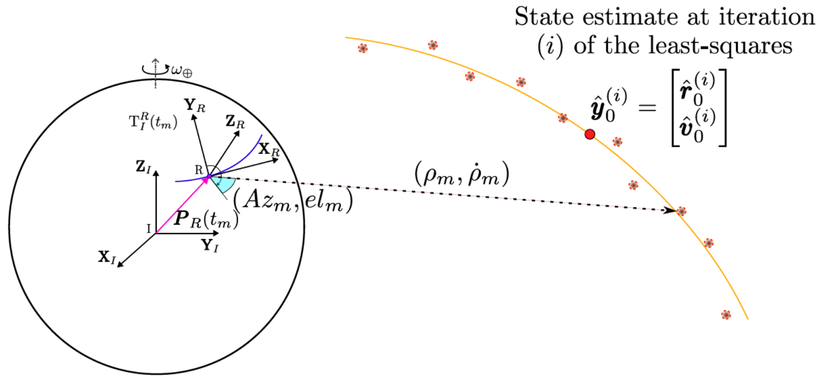
$$J_z = \min_{\Delta \hat{\boldsymbol{y}}_0^{(i)}} \boldsymbol{\xi}^\mathsf{T} W \boldsymbol{\xi}$$

*Needs **derivatives** (of observation and dynamics)*

Linear Least-Squares

Estimation from a single track

- **Keplerian** fitting of **non-weighted** Cartesian **position** – **GTDS** (Goddard Trajectory Determination System)



Fit observed *inertial position*

$$\boldsymbol{r}_m = \mathrm{OBS}_{fun}\left(\rho_m, Az_m, el_m\right)$$

$$\boldsymbol{z} = \left[\boldsymbol{r}_1^\intercal, \boldsymbol{r}_2^\intercal, \cdots, \boldsymbol{r}_n^\intercal\right]^\intercal$$

Kepler model for the dynamics → **Universal variable** χ that computes **Lagrange Coefficients**

$$\hat{\boldsymbol{y}}_m = \mathcal{P}\left(\hat{\boldsymbol{y}}_0, \Delta t_m\right) = \begin{bmatrix} \hat{\boldsymbol{r}}(\hat{\boldsymbol{y}}_0, \Delta t_m) \\ \hat{\boldsymbol{v}}(\hat{\boldsymbol{y}}_0, \Delta t_m) \end{bmatrix} = \begin{bmatrix} f\left(\hat{\boldsymbol{r}}_0, \hat{\boldsymbol{v}}_0, \Delta t_m\right)\hat{\boldsymbol{r}}_0 + g\left(\hat{\boldsymbol{r}}_0, \hat{\boldsymbol{v}}_0, \Delta t_m\right)\hat{\boldsymbol{v}}_0 \\ \dot{f}\left(\hat{\boldsymbol{r}}_0, \hat{\boldsymbol{v}}_0, \Delta t_m\right)\hat{\boldsymbol{r}}_0 + \dot{g}\left(\hat{\boldsymbol{r}}_0, \hat{\boldsymbol{v}}_0, \Delta t_m\right)\hat{\boldsymbol{v}}_0 \end{bmatrix}$$

GTDS simplification for derivatives

$$\mathrm{A} = \begin{bmatrix} f_1 & 0 & 0 & g_1 & 0 & 0 \\ 0 & f_1 & 0 & 0 & g_1 & 0 \\ 0 & 0 & f_1 & 0 & 0 & g_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_n & 0 & 0 & g_n & 0 & 0 \\ 0 & f_n & 0 & 0 & g_n & 0 \\ 0 & 0 & f_n & 0 & 0 & g_n \end{bmatrix}^{(i)}$$

$$\boldsymbol{z} = \mathrm{A}\hat{\boldsymbol{y}}_0^{(i+1)} + \boldsymbol{\xi}$$

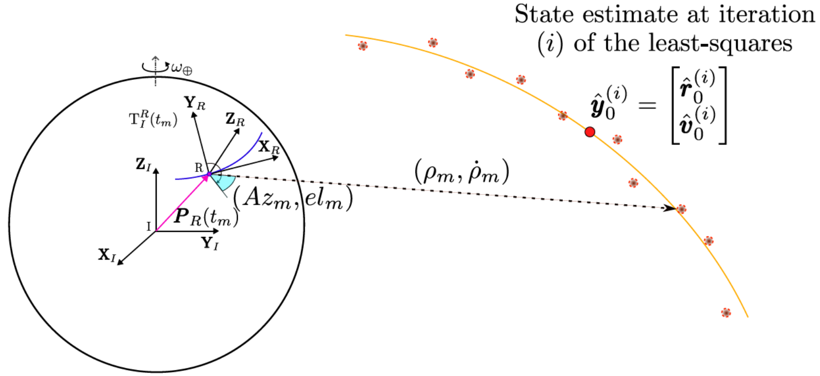
No weights applied

$$\boldsymbol{y}_0^{(i+1)} = \left(\mathrm{A}^\intercal\mathrm{A}\right)^{-1}\mathrm{A}^\intercal\boldsymbol{z} = \mathrm{H}\,\boldsymbol{z},$$
$$\mathrm{C}_y = \mathrm{H}\mathrm{C}_r\mathrm{H}^\intercal, \text{ where } \mathrm{C}_r = \mathrm{diag}\left(\mathrm{C}_{r_1}, \cdots, \mathrm{C}_{r_n}\right)$$

Long et al., 1989, Siminski, 2016

Estimation from a single track

- **Keplerian** fitting of radar observables - **KEP**



Includes range-rate

Fit *radar observables*

$$\boldsymbol{z} = [\rho_1, Az_1, el_1, \dot{\rho}_1, \cdots, \rho_n, Az_n, el_n, \dot{\rho}_n]^\mathsf{T}$$

$$[\rho, Az, el, \dot{\rho}] = h(\boldsymbol{y}, t)$$

Kepler model for the dynamics → **Universal variable** χ that computes **Lagrange Coefficients**

$$\hat{\boldsymbol{y}}_m = \mathcal{P}\left(\hat{\boldsymbol{y}}_0, \Delta t_m\right) = \begin{bmatrix} \hat{\boldsymbol{r}}(\hat{\boldsymbol{y}}_0, \Delta t_m) \\ \hat{\boldsymbol{v}}(\hat{\boldsymbol{y}}_0, \Delta t_m) \end{bmatrix} = \begin{bmatrix} f(\hat{\boldsymbol{r}}_0, \hat{\boldsymbol{v}}_0, \Delta t_m)\hat{\boldsymbol{r}}_0 + g(\hat{\boldsymbol{r}}_0, \hat{\boldsymbol{v}}_0, \Delta t_m)\hat{\boldsymbol{v}}_0 \\ \dot{f}(\hat{\boldsymbol{r}}_0, \hat{\boldsymbol{v}}_0, \Delta t_m)\hat{\boldsymbol{r}}_0 + \dot{g}(\hat{\boldsymbol{r}}_0, \hat{\boldsymbol{v}}_0, \Delta t_m)\hat{\boldsymbol{v}}_0 \end{bmatrix}$$

Weighting is necessary now!

Better approximation of the derivatives

Vallado, 2001

$$f = 1 - \frac{u}{2}\Delta t^2 - \frac{\dot{u}}{6}\Delta t^3 - \frac{\ddot{u} - u^2}{24}\Delta t^4 + \cdots$$
$$g = \Delta t - \frac{u}{6}\Delta t^3 - \frac{\dot{u}}{12}\Delta t^4 + \cdots$$
$$\dot{f} = -u\Delta t - \frac{\dot{u}}{2}\Delta t^2 + \frac{u^2 - \ddot{u}}{6}\Delta t^3 + \cdots$$
$$\dot{g} = 1 - \frac{u}{2}\Delta t^2 - \frac{\dot{u}}{3}\Delta t^3 + \cdots$$

→ Analytical derivatives

$$\partial\mathcal{P}/\partial\hat{\boldsymbol{y}}_0 = \begin{bmatrix} \partial\hat{\boldsymbol{r}}/\partial\hat{\boldsymbol{r}}_0 & \partial\hat{\boldsymbol{r}}/\partial\hat{\boldsymbol{v}}_0 \\ \partial\hat{\boldsymbol{v}}/\partial\hat{\boldsymbol{r}}_0 & \partial\hat{\boldsymbol{v}}/\partial\hat{\boldsymbol{v}}_0 \end{bmatrix}$$

Estimation from a single track

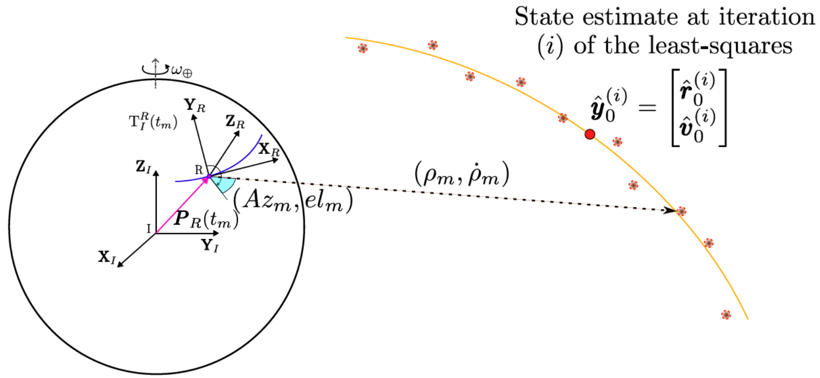
- **J2** fitting of radar observables – **KEP+J2**

Includes range-rate

Fit *radar observables*

$$\boldsymbol{z} = [\rho_1, Az_1, el_1, \dot{\rho}_1, \cdots, \rho_n, Az_n, el_n, \dot{\rho}_n]^\mathsf{T}$$

$$[\rho, Az, el, \dot{\rho}] = h(\boldsymbol{y}, t)$$



***Kepler+J2** model* for the dynamics → Taylor expansión of **Generalized Equinoctial Orbital Elements (GEqOE)**

Bau et al., 2021

$$* \begin{cases} \dot{\nu} = 0 & \textit{Generalized mean motion} \\ \dot{p}_1 = p_2\left(\frac{h-c}{r^2} - I\hat{z}\right) - \frac{1}{c}\left(\frac{X}{a} + 2p_2\right)\mathscr{U} \\ \dot{p}_2 = p_1\left(I\hat{z} - \frac{h-c}{r^2}\right) + \frac{1}{c}\left(\frac{Y}{a} + 2p_1\right)\mathscr{U} \\ \dot{q}_1 = -I\frac{Y}{r} \\ \dot{q}_2 = -I\frac{X}{r} \\ \dot{\mathcal{L}} = \nu + \frac{h-c}{r^2} - I\hat{z} - \frac{1}{c}\left[\frac{1}{\alpha} + \alpha\left(1 - \frac{r}{a}\right)\right]\mathscr{U} \end{cases}$$

Generalized Laplace vector ($\dot{p}_1$, $\dot{p}_2$)

μg

Generalized mean longitude ($\dot{\mathcal{L}}$)

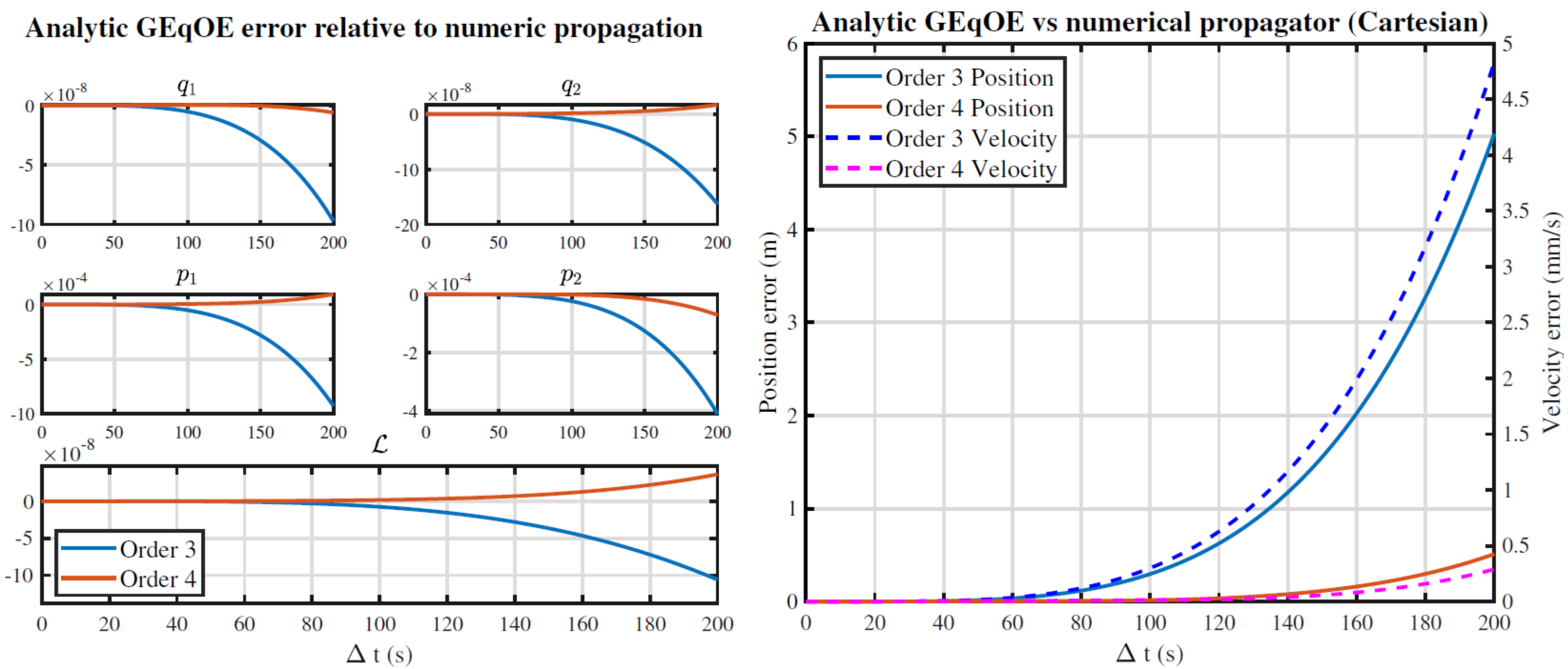
$$\chi = [\nu, p_1, p_2, q_1, q_2, \mathcal{L}]$$

No dissipative forces and **only J2 in the perturbing potential** $\mathscr{U}$

Estimation from a single track

Analytical orbit determination from radar data

- **Taylor expansion in GEqOE**



Estimation from a single track

- **J2** approximate **analytical** propagator (with derivatives)

GEqOE-Cartesian

Polynomial evaluation

$$\chi\left(\chi_0, \Delta t_m\right) = \left[\nu, p_1, p_2, q_1, q_2, \mathcal{L}\right]^\mathsf{T} \longrightarrow \boldsymbol{X}$$

$$\frac{\partial \chi}{\partial \chi_0} \longrightarrow \frac{\partial \mathbf{X}}{\partial \mathbf{X}_0} = \left.\frac{\partial \mathbf{X}}{\partial \chi}\right]_t \left(\frac{\partial \chi}{\partial \chi_0}\right) \left.\frac{\partial \chi}{\partial \mathbf{X}}\right]_{t_0}$$

Validated with AD (Hipparchus)

$$\partial \mathcal{P} / \partial \hat{\boldsymbol{y}}_0 = \begin{bmatrix} \partial \hat{\boldsymbol{r}} / \partial \hat{\boldsymbol{r}}_0 & \partial \hat{\boldsymbol{r}} / \partial \hat{\boldsymbol{v}}_0 \\ \partial \hat{\boldsymbol{v}} / \partial \hat{\boldsymbol{r}}_0 & \partial \hat{\boldsymbol{v}} / \partial \hat{\boldsymbol{v}}_0 \end{bmatrix}$$

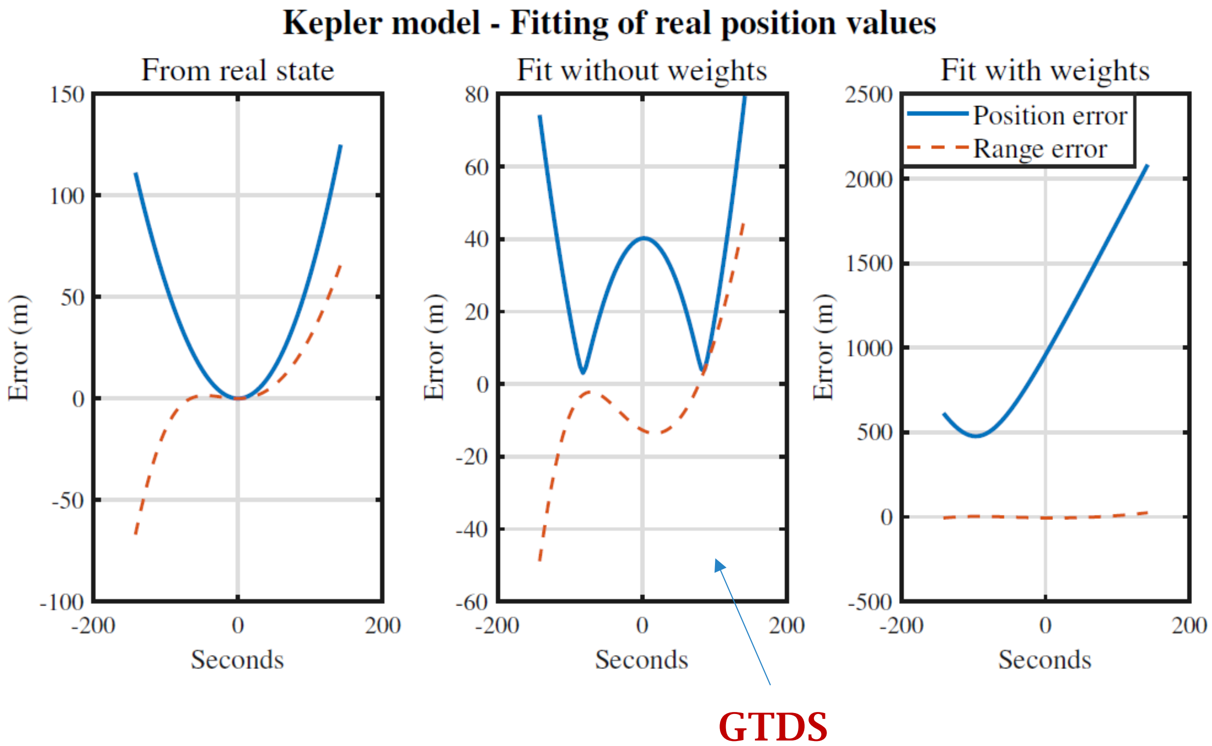
Estimation from a single track

- Adding **extra information**: Orbital Plane-based Orbit Determination – **OPOD**
  - LEO radar track: scarcity of information
  - Changes in orbital plane are very expensive in LEO: *Inclination* (i) *and RAAN* (Ω) *should be easy to **predict** with precision*
  - Add **predicted** i and Ω as **virtual measurements** to the radar observables fitting
    - How much *uncertainty* consider? A maneuver could have happened

Simulated results for IOD

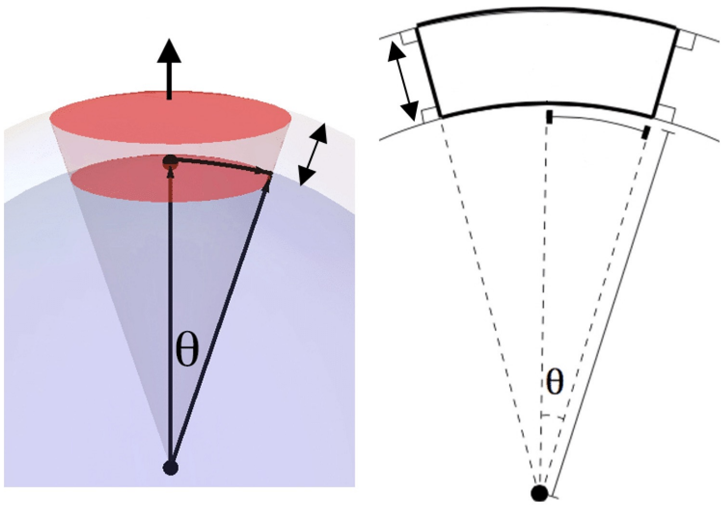
- Preliminary testing of the IOD algorithms
  - Use of radar 3 to test with a *very long track* (72 measurements over 284 seconds)

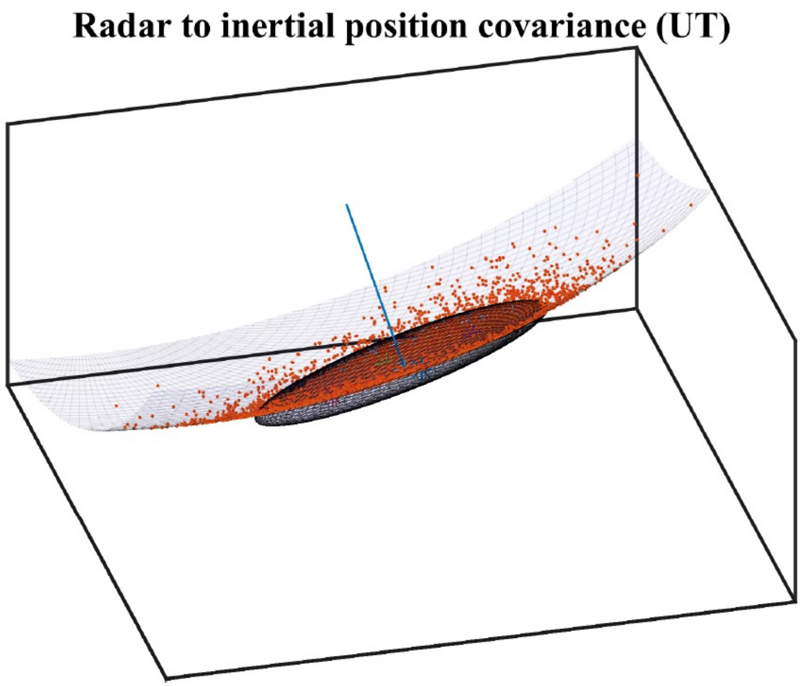
Considering noiseless measurements



Simulated results for IOD

- Preliminary testing of the IOD algorithms
  - Use of a simulated radar to test with a *very long track* (72 measurements over 284 seconds)





Simulated results for IOD

- Preliminary testing of the IOD algorithms
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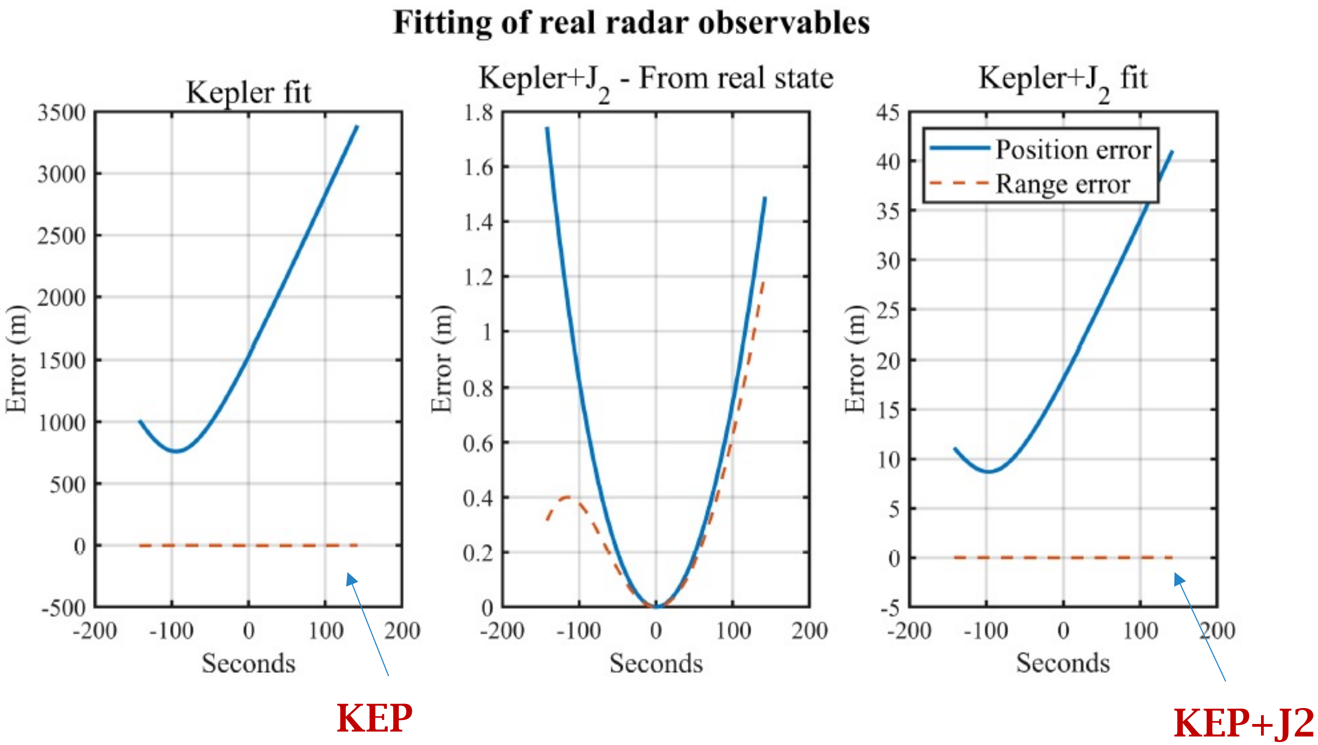
Exacerbated when fitting radar observables (*using Kepler*)

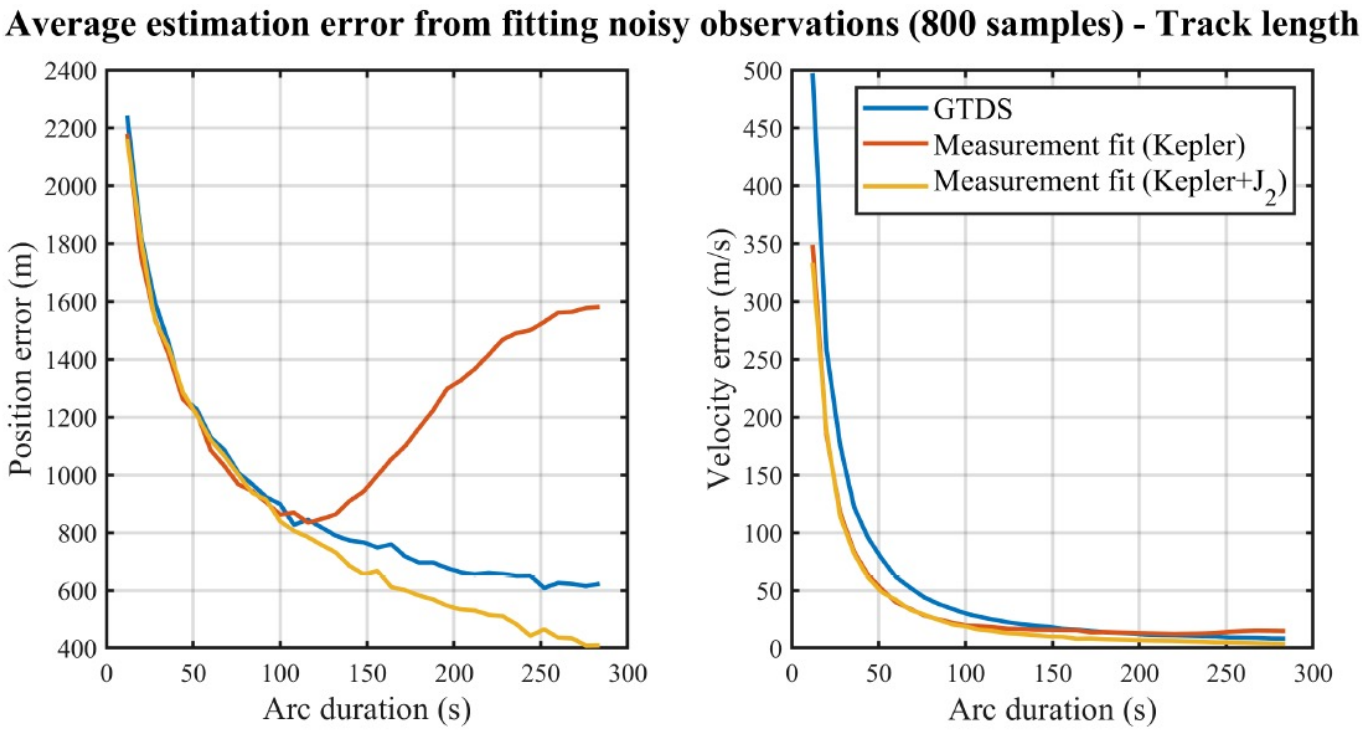


The addition of J2 solves the long track issue

Simulated results for IOD

- Preliminary testing of the IOD algorithms
  - Use of radar 3 to test with a *very long track* (72 measurements over 284 seconds)

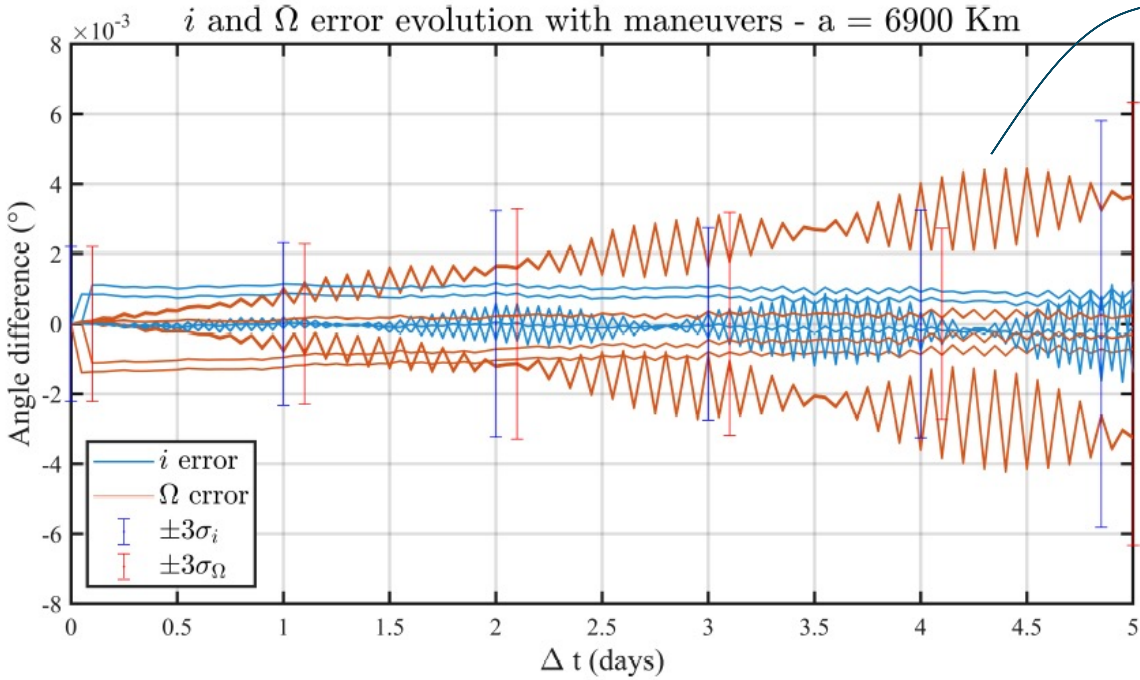
Adding noise to measurements and sampling



Simulated results for IOD

- Preliminary testing of the OPOD method
  - Orbital plane predictability check **with maneuvers**

$0.2\ m/s$ impulse in **5 directions** and **2 instants**
- Prograde
- Retrograde
- Radial-in
- Radial-out
- Out-of-plane



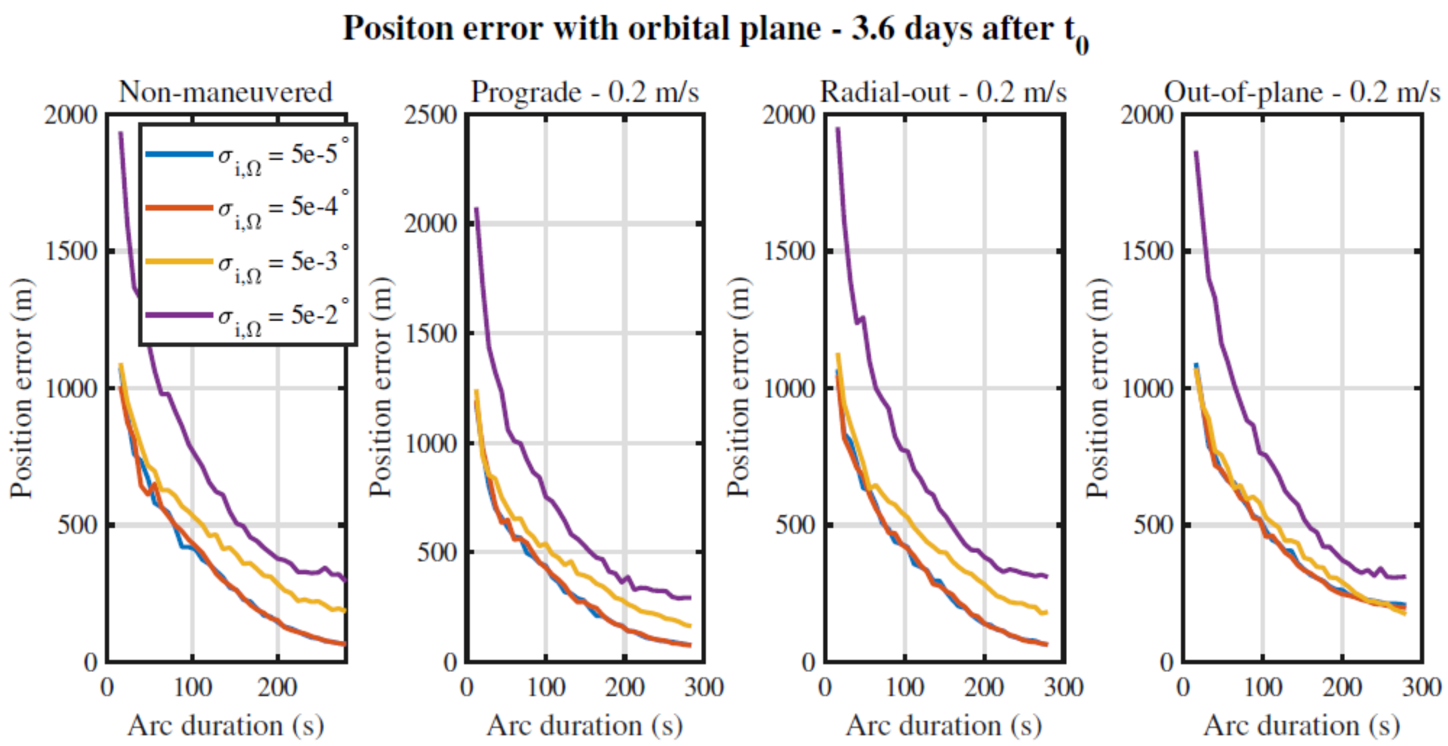
Prediction errors up to $5 \cdot 10^{-3}$ degrees after 5 days

$\sigma_{i,\Omega} = 5 \cdot 10^{-3}\ degrees$

For orbital plane virtual measurement fit (conservative?)

Simulated results for IOD

- Preliminary testing of the OPOD method (KEP+J2)
  - Effect of including orbital plane in the **estimation error**



Simulated results for IOD

- Synthetic scenarios
  - k^2 statistics on the IOD methods

$$k^2 = \boldsymbol{d}^T C_y \boldsymbol{d}$$

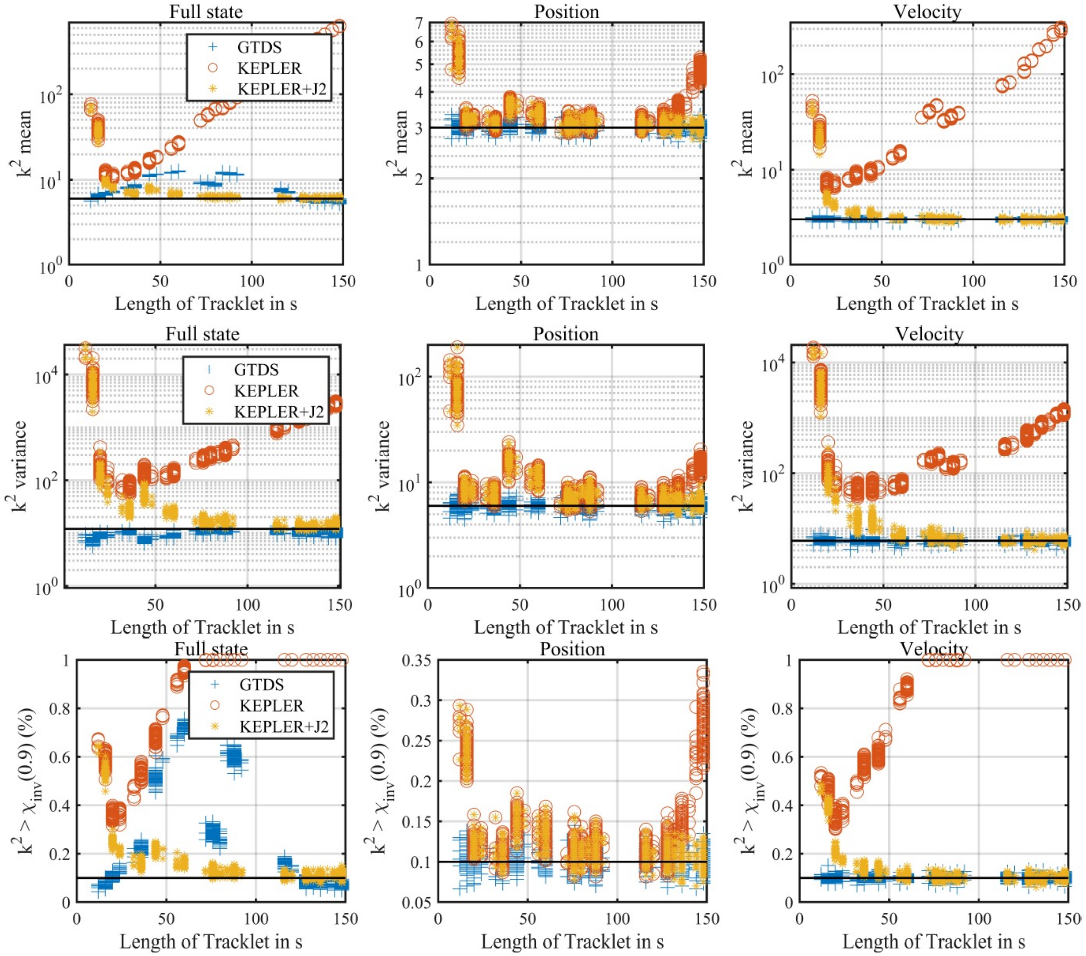
Estimation error (against real values)

Estimation covariance

k^2 statistics $\rightarrow \chi^2$ distributed

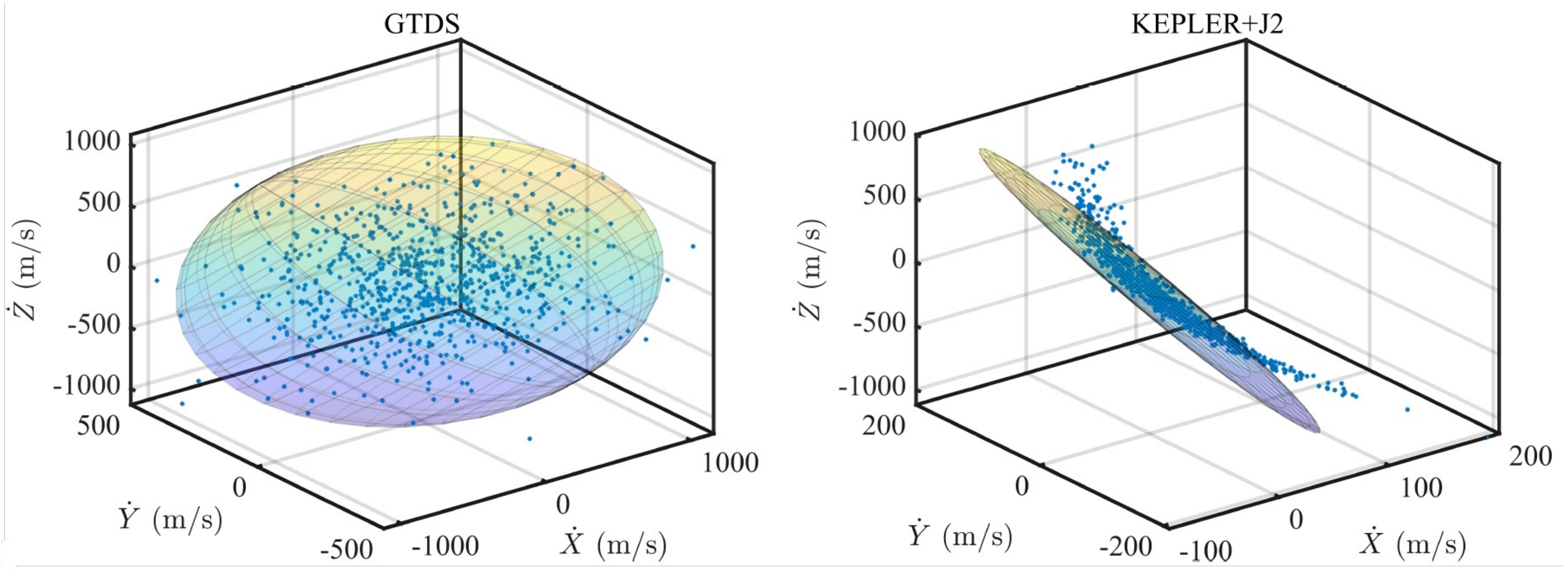
$$\mu_{k^2} = p$$
$$\sigma^2_{k^2} = 2p$$
$$k^2 > \chi^2_{inv}(0.9, p) \rightarrow 10\%$$

p degrees of freedom (dimension of y)



Simulated results for IOD

- Synthetic scenarios
  - k^2 statistics on the IOD methods



Simulated results for IOD

- Synthetic scenarios
  - k^2 statistics on the **OPOD** method (orbital plane data included in the measurement fittings)

$$k^2 = \boldsymbol{d}^T C_y \boldsymbol{d}$$

Estimation error (against real values)

Estimation covariance

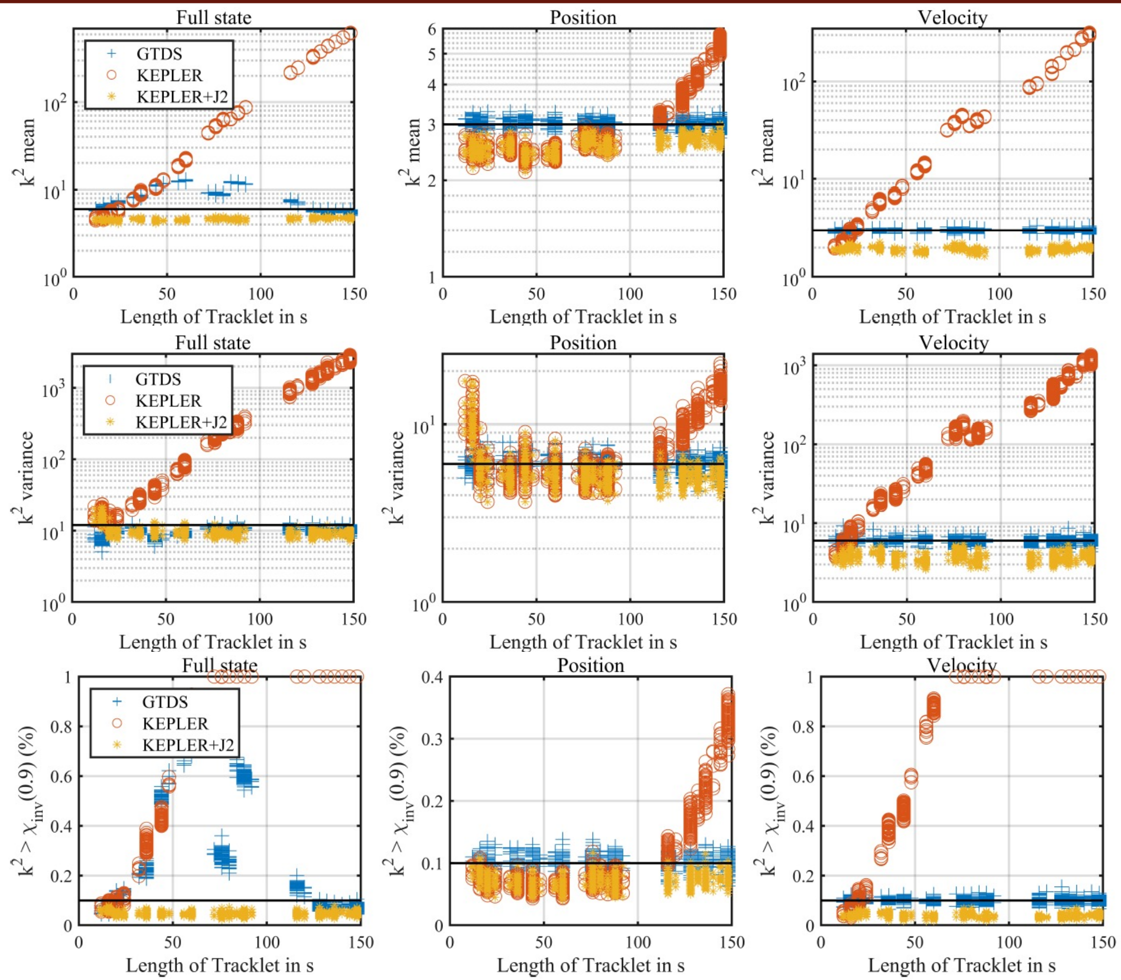
k^2 statistics → χ^2 distributed

$\mu_{k^2} = p$

$\sigma^2_{k^2} = 2p$

$k^2 > \chi^2_{inv}(0.9, p) \rightarrow 10\%$

p degrees of freedom (dimension of y)



Conclusions and Future work

- Novel approach to initial orbit determination (IOD) using single radar track data and **predicted orbital plane information**
- Least-squares fitting procedure developed, incorporating **analytically formulated J2 perturbation** dynamics and range-rate observables
- Full state estimation using radar observables directly with **Keplerian dynamics** fails due to **modeling errors** and precise range measurements
- Issues with **shorter arcs** due to **non-linear** relation between estimate and radar observables affecting velocity estimation
- Inclusion of predicted state parts (**i** and **Ω**) as virtual measurements improves accuracy and reliability (**OPOD** method)
- Future research: direct fitting of GEqOEs to incorporate range-rate information and improve uncertainty representation for very short tracks

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In this seminar:

- Overview of our recent activities in maneuver detection (JM Montilla PhD).
- In more detail: Single track IOD for low Earth orbit with approximated J2 dynamics (JM Montilla, Jan Siminski, R. Vazquez).
- **Overview of Space Occupancy Filter (A. S. Rivero PhD)**
- In more detail: Space Ocuppancy Path Conjunction Filter (A. S. Rivero, C. Bombardelli, G. Bau, R. Vazquez)
- Overview of other work

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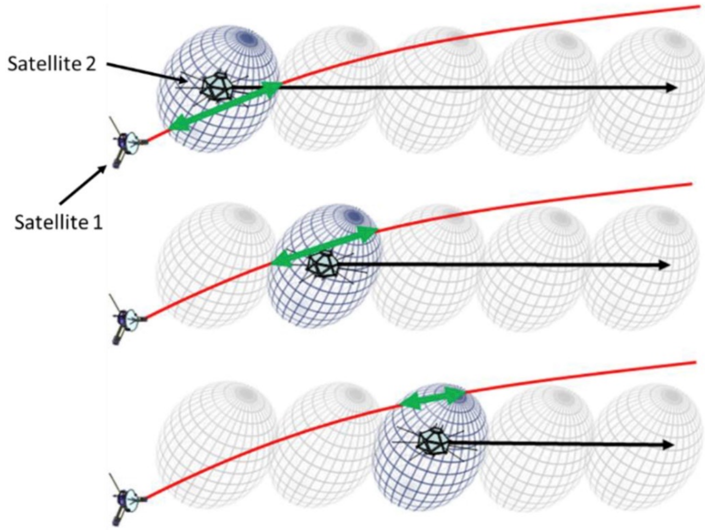
SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- Ongoing project in collaboration with Prof. Claudio Bombardelli at UPM and G. Bau at U Pisa
- Conjuntion Screening is becoming an increasingly challenging task:
    - Large constellations being deployed
    - Growing numbers of space debris
    - Risk of losing a satellite in a collision not negligible!
- Development precise and fast propagators to predict close encounters over a population of objects in orbit



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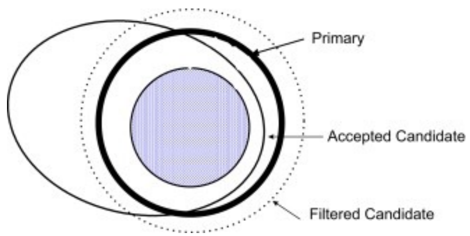
Classical filters

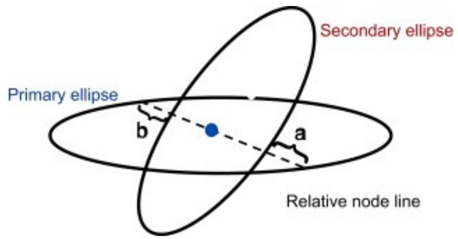
All vs. all problem: Conjunction filters to discard pairs of objects that can never collide.

Classical conjunction filters[1]

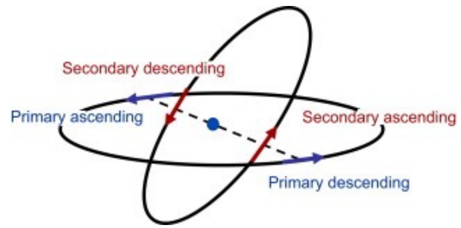
Apogee-perigee filter



Orbit path Filter



Time Filter



[1]F. R. Hoots, L. L. Crawford, and R. L. Roehrich. “An analytic method to determine future close approaches between satellites”. In: Celestial Mechanics 33 (1984), pp. 143–158.

Classical filters

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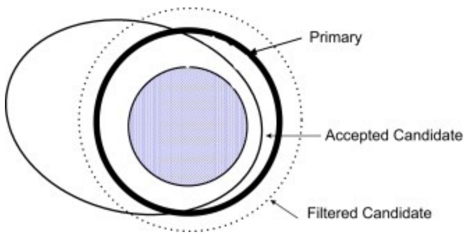
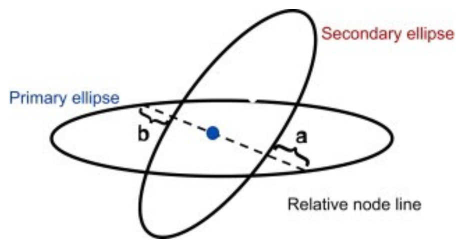
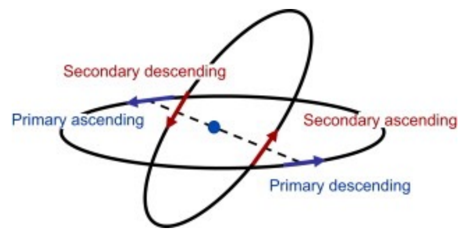


Orbit path Filter



Geometric Filters

Time Filter



[1]F. R. Hoots, L. L. Crawford, and R. L. Roehrich. "An analytic method to determine future close approaches between satellites". In: Celestial Mechanics 33 (1984), pp. 143–158.

Main problems

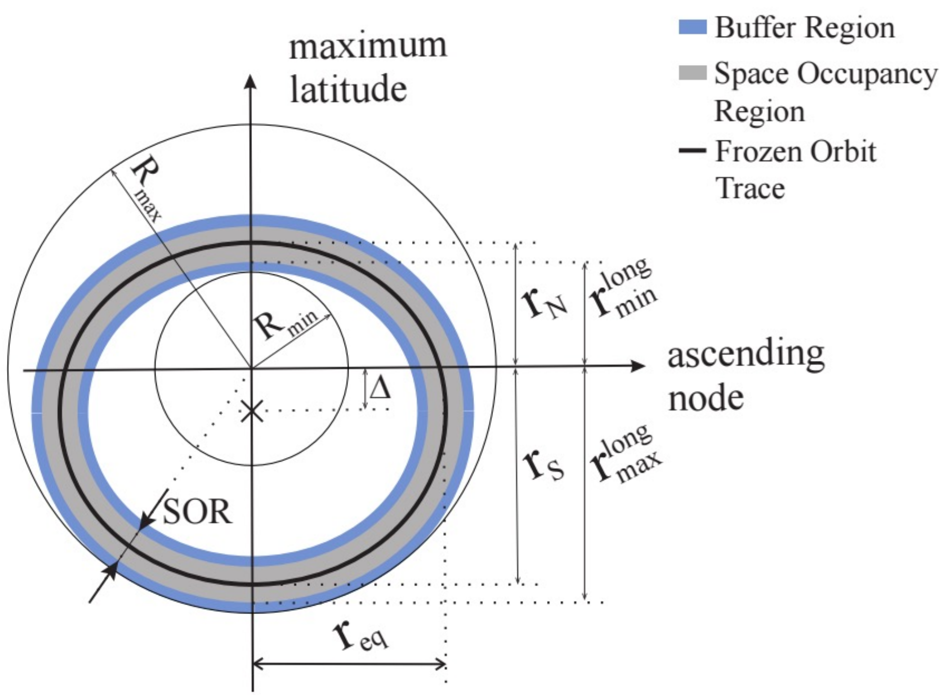
Main problems of geometric filters:

- Based on a Keplerian model → do not take into account the effect of perturbations
- Do not consider object manoeuvres during the prediction time span.
- In previous work, the classical AP-filter and OP-filter were modified to account for the effect of zonal harmonics: the SO-filter and SOP-filter.

Space Occupancy Filter

Proposal to Space Occupancy Filters

- Effect of zonal harmonics using classical Cook's model for near-circular orbits [2]
- Based on a <u>Zonal-perturbed</u> two-body-problem model, employing the concept of Space Occupancy[3]



[2]G.E. Cook. "Perturbations of near-circular orbits by the Earth's gravitational potential". In: Planetary and Space Science 14 (1966), pp. 433 –444.

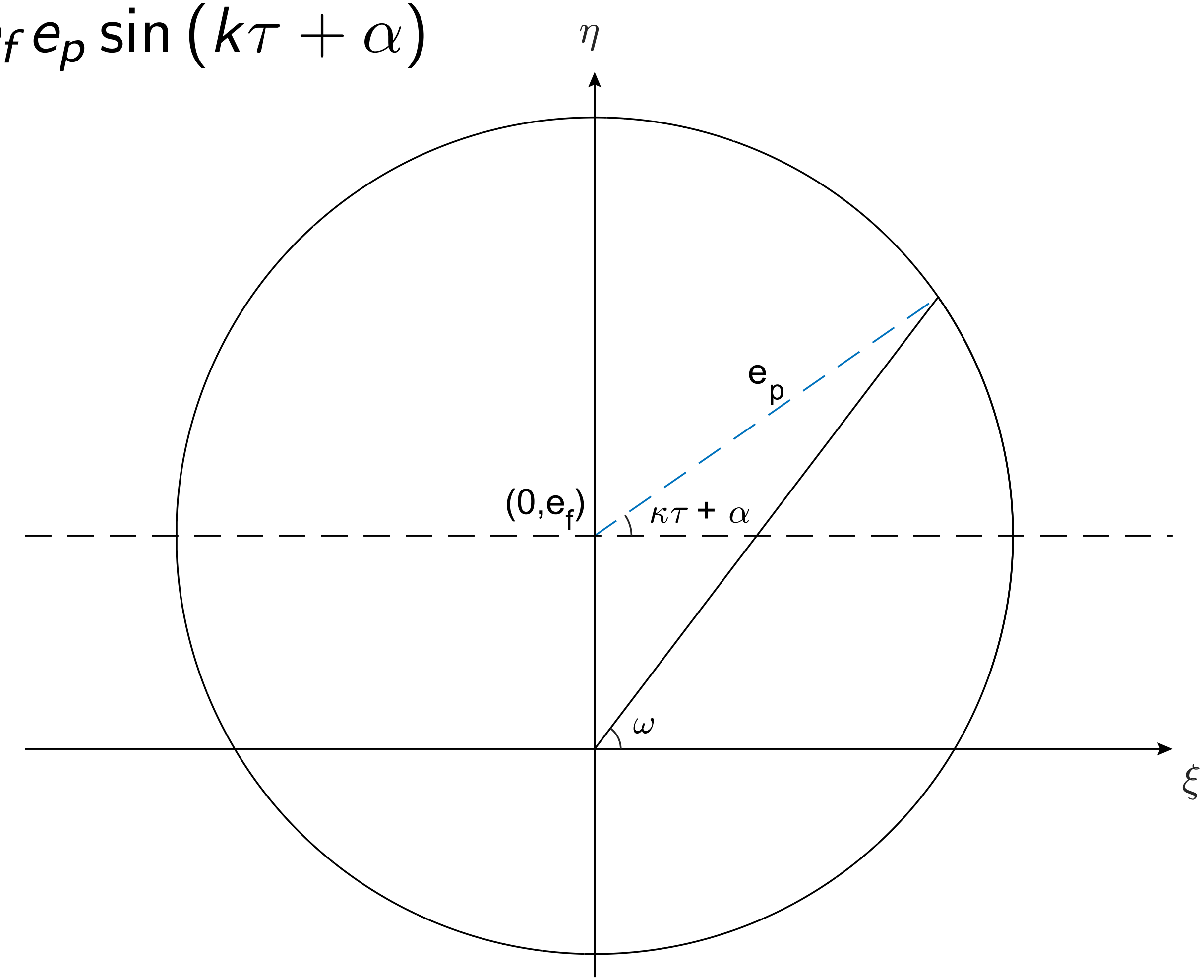
[3]C. Bombardelli et al. "Space occupancy in Low-Earth Orbit". In: Journal of Guidance, Control, and Dynamics 44 (2021), pp. 684–700.

Short-term Space Occupancy Estimation

Conjunction analysis done in the span of a few days: short-term filter becomes necessary. Mean eccentricity evolution:

$$\hat{e}(\tau) = \sqrt{e_f^2 + e_p^2 + 2e_f e_p \sin(k\tau + \alpha)}$$

Cook's long-periodic evolution of eccentricity vector takes months: one can be in a "large" or small eccentricity arc



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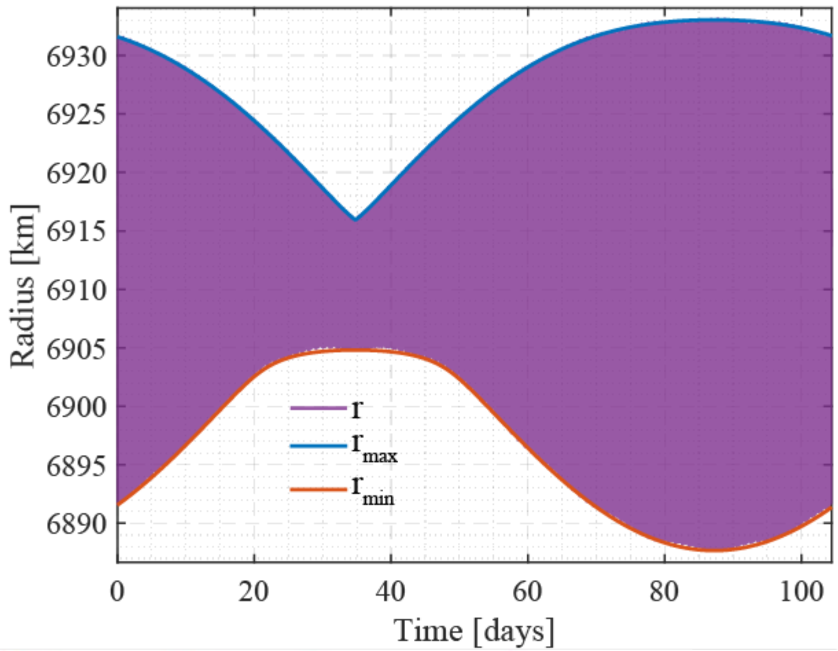
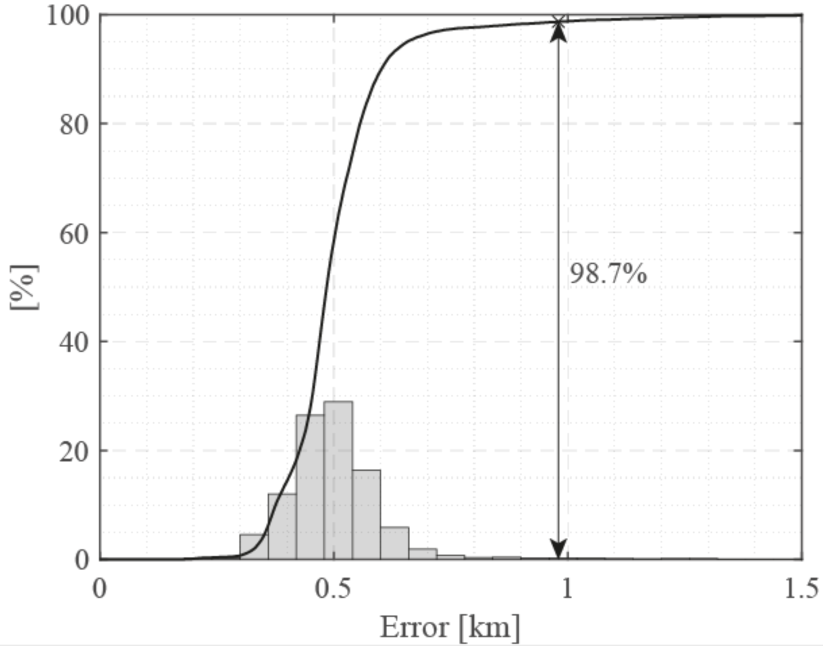
Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- SO-filter: Improves the **classical apogee-perigee** filter at very low computational cost





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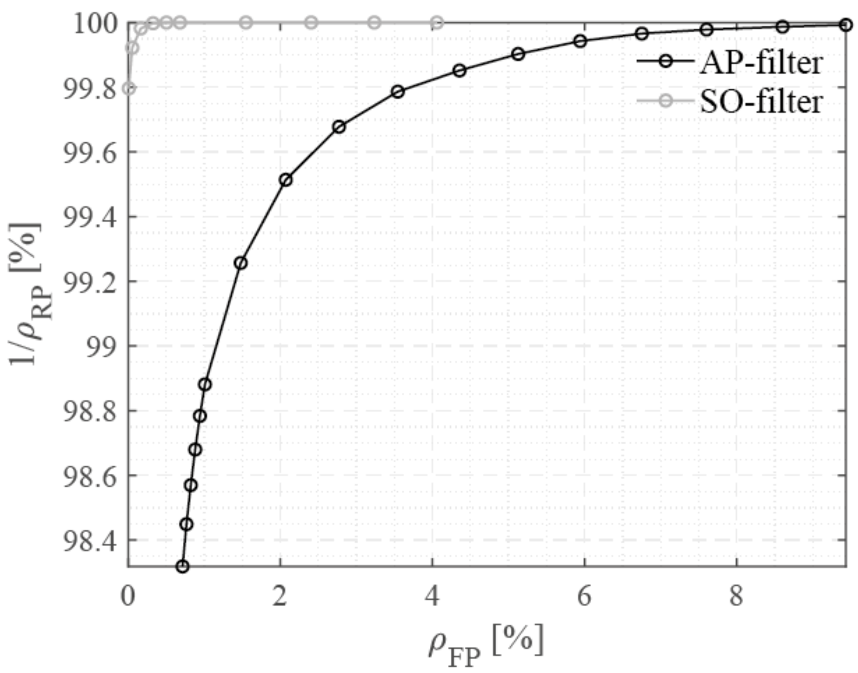
Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- SO-filter: Improves the classical apogee-perigee filter at very low computational cost
- False positives to detected real positives ratio:

$$\rho_{FP} = \frac{N_{FP}}{N_{RP} - N_{FN}}$$

- Fraction of detected real positives:

$$\frac{1}{\rho_{RP}} = \frac{N_{RP} - N_{FN}}{N_{RP}} = \frac{1}{1 + \rho_{FN}}$$



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SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- Comparison of the filter performance considering the SO-filter and the classical AP-filter on a population of **16 972** orbits:
  144 151 710 potential pairs to check for conjunctions.

Filter	False Positives	False Negatives	ρ_{FP}	η^*
AP-filter	5 507 984	0	17.239 %	73.990 %
SO-filter	530 720	0	1.661 %	77.446 %
SO-filter exact	524 183	0	1.641 %	77.451 %
SO-filter raw	2 145 317	0	6.714 %	76.325 %

- Pairs to check after the SO-filter: 32 481 309 pairs

* η: Percentage of pairs eliminated by the filter compared to the total number of pairs.

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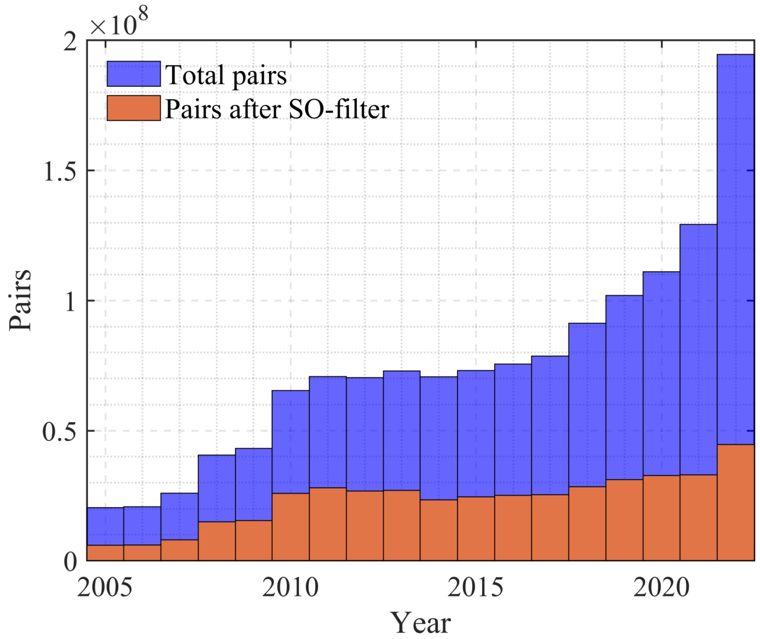
SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- Same tools can be used to study how space population has evolved over the years.
- Analysis of the evolution from 2005 to the present using the space occupancy tools
- The number of objects in space has been rapidly increasing. However, the number of objects sharing their space occupancy has not evolved in the same way!
- More details in our preprint: https://arxiv.org/abs/2309.02379



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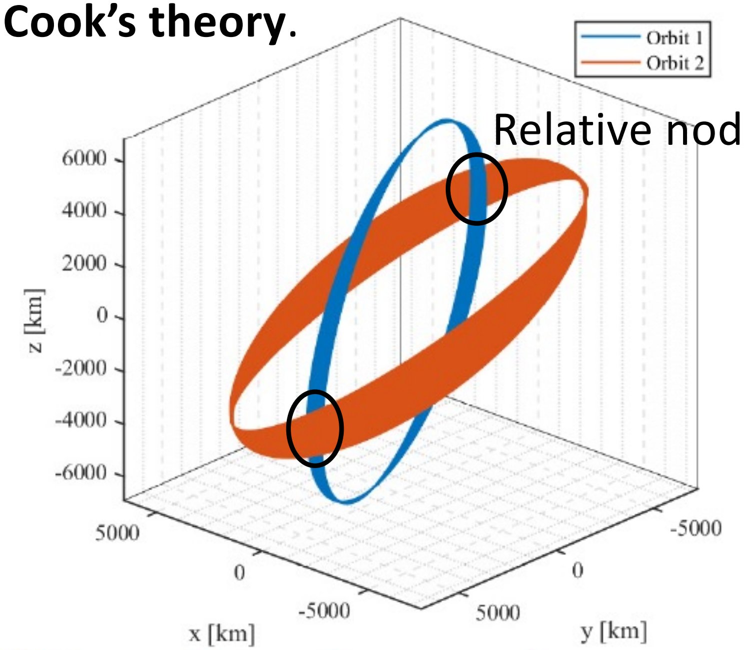
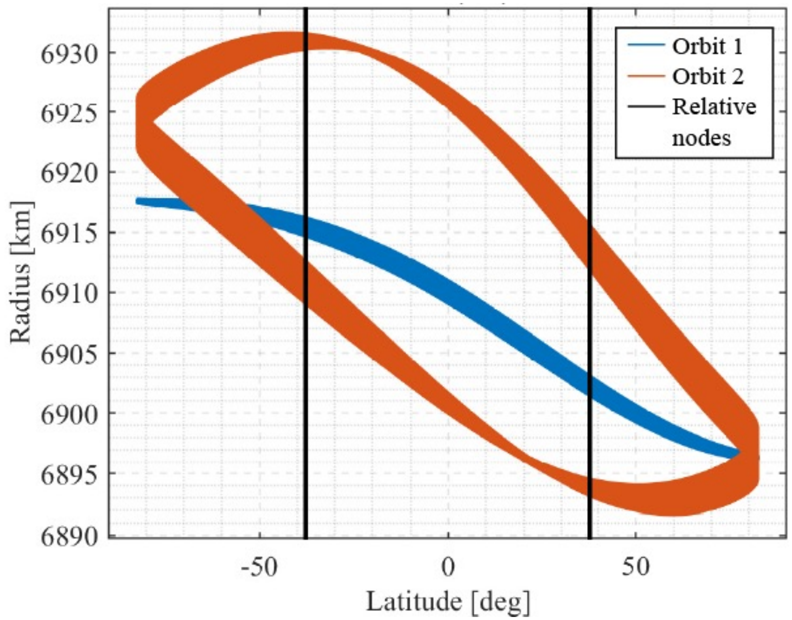
Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- Collaboration at the University of Pisa (Giulio Baù): Development of the second conjunction filter, classical **orbit path filter**, based on the **Space Occupancy concept** and **Cook's theory**.





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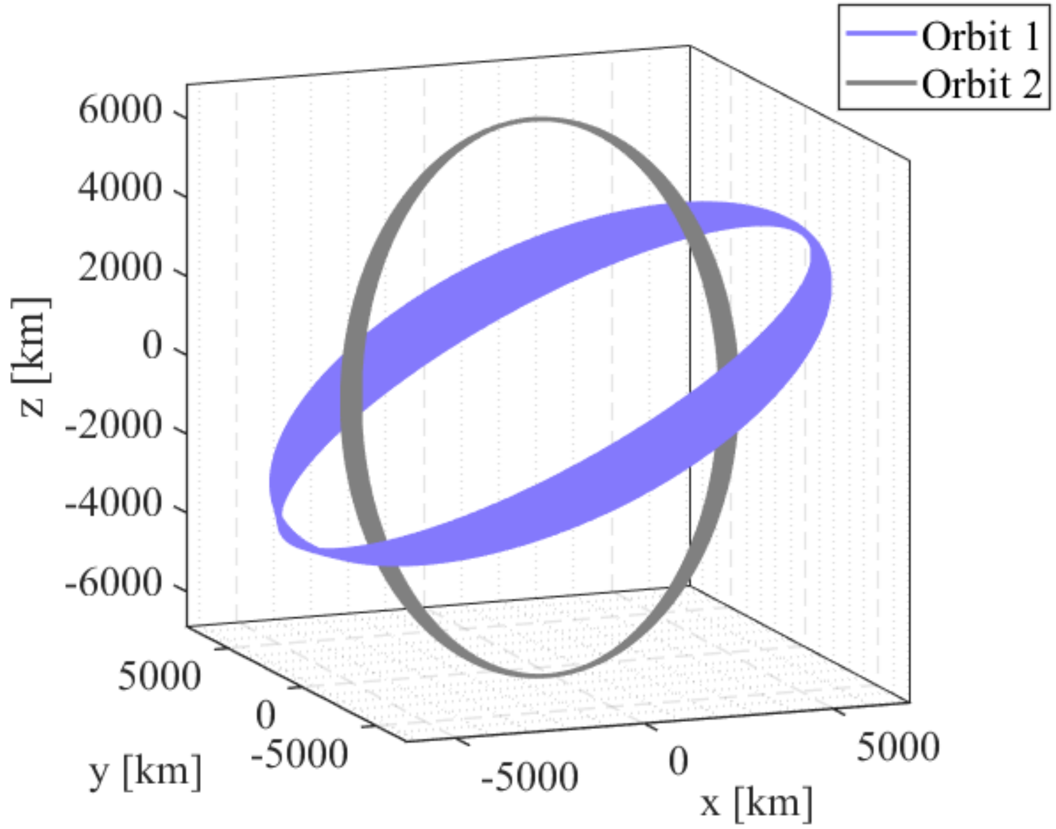
Space Occupancy Path Filter

- Minimum distance between two orbits exceeds a certain threshold distance
- The minimum orbit intersection distance is achieved at two points close to a mutual node[8]
- Check the distance between the altitude bands at the relative nodes



[8] G. F. Gronchi. "On the stationary points of the squared distance between two ellipses with a common focus". In: *SIAM Journal on Scientific Computing* 24.1 (2002), pp. 61–80.

Space Occupancy Path Filter

- Increase in the short-term SO error by restricting the domain to a section of the orbit, due to the first-order expansion:

$$r \approx a \left[1 - e \cos M + \frac{1}{2} e^2 (1 - \cos 2M) + \frac{3}{8} e^3 (\cos M - \cos 3M) \right]$$

Space Occupancy Path Filter

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- Develop an approximation of the orbital radius evolution up to third order in eccentricity and first order in J_2.
- Third-order approximation of the J_2 contribution in Cook's theory

Space Occupancy Path Filter

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- Develop an approximation of the orbital radius evolution up to third order in eccentricity and first order in J_2.
- Third-order approximation of the J_2 contribution in Cook's theory
- Computational cost: four polynomials of degree 6

Space Occupancy Path Filter Implementation

- TLEs from cataloged RSOs with $r_a < 40,000$ km and $e \leq 0.1$: 16,972 orbits
- Mean orbital elements using Kozai-Lyddane equations at a common initial time t_0 (11/02/2022 09:18:20)
- Radius evolution over a time span of five days has been computed numerically with a *high-fidelity* model, including 23×23 harmonics, lunisolar third body effects and Earth's geoid precession-nutation and polar motion

Space Occupancy Path Filter Implementation

- First step: apply the SO-filter: 32,481,309 pairs require further analysis[9]
- Nearly coplanar orbit cannot be handled because the minimum distance may not occur in the vicinity of the nodal points[10]. Therefore, mutual inclination less than 10 degrees or greater than 170 degrees are considered coplanar: 1,649,843 coplanar pairs (5% of the sample)
- The filter struggles to handle orbits with low inclination, as the relative nodes cannot be precisely computed: 30 orbits, 9,212 pairs
- The SOP-filter has been tested over 30,818,061 pairs

[9] Rivero, Bombardelli, and Vazquez, *Short-Term Space Occupancy and Conjunction Filter*.

[10] Gronchi, "On the stationary points of the squared distance between two ellipses with a common focus".

Space Occupancy Path Filter Testing, without buffer

Performance of the SOP-filter, on a population of 16,972 orbits:
32,481,309 potential pairs to check for conjunctions

Real scenarios

Coplanar pairs	1,649,843
Equatorial pairs	9,212
Positives	**6,446,647**
Negatives	**24,371,414**

Summary of Filter performance without buffer

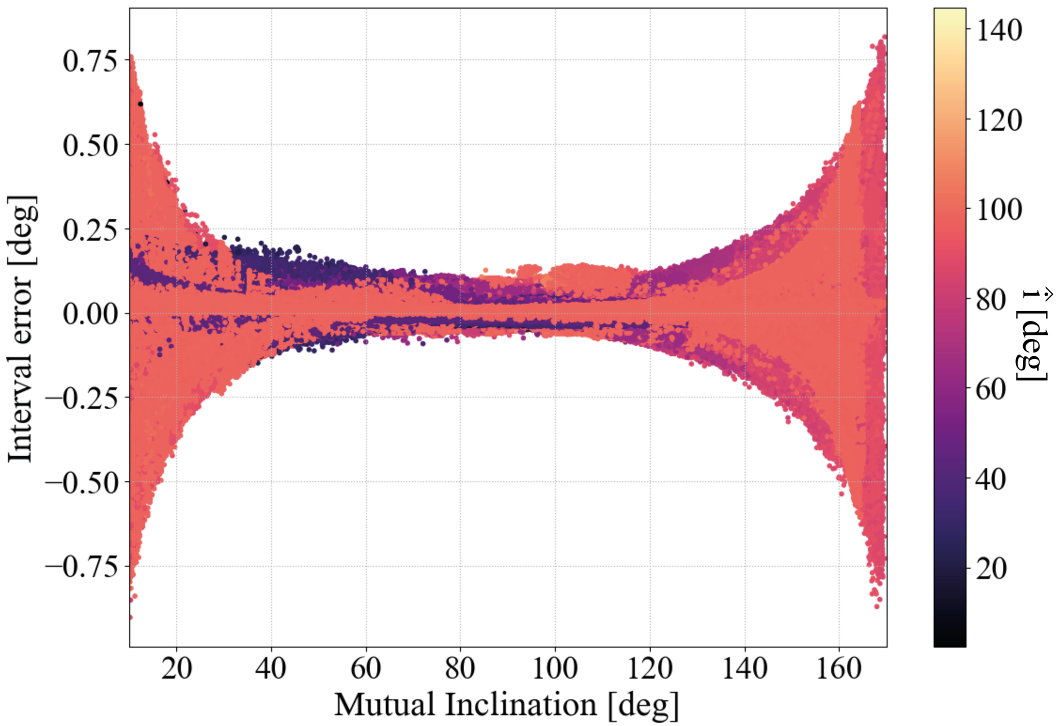
False Positives	47,814
False Negatives	434,983
% False Positives	0.789%
% False Negative	1.757%

80.34% pairs were eliminated from further analysis

Interval buffer

- Interval buffer: minimum angle required to ensure that the calculated interval encompasses the entirety of the actual relative node
- Dependence on the mutual inclination and the inclination of the orbit

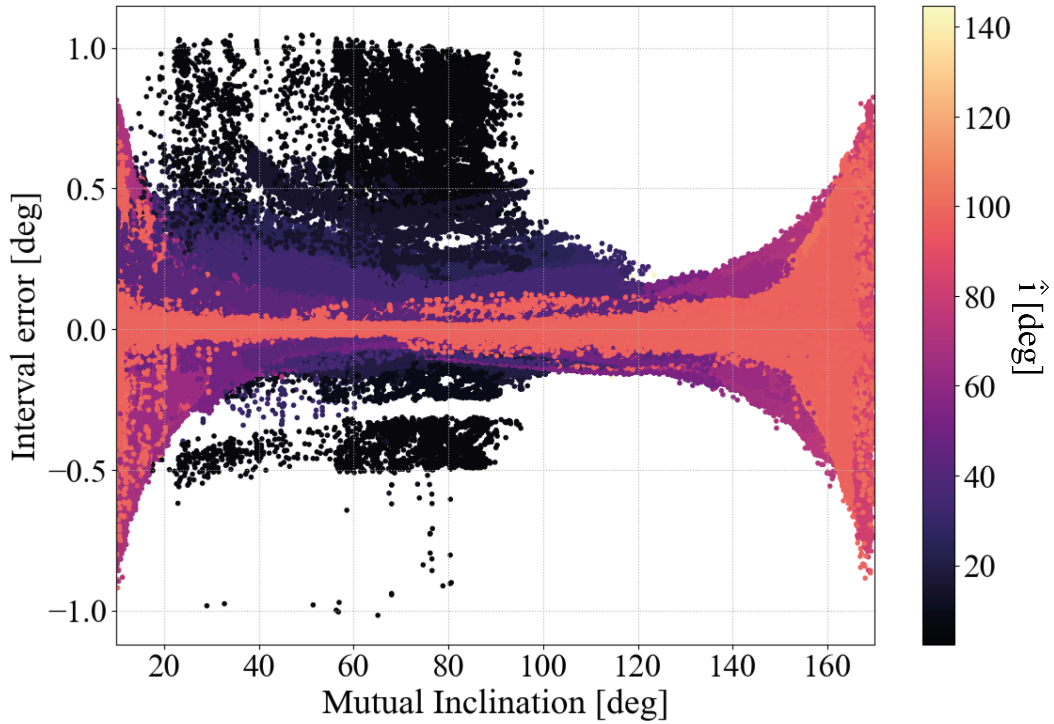
- Errors on the initial instant of time



Interval buffer

- Interval buffer: minimum angle required to ensure that the calculated interval encompasses the entirety of the actual relative node
- Dependence on the mutual inclination and the inclination of the orbit

- Errors on the last instant of time



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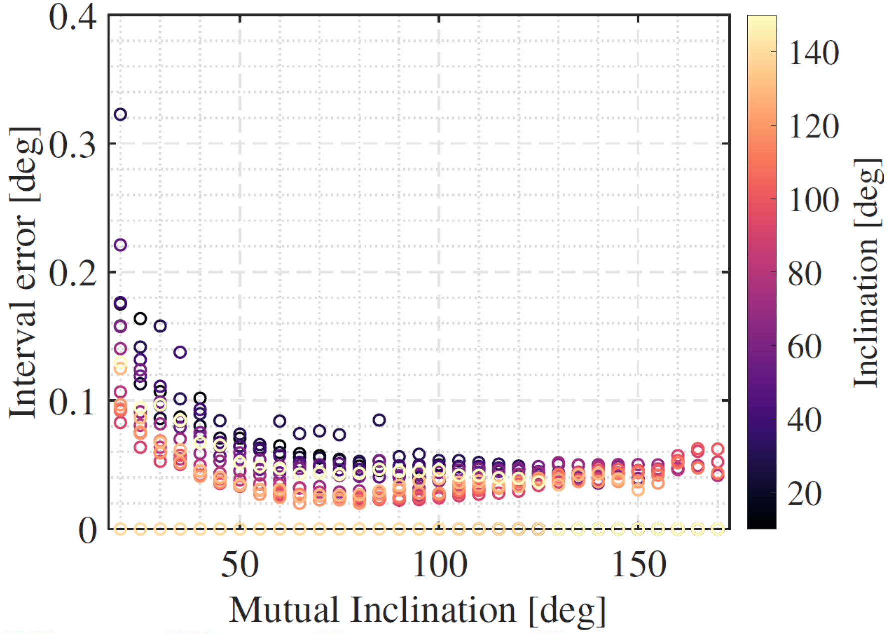
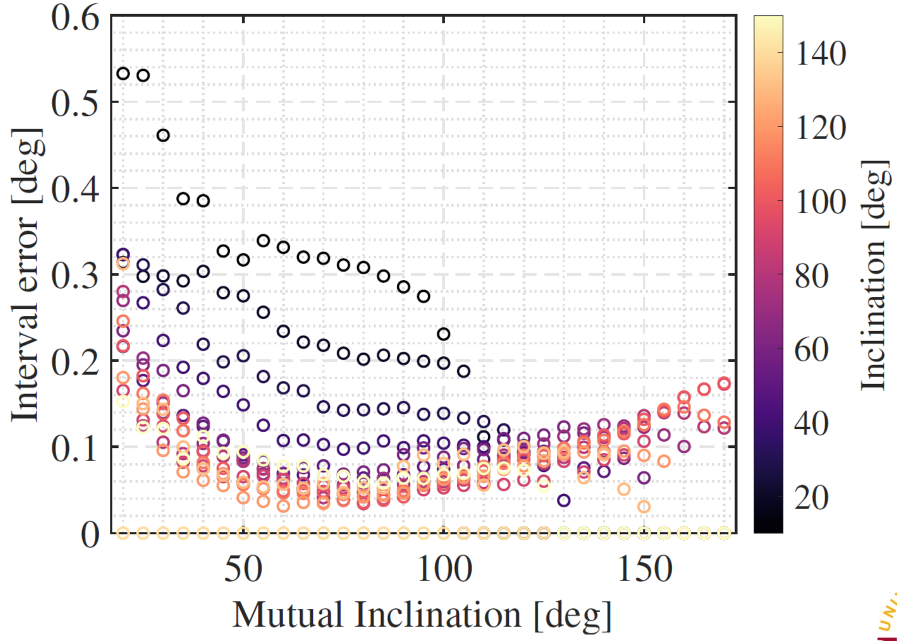
We were able to improve the errors by taking into account a correction due to precession-nutation of the Earth axis!





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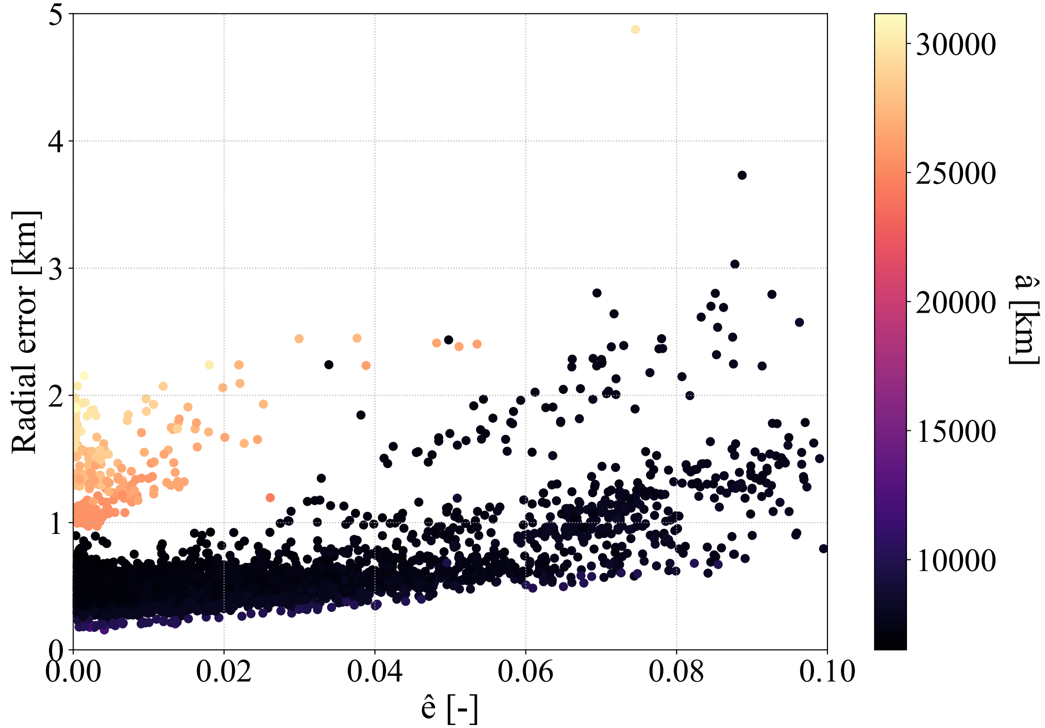
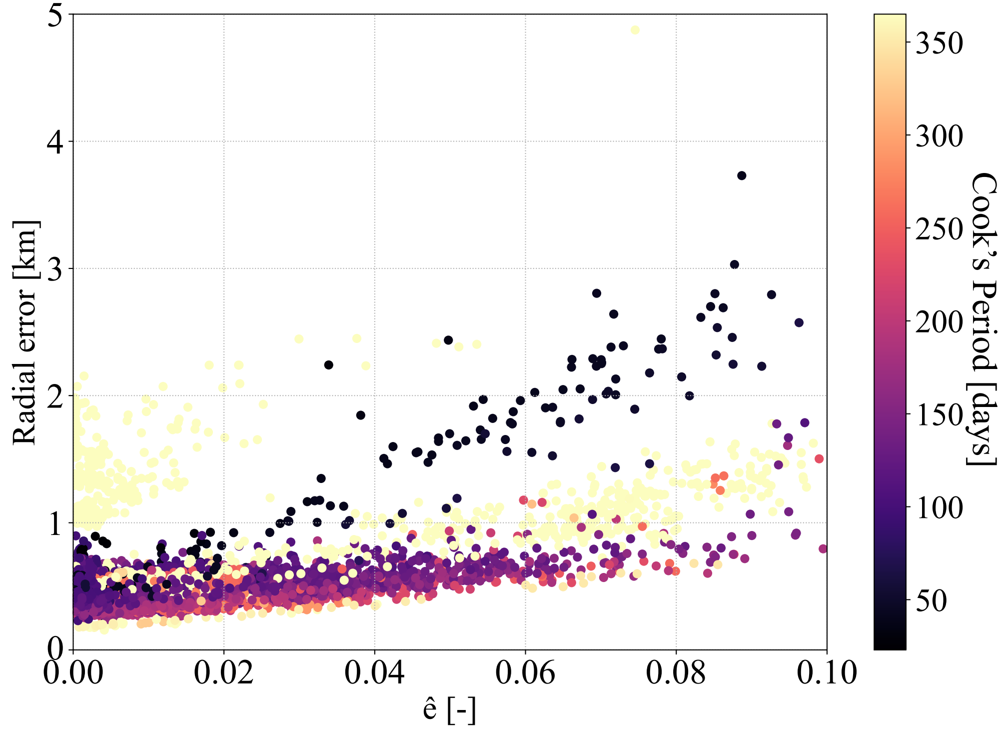
Radial buffer

- Radial buffer: minimum distance required to not understimate any radial bound
- Consider the complete precession period and each possible position of the relative node
- Dependence on the eccentricity and the semimajor-axis



Radial buffer

- Radial buffer: minimum distance required to not understimate any radial bound
- Consider the complete precession period and each possible position of the relative node
- Dependence on the eccentricity and Cook's period



Space Occupancy Path Filter Testing, with buffers

Summary of Filter Performance with buffers

False Positives	1,290,582
False Negatives	0
% False Positive	16.680%

74.89% pairs were eliminated from further analysis.

Adding a threshold for the distance: 4 km
69.00% pairs were eliminated from further analysis.

The SO-filter and the SOP-filter eliminated 92.22% of the input pairs.

Conclusions

- Filter process based on the concept of space occupancy to consider the effect of zonal-harmonics
- The SOP-filter improves the classical approach based on *Space Occupancy* theory without much computational cost
- The strong dependence of the error on the orbital elements allows to optimize the required buffer
- The SO filter process can reduce the number of pairs to less than 8%
- Future work: Complete the conjunction filter process with the time filter

SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- Recent stay at Space Debris Office (Klaus Merz): Reformulation of the SO filter to **account for satellite manoeuvres and atmospheric**.

  - Ephemeris sampling to update orbital elements and capture the effect of manoeuvres during prediction time span.

  - Implementation of Desmond King-Hele theory (considering spherical symmetrical exponential atmosphere) to model the decedent of the semimajor axis

$$\Delta a = -2\pi\ F \cdot BCa^2 \rho_{p0} \exp\left(\beta(a_0 - a - x_0)\right) \Big[I_0 + 2eI_1 + \frac{3}{4}e^2(I_0 + I_2) + \frac{1}{4}e^3(3I_1 + I_3) + O(e^4) \Big], \quad (1)$$

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Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- Validation of the reformulated SO process

    - Ephemerides from the Starlink constellation at epoch 11/07/2024 05:00:42: 5,369 orbits. Initial number of pairs: 14,410,396 pairs

    - Test cases:
        - Original configuration
        - Start-end (2 days, no drag)
        - Start-end (2 days, with drag)
        - Daily (2 intervals, with drag)
        - 12-hourly (4 intervals, with drag)

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Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- Performance of the SO-filter, on Starlink population: 14,410,396 potential pairs to check for conjunctions

Table: Filter Performance Comparison.

Filter	False Positives	False Negative	Pairs after filter	η
Original	118,561	9,966	3,518,113	75.59%
2d no drag	125,901	0	3,535,419	75.47%
2d drag	132,986	0	3,542,504	75.42%
Daily drag	167,393	0	3,521,660	75.56%
12h drag	238,178	0	3,508,831	75.65%

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Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- Performance of the SOP-filter, on Starlink population: 3,303,697 potential pairs to check for conjunctions

Table: Filter Performance Comparison.

Filter	False Positives	False Negative	Pairs after filter	η
Original	36,393	12,732	2,328,489	36.75%
2d no drag	56,208	7,135	2,353,901	35.98%
2d drag	137,873	0	2,442,701	33.29%
Daily drag	118,122	0	2,419,000	34.01%
12h drag	107,658	0	2,400,300	34.57%

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- **Overview of other work**

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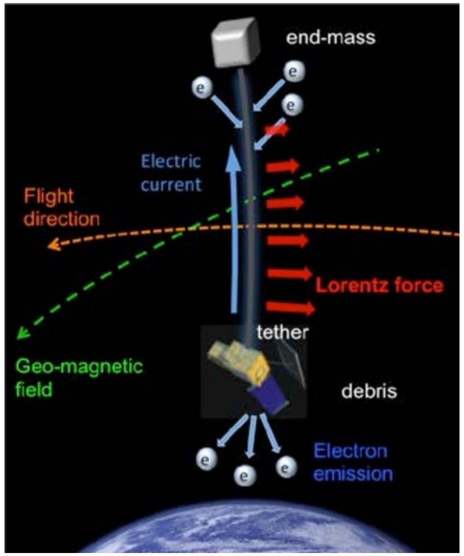
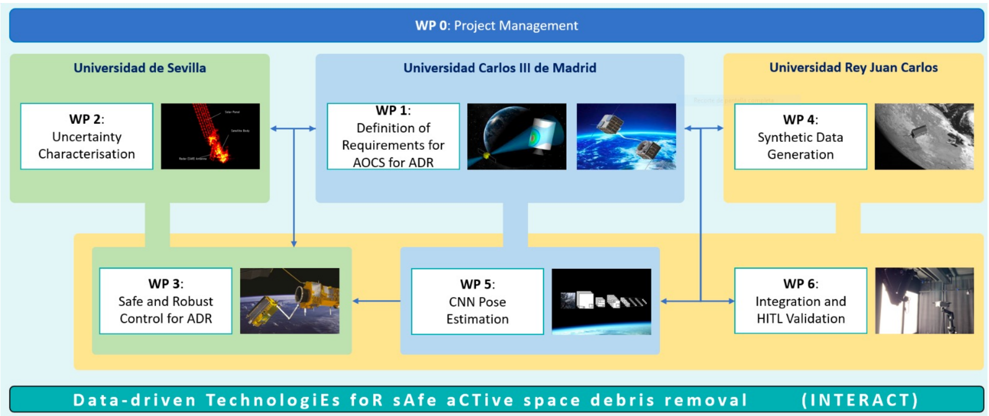
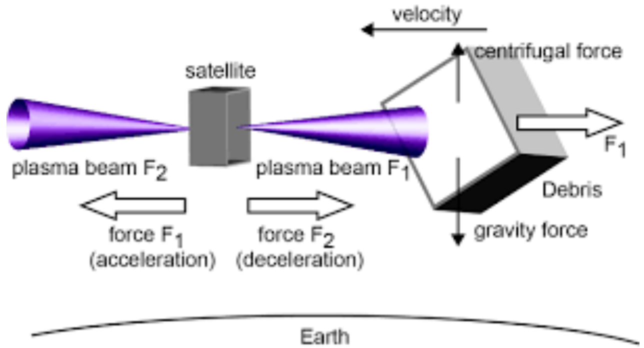
- Active Debris Removal (ADR) using electrodynamic tethers (ET) or Ion Shepherd Beam (IBS)
- Joint project with UC3M/UPM and URJC (Madrid, Spain).
- In charge of uncertainty characterization (rotational dynamics of debris) + GNC

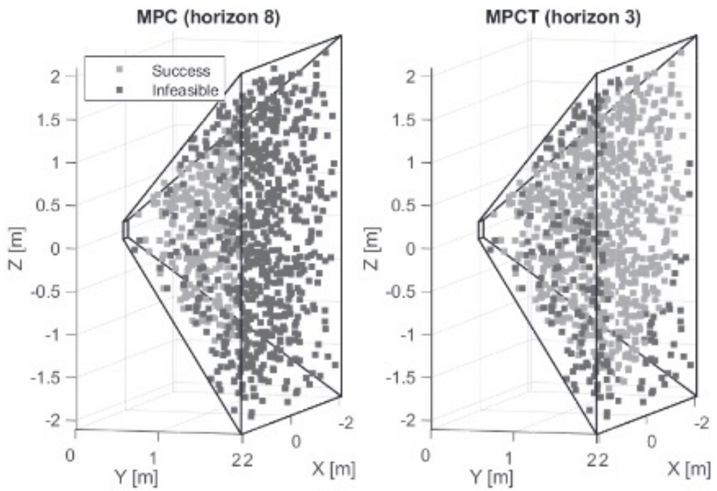


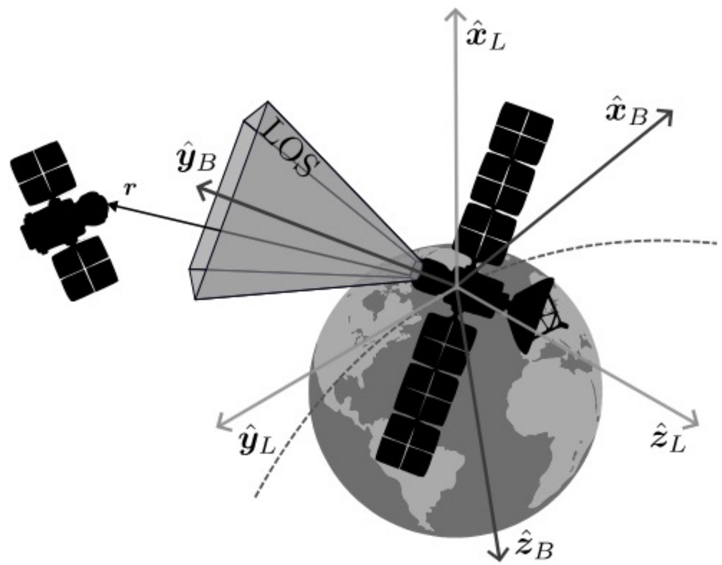




Non-cooperative rendezvous with a tumbling target

- JA Rebollo

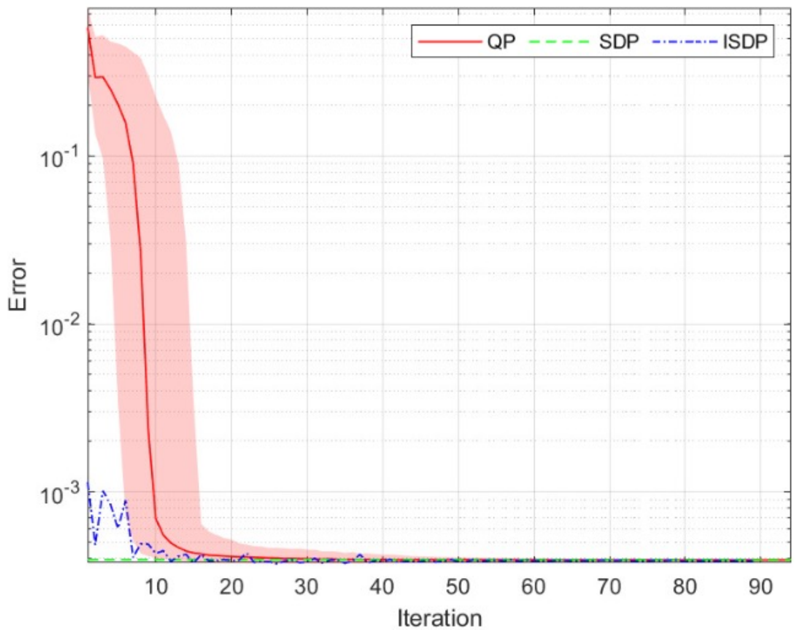


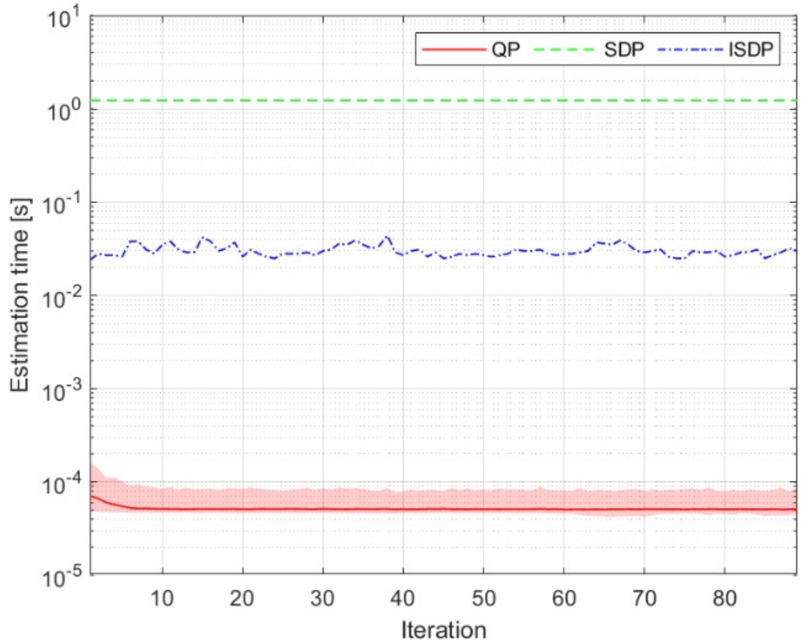


J.A. Rebollo, R. Vazquez, I. Alvarado, D. Limon, "MPC for Tracking applied to rendezvous with non-cooperative tumbling targets ensuring stability and feasibility," CDC 2024.

Inertia estimation for a tumbling target from pose data

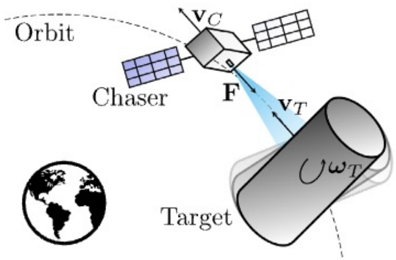
- JA Rebollo



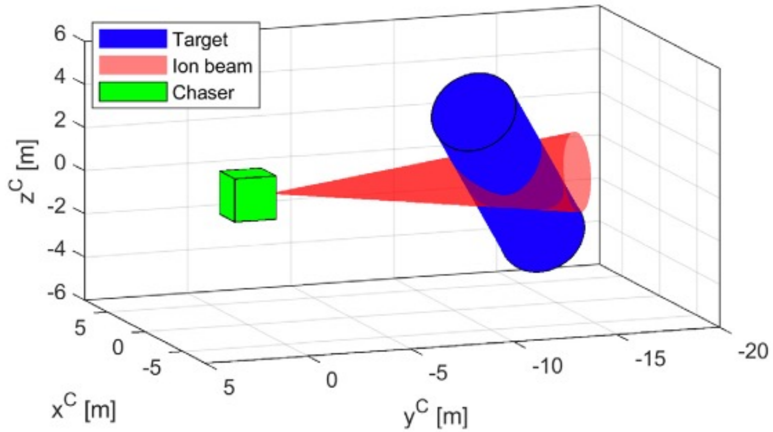


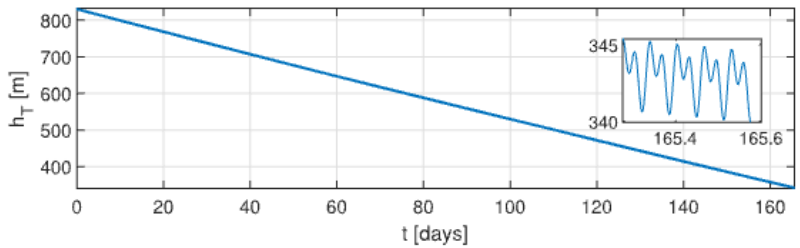
J.A. Rebollo, F. Gavilan, R. Vazquez, D. Limon, "A QP-based iterative approach to on-line inertia estimation for non-cooperative tumbling spacecraft," CDC 2024.

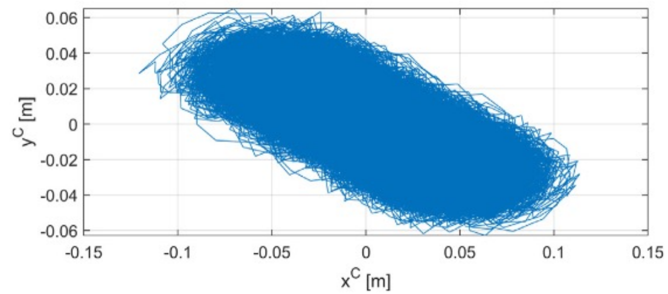
De-orbiting using Ion Beam Shepherd using MPCT with improved modeling



- J Urrios







J. Urrios, R. Vazquez, F. Gavilan, I. Alvarado, "**Robust Model Predictive Control for an Ion Beam Shepherd in a large-debris removal mission**," accepted in Acta Astronautica, 2024.

New/Potential Projects (Space Surveillance Chair)

In collaboration with Indra-Deimos, just started:

- Machine learning algorithms for collision avoidance maneuvers (J Urrios)
- Covariance abacuses and covariance realism (JA Rebollo)

S3T Projects (ESA-AEE), granted, to start 2025:

- Enhancing the S3T Ground Segment through Coordinated Multi-Sensor Observation using Stare and Chase Techniques and Data Fusion Analysis (with ROA plus several S3T sensors)
- Atmospheric density model calibration using S3TSR radar

Under evaluation: **Proposal to AFOSR** on machine learning and uncertainty propagation

cve

Contact: Rafael Vazquez (Full Professor of Orbital Mechanics)

rvazquez1@us.es

Thanks!

ESCUELA TÉCNICA SUPERIOR DE INGENIEROS

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Some recent publications (some of them downloadable at http://aero.us.es/rvazquez)

- J.M. Montilla, R. Vazquez and P. Di Lizia, "Maneuver detection with two mixture-based metrics for radar track data," accepted in Journal of Guidance, Control, and Dynamics, 2024.
- J.M. Montilla, J. Siminski and R. Vazquez, “Single track orbit determination analysis for low Earth orbit with approximated J2 dynamics”, in Press, Advances in Space Research, 2024
- A.S. Rivero, C. Bombardelli, and R. Vazquez "Space-Occupancy Conjunction Filter," submitted, 2024.
- J.M. Montilla, J.C. Sanchez, R. Vazquez, J. Galan-Vioque, J. Rey Benayas, J. Siminski, "Manoeuvre detection in Low Earth Orbit with Radar Data," Advances in Space Research, 2022.
- J. Urrios, R. Vazquez, F. Gavilan, I. Alvarado, "Robust Model Predictive Control for an Ion Beam Shepherd in a large-debris removal mission," accepted in Acta Astronautica, 2024.
- A.S. Rivero, G. Baù, R. Vazquez and C. Bombardelli, "A novel conjunction filter based on the minimum distance between perturbed trajectories,“ submitted, 2024.
- J.A. Rebollo, R. Vazquez, I. Alvarado, D. Limon, "MPC for Tracking applied to rendezvous with non-cooperative tumbling targets ensuring stability and feasibility," submitted, 2024.
- J.A. Rebollo, F. Gavilan, R. Vazquez, D. Limon, "A QP-Based Iterative Approach to On-Line Inertia Estimation for Non-Cooperative Tumbling Spacecraft," submitted, 2024.

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