

# Actividades de Vigilancia Espacial en la Universidad de Sevilla: Detección de Maniobras y Análisis de Conjunciones

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Cátedra de Vigilancia Espacial



# Aerospace Engineering at University of Seville

**University of Seville:** One of Spain oldest (>500 years) and largest (3<sup>rd</sup> after UNED and Complutense) with more than 70K students and 4K professors covering all areas of knowledge.

- ETSI (Escuela Técnica Superior de Ingeniería) created in the 60s, strong tradition in physical/mathematical background and in technology transfer with industry through AICIA (Asociación de Investigación y Cooperación Industrial de Andalucía)
- Aerospace Engineering started in 2002 (2<sup>nd</sup> oldest in Spain after UPM)



# Aerospace Engineering at University of Seville

- Teaches Undergraduate Degree (120, specializations: Aerospace Vehicles, Aerial Navigation and Airports), Master's Degree (80) and also PhD, high mark required to enter -> **high-level students**
- Mainly focused in aeronautics (a traditional industry in Seville) but interest in Space growing and expected to grow even more (**Spanish Space Agency created in Seville!**), possibly with a specialized Master in Space Engineering soon to be created
- The Department of Aerospace Engineering was created in 2006 of which GIA (Grupo de Ingeniería Aeroespacial) is part of. We teach Orbital Mechanics (Basic and Applied), Spacecraft Attitude Dynamics, Spacecraft Systems.



# Space Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla



Three main topics of research:

- **Space Surveillance and Awareness (SSA)**
- **Guidance, Navigation and Control (GNC) for spacecraft**
- **Optimization applied to Scheduling for Space**

Team: small group of 3 professors + 2 PhDs + several MSc students **fully devoted to space activities** (with the collaboration of the rest of the GIA group, and a professor from Applied Math, **as well as several national/international collaborations**)

**Rafael Vazquez:** full professor of orbital mechanics. Background in control theory, applied mathematics.

# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## In this talk:

- Manoeuvre detection based on S3TSR data. Project with Indra.
- Further advances on Manoeuvre detection (JM Montilla PhD).
- Filtering for conjunction analysis (A Sanchez PhD)
- Optimization applied to Scheduling for Space

# Manoeuvre detection based on S3TSR data



**US:** Jose M. Montilla, Julio C. Sanchez, Rafael Vazquez, Jorge Galan-Vioque

**Indra:** Javier Rey Benayas

**ESA:** Jan Siminski



@CDTIoficial





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# Motivation and Introduction



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# Motivation.

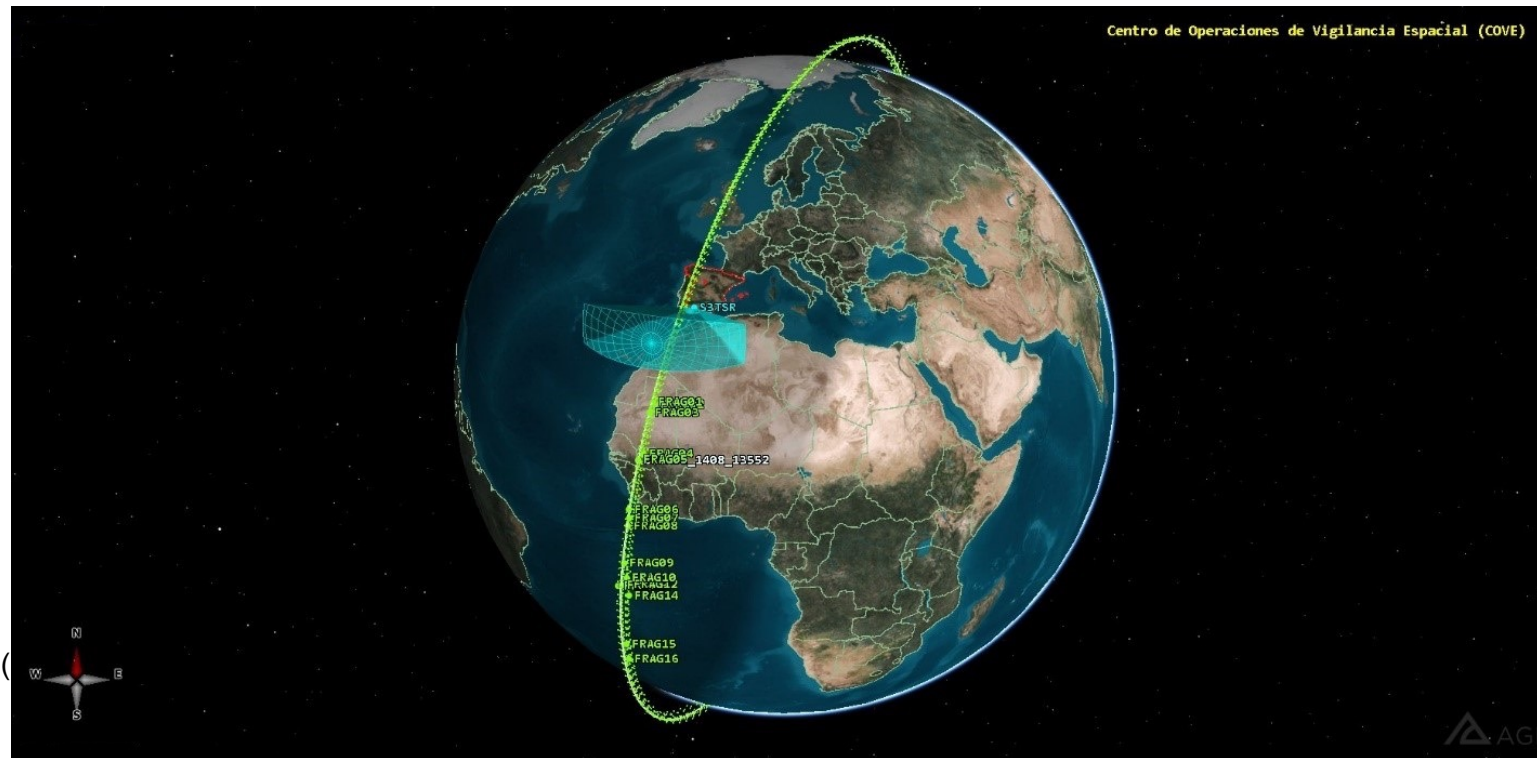
- In Space Surveillance and Tracking (SST), accurate orbital determination and manoeuvre detection crucial to infer objects' orbital information and future behaviour.
- Satellites performing **unknown** manoeuvres pose a challenge when trying to associate the new collected observations (laser/radar/optical) with the catalogued reference orbits
- Manoeuvre detection can significantly reduce the number of uncorrelated targets detected by the SST sensors infrastructure
- **A considerable number of uncorrelated objects are just known satellites, that have performed unpublished manoeuvres**

# Introduction.


- This work presents two metrics for the **detection of manoeuvres** in Low Earth Orbit (LEO) from radar data. These were initially introduced in “Vazquez et al., Manoeuvre detection for near-orbiting objects. In *8th European Conference on Space Debris*, (2021),” and validated with simulations.
- Here, additional validation is provided with real tracks from S3TSR, the Spanish surveillance radar developed, installed and validated by Indra with the funding of CDTI under the technical and contractual management of ESA.
- Manoeuvre information and ephemerides are obtained from ESA/ESOC and DLR/GSOC to assess the results, for a number of selected scenarios.

# Introduction.

- S3TSR: field of visibility



# Introduction.

- Development carried out on OREKIT & MATLAB
  - OREKIT: in JAVA, can be interfaced (with some difficulty  ) with MATLAB
  - MATLAB: Great for data analysis and plotting
- OREKIT is validated software, already used in real missions, and provides:
  - Handling of all required frames at all dates
  - Absolute propagators in cartesian coordinates, modified equinoctial elements or Keplerian classical elements using variable-step numerical methods
  - Perturbations, including non-spherical harmonics of arbitrary order, drag and atmospheric models, solar radiation pressure and third-body effects
  - A Taylor differential algebra framework
  - Handling of common orbital formats such as TLEs and OEMs

# Manoeuvre Detection Metrics

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# Manoeuvre Detection Metrics

- The problem can be mathematically formulated as follows:

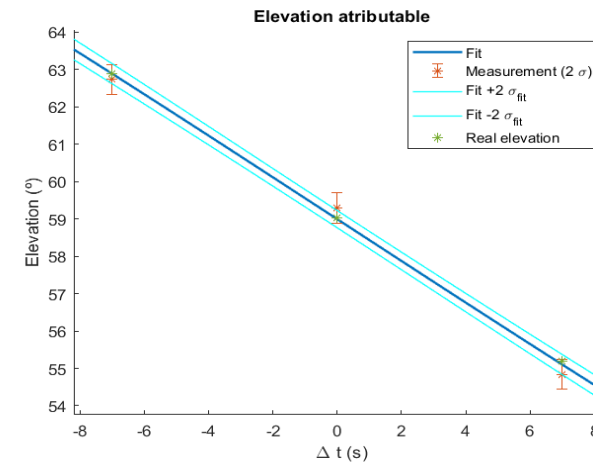
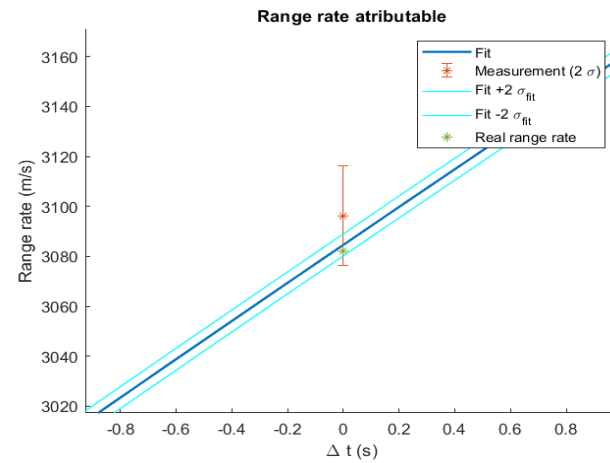
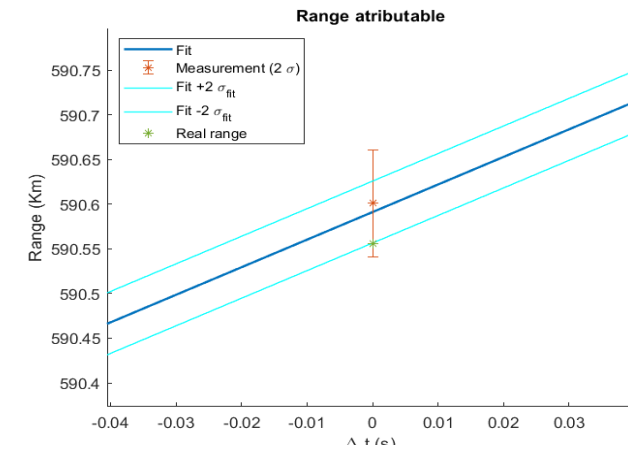
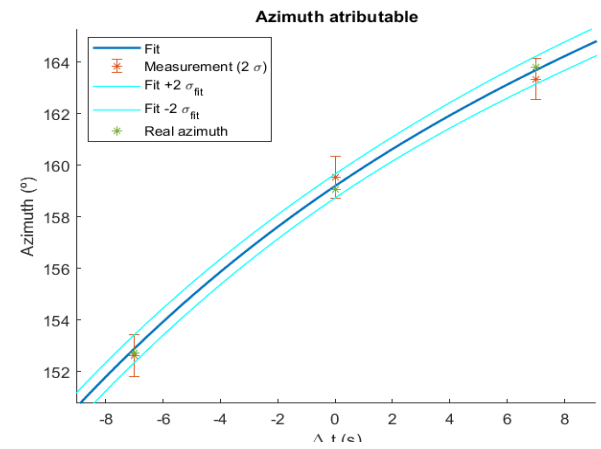
***Given an starting point  $x_0(t_0)$  with associated covariance  $\Sigma_0$  and a radar track (several measurements of range, range-rate, azimuth and elevation with their associated uncertainties) obtained at a time  $t_f > t_0$ , decide whether that a manoeuvre has been carried out or not.***

- To address this problem:
  - Use of attributables to “compress” radar measurements; keep only range and range-rate attributable
  - Metric 1: Mahalanobis distance metric
  - Metric 2: Control distance metric ( $\Delta V$ )

# Attributables

- Attributables example and reduction of error.

Reihs et al., Application of attributables to the correlation of surveillance radar measurements. *Acta Astronautica* 182, 399–415 (2021).



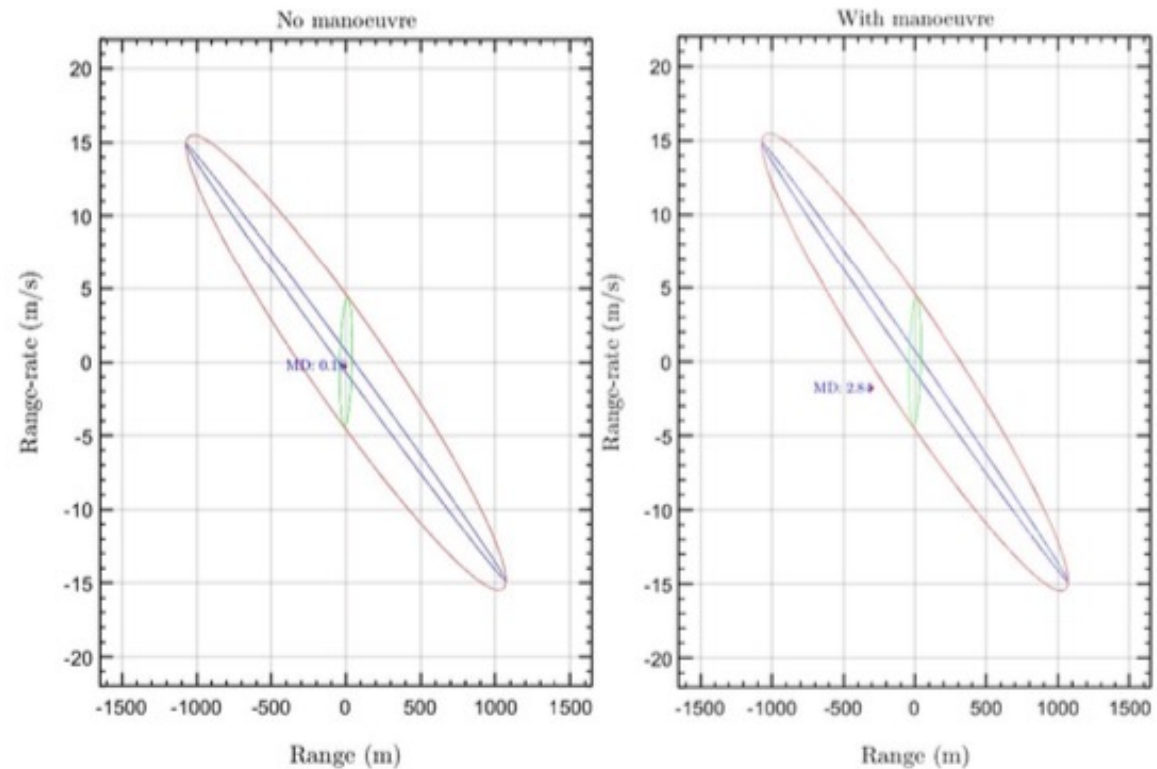


# Mahalanobis distance metric

- Metric 1: Use of Mahalanobis distance to compare real and projected attributable
  - Sample the initial condition of the PDF
  - Propagate the sampled points using an OREKIT propagator up to attributable time with Taylor differential algebra methods, see e.g. Andrea, A. and Maisonobe, L. Automatic differentiation for propagation of orbit uncertainties on OREKIT. In Stardust Conference, (2016).
  - Projected values at attributable time are obtained as a cloud of points.
  - Now for each sampled orbit, one can compute the range and range-rate measurements at the attributable time, obtaining a “cloud of measurements”, and from it mean and covariance.
  - Finally, the attributables and the mean of projected measurements can be compared. If no manoeuvre has been performed, one would expect that both values should more or less agree, thus their difference should be close to zero, and we know an estimation of its covariance.
  - We can then obtain the Mahalanobis distance which will be a metric for manoeuvre detection.

# Mahalanobis distance metric

- Use of Mahalanobis distance(MD)/confidence interval example: no manoeuvre (top) vs. manoeuvre (bottom). Green: measurement uncertainty; blue: orbit uncertainty; red: combination of both.



- Metric 1:** Based on the MD, a probability measure has been computed based on the fact that the square of MD is distributed as chi-square:

$$P_{MD} = \max\{0, 2(\chi^2(MD^2; 2) - 0.5)\}$$

# Control Distance metric

- Metric 2: Use of  $\Delta V$  metric to detect manoeuvres (Singh et al., Holzinger et al.)
  - Consider the following optimal control problem

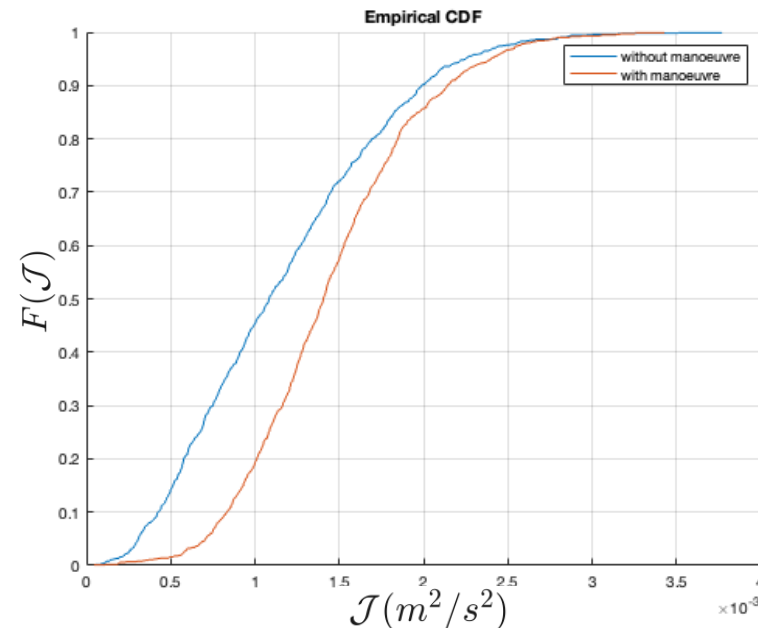
$$\begin{aligned}\mathcal{J} &= \min_{\Delta V_i} \sum_{i=1}^N \Delta V_i^2, \\ \text{s.t. } x'(t) &= f(x(t), u(t), t), \\ x(t_0) &= x_0, \\ h(x(t_f)) &= [\rho \dot{\rho}]^T.\end{aligned}$$

where  $f$  represents the orbital dynamics, with possibly some manoeuvres represented by  $\Delta V_i$ , and  $h$  represents the range and range rate measurement at the time of the attributable.

- The value of  $\mathcal{J}$  represents a metric as shown in the literature which can be used to detect manoeuvres.

# Control Distance metric

- Software package “casadi” (Matlab, interface to the well-known IPOPT solver) used to find the solution to the deterministic problem. Uncertainty added by solving it a number of times (Monte Carlo-style simulation), thus one obtains a distribution of  $\Delta V$  represented as a CDF.
- Example of CDF with/without manoeuvre:
- **Metric 2:** First compute a baseline for each scenario doing a Monte Carlo analysis without manoeuvre. Percentiles 0.1,0.5,0.8 computed for both the mean distribution (M) and the mean + 2 sigma (D) distributions.



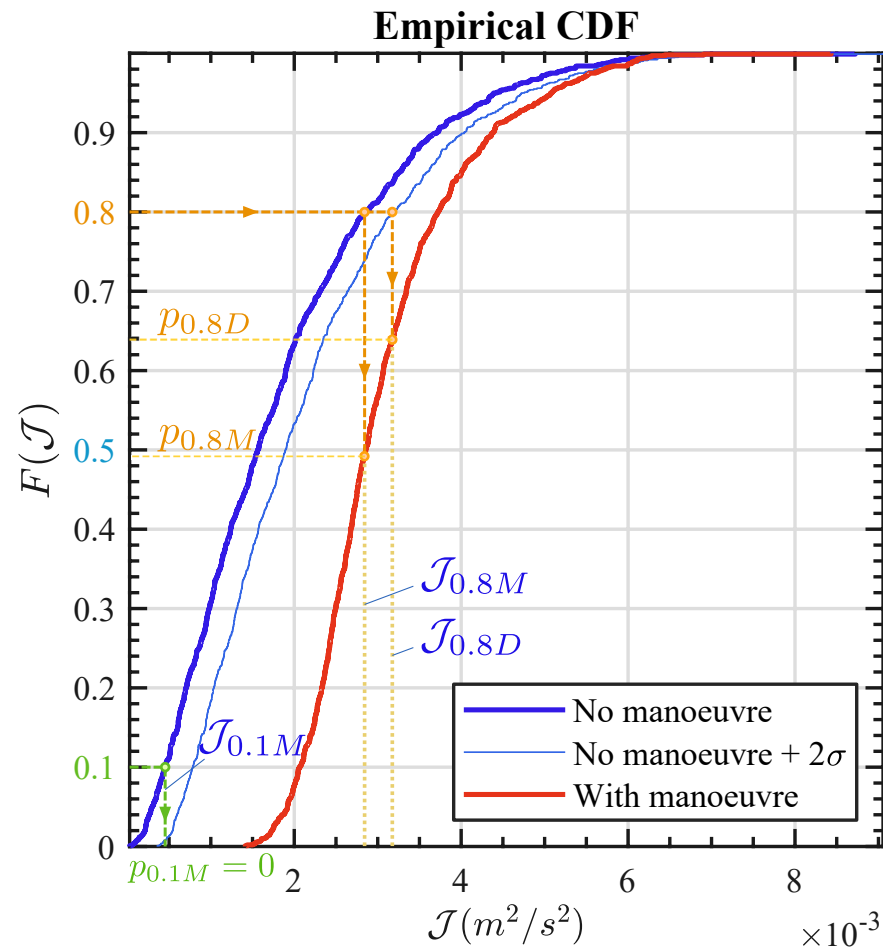
# Probability Metric 2 (based on $\Delta V$ )

- Algorithm 2: Six metrics were considered:
  - For P1M (and P1D), compute the probability of  $\Delta V$  being below the percentile 0,1 of mean (respectively, 2-sigma) non-manoeuved distribution. Then  $P1 = \max\{0, 10(0.1 - p)\}$ . If  $p$  is more than 0.1 the probability becomes zero and if it is below 0.1, the difference is multiplied by 10 (thus having zero probability would represent a 100% probability of manoeuvre).
  - P5M and P5D are computed similarly for the 0.5 percentile.
  - P8M and P8D are computed similarly for the 0.8 percentile.

In simulations it was found that P1M and P1D perform best.

# Probability Metric 2 (based on $\Delta V$ )

- Example:



# Results



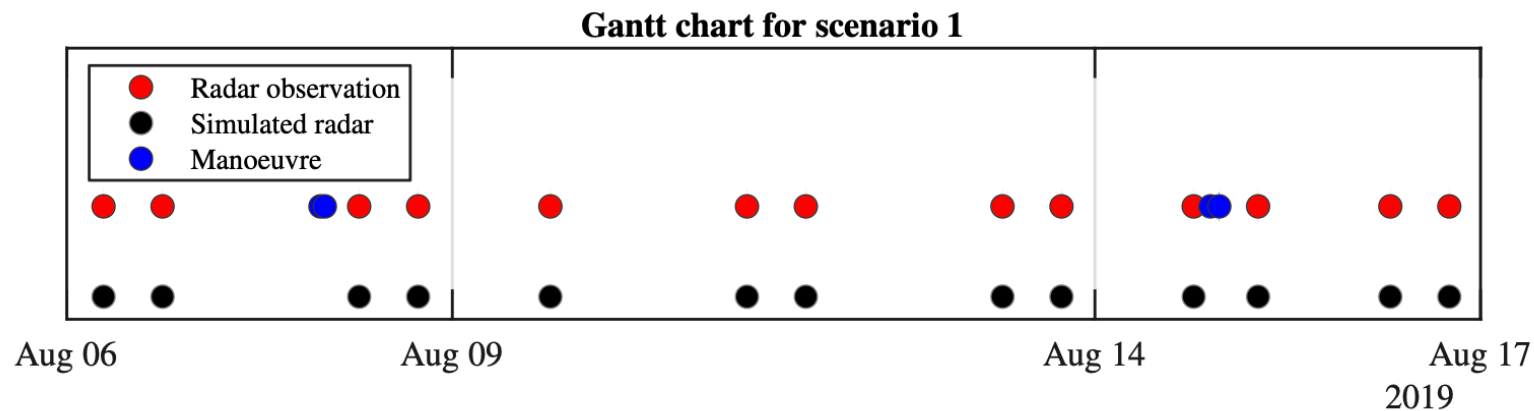
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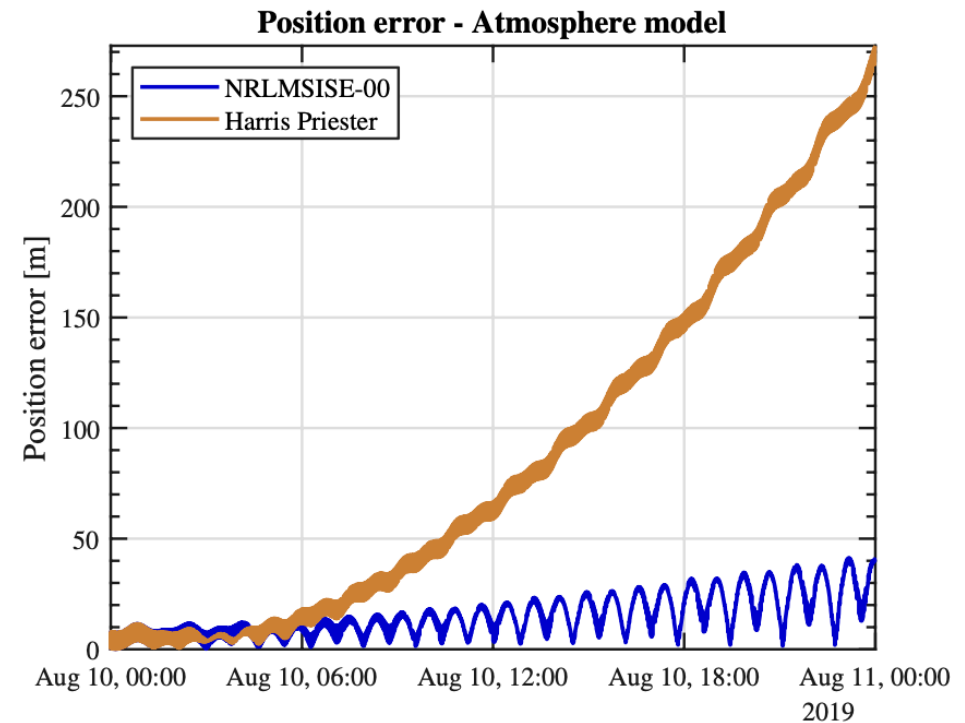
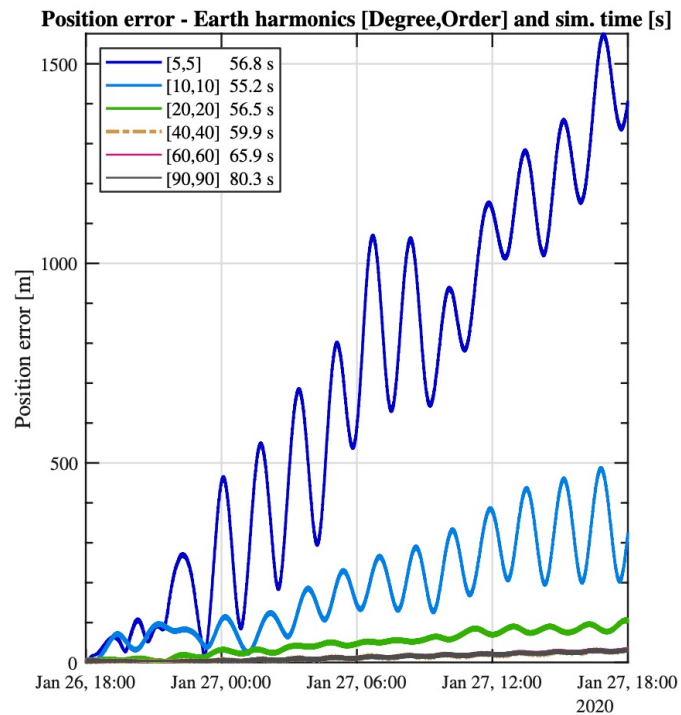
# Scenarios

- A total of 15 scenarios were selected, using real radar data obtained from S3TSR, and manoeuvre information and precise ephemerides are obtained from ESA/ESOC and DLR/GSOC.
- The satellites under consideration were Sentinel-1A, Sentinel-1B, Sentinel-2A, Sentinel-2B, Swarm-C, TerraSAR-X and TanDEM-X (the last two are formation-flying -> challenge for the radar).
- In total, 158 segments (one radar track up to the next -- from 12 hours to more than 48 hours in some cases), 24 of them with manoeuvres and 134 without manoeuvres. The manoeuvres are rather small (0.1 m/s or less).



# Scenarios – propagation for real orbits

- From the precise ephemerides, an OREKIT propagator was developed to best fit the OEM ephemerides by taking into account gravity harmonics (up to order 40), third-body perturbations (Sun and Moon), solar radiation pressure and drag.



# Results: Metric 1 (Mahalanobis distance)

If the metric is equal or higher than 0.5 a manoeuvre is considered to have been performed.

% Man. detected	% False positives	% False negatives
41.66	2.98	58.34

Analysing the results, the causes of errors are as follows:

- From a total of 5 false positives, all except one present less than 10 plots in the track following the manoeuvre. **Thus, the main cause of false positives is tracks with fewer plots.**
- From a total of 14 false negatives, all except one were segments of length equal to or larger than one day. **Thus, the main cause of false negatives is longer propagations accumulating additional propagation error.** Sometimes these longer propagation periods are due to missed radar observations right after the manoeuvre.

# Results: Metric 2 ( $\Delta V$ )

If the metric is equal or higher than 0.5 a manoeuvre is considered to have been performed.

#Manoeuvres	#Segments w/o Manoeuvre	P1M % Man. detected	P1M % False positives	P1M % False negatives
24	134	54.16	45.84	8.21

Better results than metric 1, but the price to pay is more false positives.

Very similar conclusions as metric 1 with regards to causes of error.

# Conclusions & Future Work



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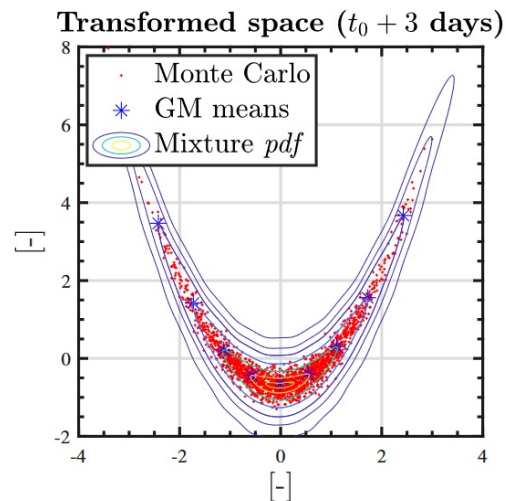
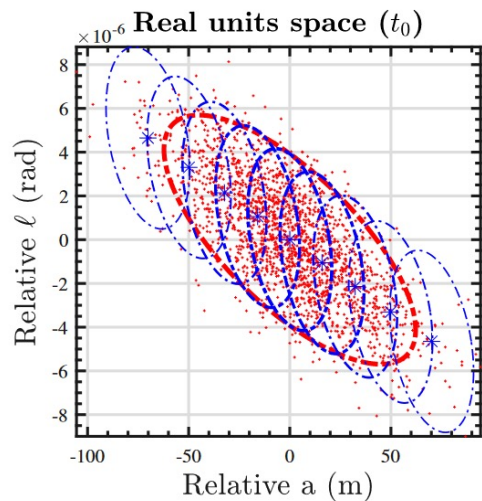
# Conclusions & future work

- The obtained results were very good in simulation, but the real data results are in need of improvement. The main identified difficulty was the scarcity of measurements (low number of tracks and/or low number of plots) which was to be expected as we had a single data source. Future expansion of the capabilities of S3TSR may improve the quality of the metrics, as well as considering other sources of data (laser).
- For more details: **Montilla, Jose M., et al. "Manoeuvre detection in Low Earth Orbit with Radar Data." Advances in Space Research (2024).**
- Extensions on the algorithms: other distances (e.g. Bhattacharyya distance), directly addressing the stochastic optimal control problem of metric 2... and see next slides!
- Manoeuvre detection filter: not presented here, but also developed in the project, combines orbit determination and manoeuvre detection
- The long-term aim would be to have these metrics **integrated in the S3T Cataloguing System**

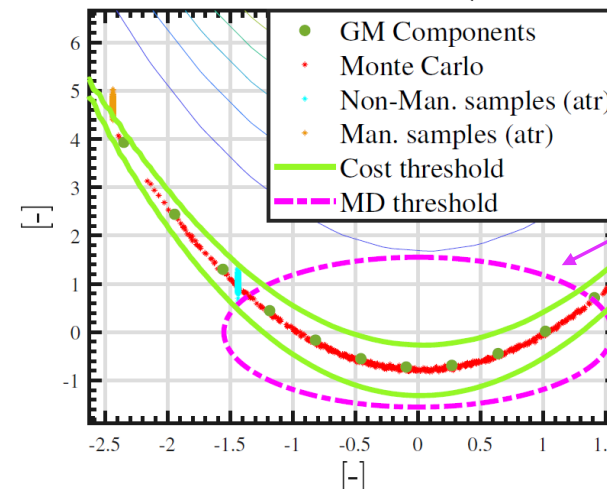
# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Manoeuvre Detection (JM Montilla PhD)

- Collaboration at Polimi (Pierluigi de Lizia): use of **Gaussian Mixtures** to improve covariance realism. Projecting prediction on measurement space and compare with **attributable** of radar track: **cost metric**.



Cost metric distribution ( $t_0 + 3$  days)



More realistic than MD

$$\mathcal{PE} = \int p_1(\mu)p_2(\mu)d\mu = \sum_{i=1}^N \omega_i \mathcal{N}(\hat{\mu}_i; \mu_{\mathcal{A}}, \hat{\Sigma}_i + \Sigma_{\mathcal{A}})$$

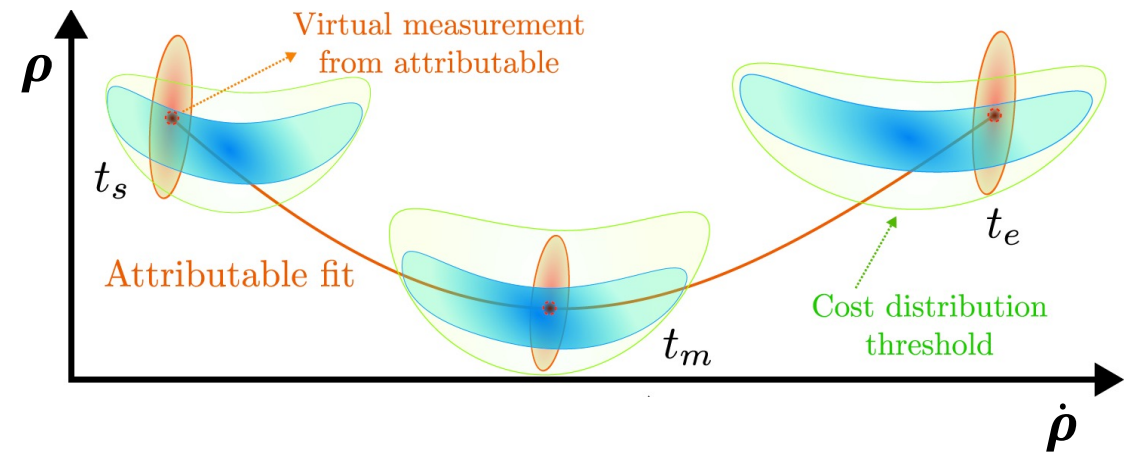
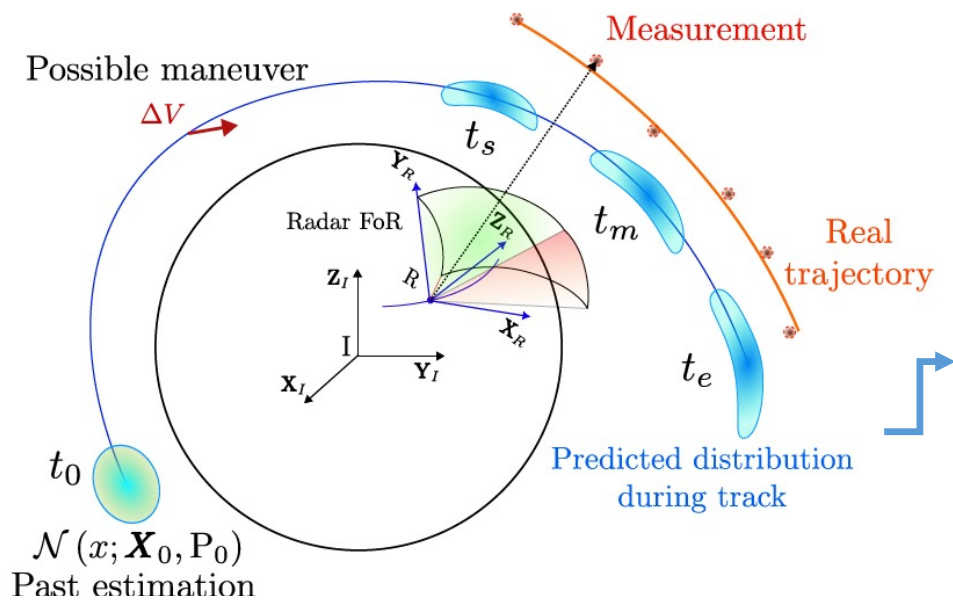
$$c = -\ln \mathcal{PE}$$



# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Manoeuvre Detection (JM Montilla PhD)

- Collaboration at Polimi (Pierluigi de Lizia): the **evolution of the cost** metric is analysed to overcome dimensionality loss.

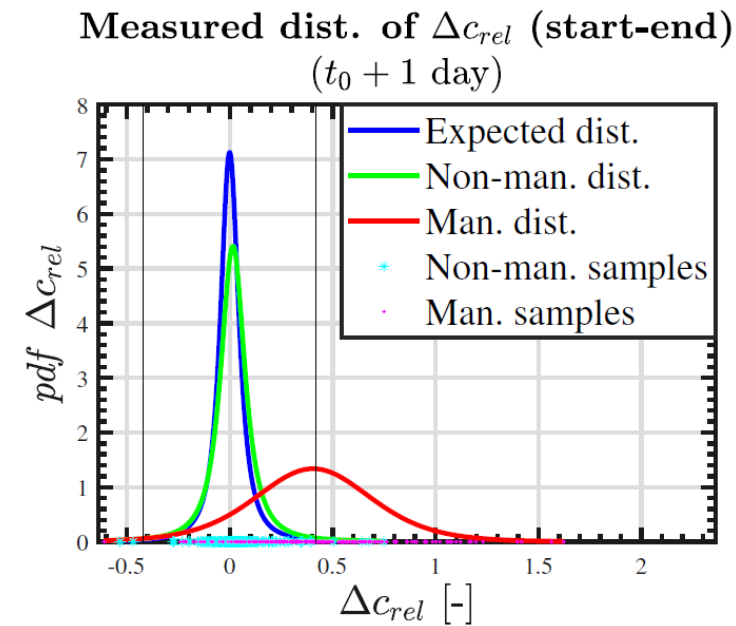
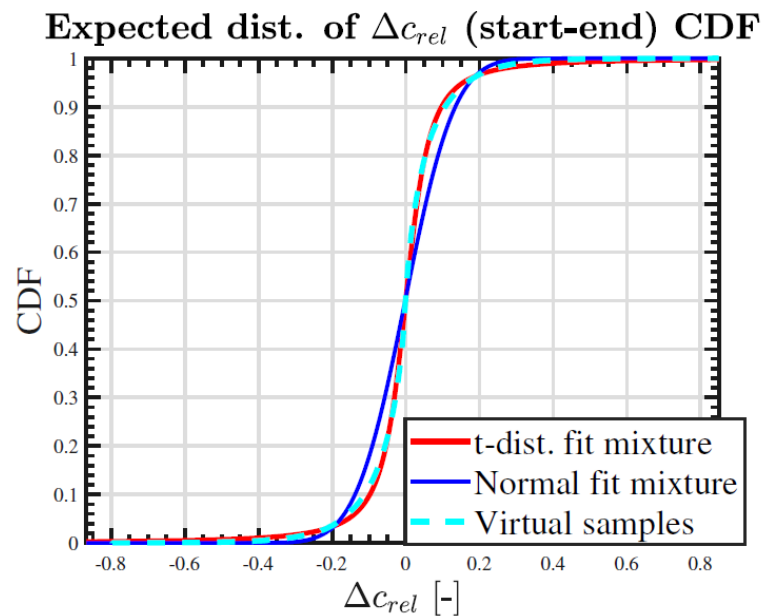


$$\Delta c_{rel}(t_a, t_b) = \frac{c(t_b)}{c_{gm}(t_b)} - \frac{c(t_a)}{c_{gm}(t_a)}$$

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## Manoeuvre Detection (JM Montilla PhD)

- Collaboration at Polimi (Pierluigi de Lizia): the **evolution of the cost** metric is analysed to overcome the dimensionality loss.



# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Manoeuvre Detection (JM Montilla PhD)

- Collaboration at Polimi (Pierluigi de Lizia): High-fidelity synthetic **simulations** of Sentinel-1A like orbit

Combining cost and  $\Delta C$

[%]	$MD_{0.7}$	$c_{0.7}$	$\Delta c_{0.98}^{se}$	$\Delta c_{0.98}^{sm}$	$\Delta c_{0.98}^{me}$	$\Delta c_{0.98}^{all}$	$c + \Delta c$
TRUE POSITIVES	65.72	60.4	53.68	46.31	48.39	60.38	65.94
FALSE POSITIVES	20.42	1.5	0.73	1.02	0.69	1.65	2.79

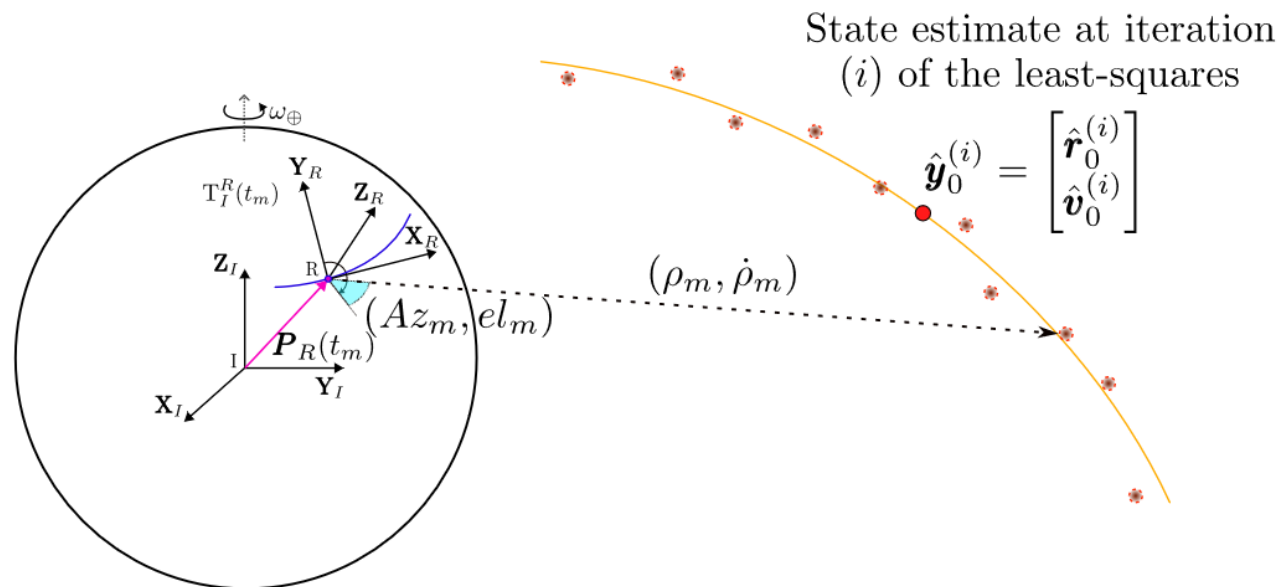
[%]	$c_{0.4}$	$c_{0.45}$	$c_{0.5}$	$c_{0.55}$	$c_{0.6}$	$c_{0.65}$	$c_{0.7}$
TRUE POSITIVES	70.32	68.33	66.45	64.81	63.21	61.76	60.4
FALSE POSITIVES	8.64	6.65	4.93	3.967	2.84	2.05	1.5

Cost threshold change gets **worse** FP rate

# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Manoeuvre Detection (JM Montilla PhD)

- Recent stay at Space Debris Office (Jan Siminski): Improving **IOD** using single radar tracks by incorporating range-rate information and using non-Keplerian models.

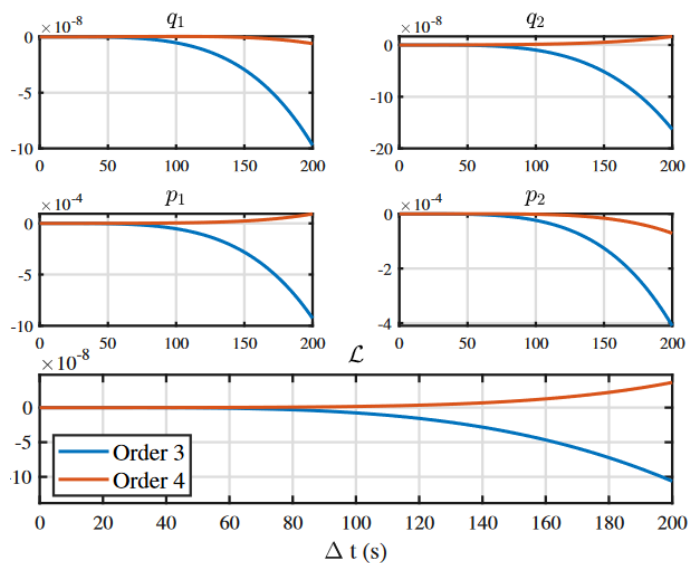


# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

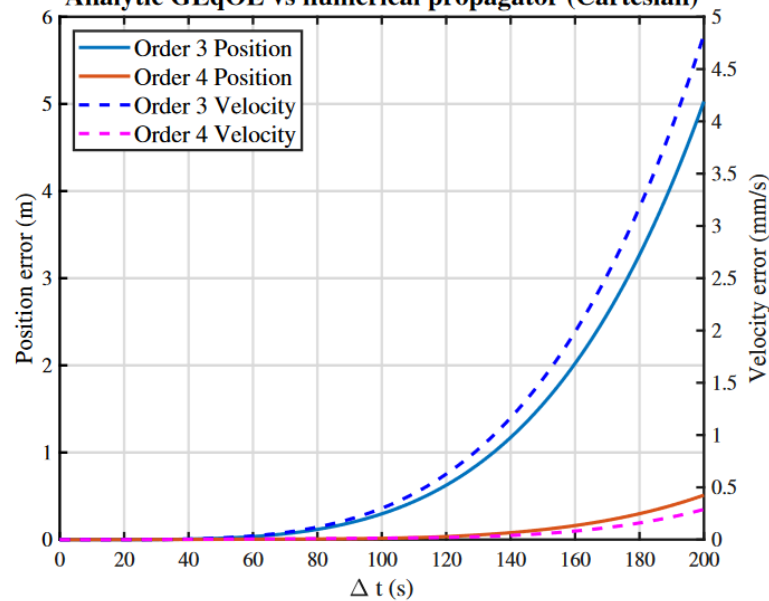
## Manoeuvre Detection (JM Montilla PhD)

- Recent stay at Space Debris Office (Jan Siminski): Implementation of analytic **General Equinoctial Orbital Elements** Taylor expansion propagator with **J2** (short-term validity).

Analytic GEqOE error relative to numeric propagation



Analytic GEqOE vs numerical propagator (Cartesian)



Analytical **error-state transition matrix** from the polynomial needed for least-squares fitting

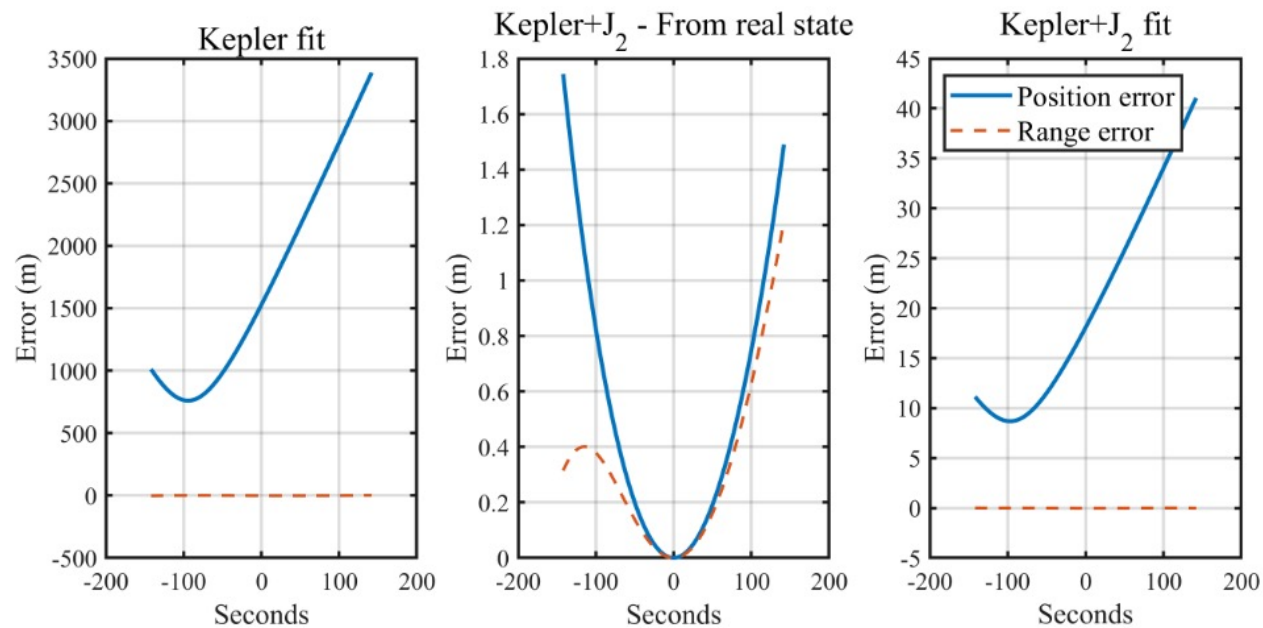
$$\frac{\partial \mathbf{X}}{\partial \mathbf{X}_0} = \frac{\partial \mathbf{X}}{\partial \boldsymbol{\chi}} \Big|_t \left( \frac{\partial \boldsymbol{\chi}}{\partial \boldsymbol{\chi}_0} \right) \frac{\partial \boldsymbol{\chi}}{\partial \mathbf{X}} \Big|_{t_0}$$

# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Manoeuvre Detection (JM Montilla PhD)

- Fitting radar observables directly needs  $J_2$  perturbation for long tracks

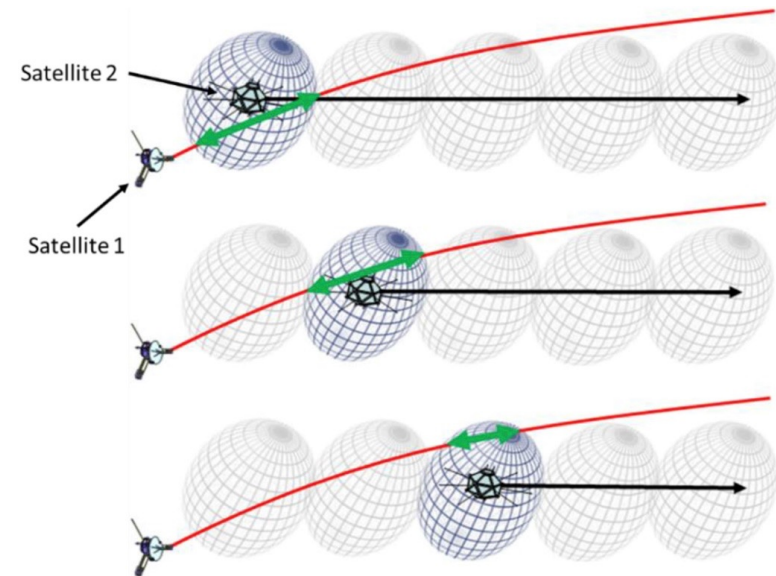
Fitting of real radar observables



# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- Ongoing project in collaboration with Prof. Claudio Bombardelli at UPM
- Conjunction Screening is becoming an increasingly challenging task:
  - Large constellations being deployed
  - Growing numbers of space debris
  - Risk of losing a satellite in a collision not negligible!
- Development precise and fast propagators to predict close encounters over a population of objects in orbit



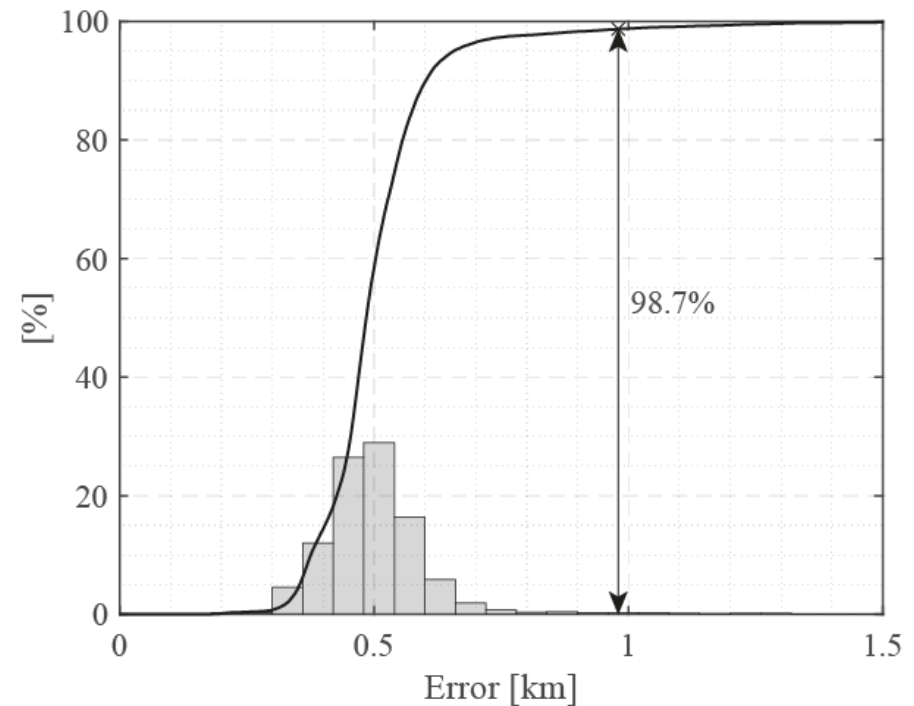
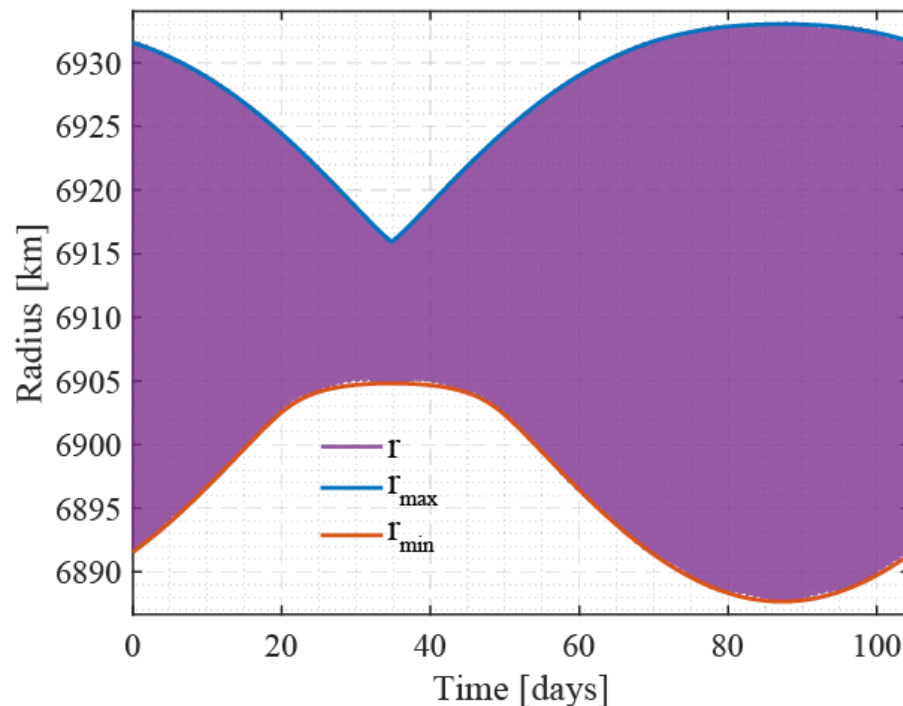




# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- SO-filter: Improves the **classical apogee-perigee** filter at very low computational cost



# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

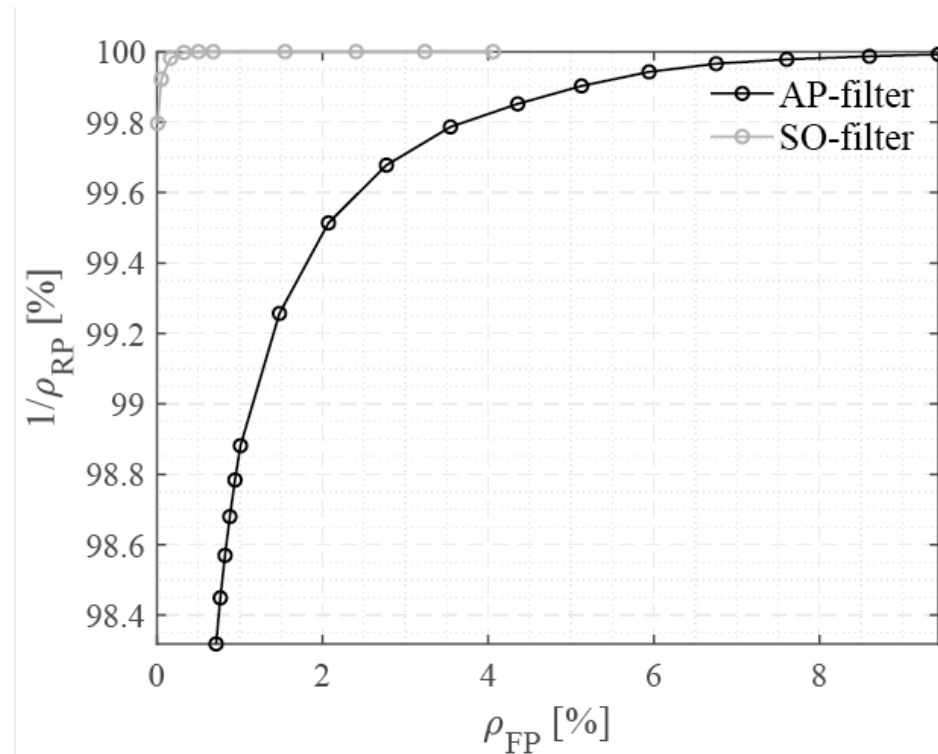
## Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- SO-filter: Improves the classical apogee-perigee filter at very low computational cost
- False positives to detected real positives ratio:

$$\rho_{FP} = \frac{N_{FP}}{N_{RP} - N_{FN}}$$

- Fraction of detected real positives:

$$\frac{1}{\rho_{RP}} = \frac{N_{RP} - N_{FN}}{N_{RP}} = \frac{1}{1 + \rho_{FP}}$$



# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- Comparison of the filter performance considering the SO-filter and the classical AP-filter on a population of **16 972** orbits:  
**144 151 710** potential pairs to check for conjunctions.

Filter	False Positives	False Negatives	$\rho_{FP}$	$\eta^*$
AP-filter	5 507 984	0	17.239 %	73.990 %
SO-filter	530 720	0	1.661 %	77.446 %
SO-filter exact	524 183	0	1.641 %	77.451 %
SO-filter raw	2 145 317	0	6.714 %	76.325 %

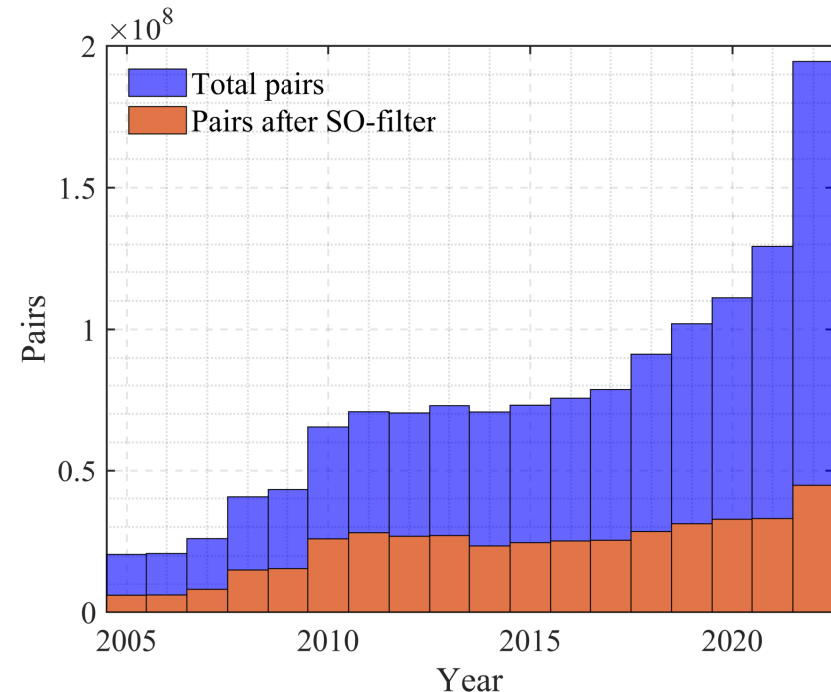
- Pairs to check after the SO-filter: **32 481 309** pairs

\*  $\eta$  Percentage of pairs eliminated by the filter compared to the total number of pairs.

# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

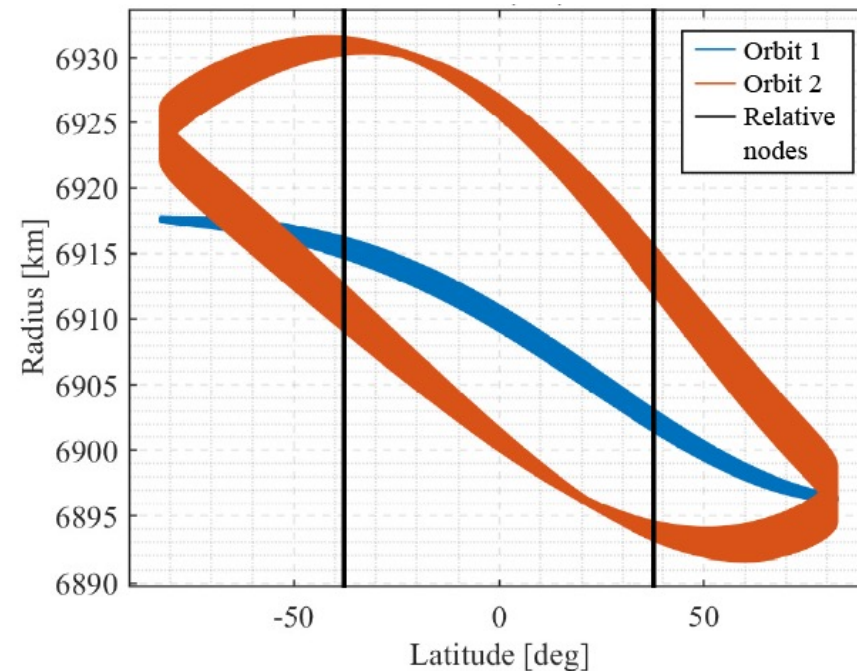
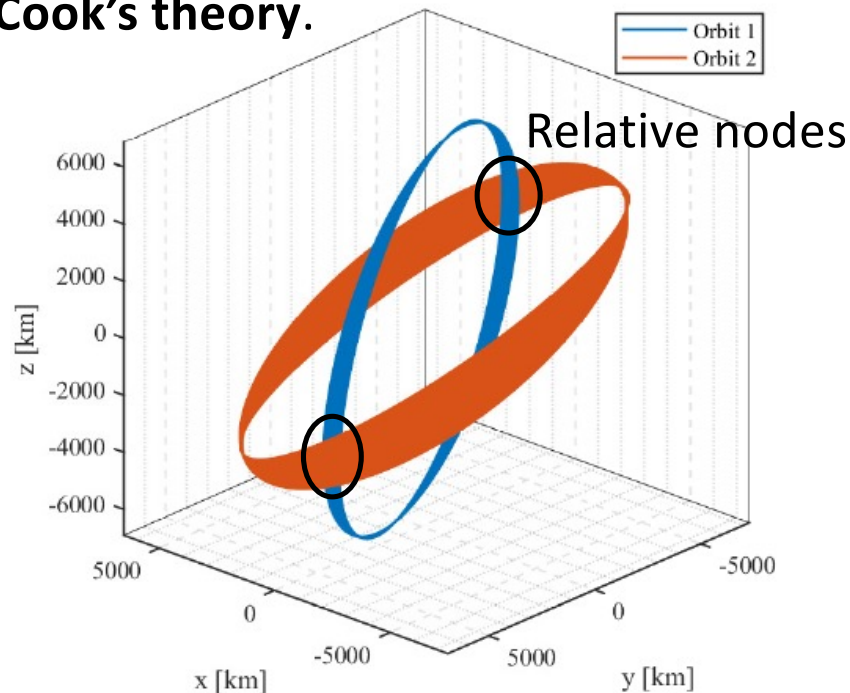
- Same tools can be used to study how space population has evolved over the years.
- Analysis of the evolution from 2005 to the present using the space occupancy tools
- The number of objects in space has been rapidly increasing. However, the number of objects sharing their space occupancy has not evolved in the same way!
- More details in our preprint:  
<https://arxiv.org/abs/2309.02379>



# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

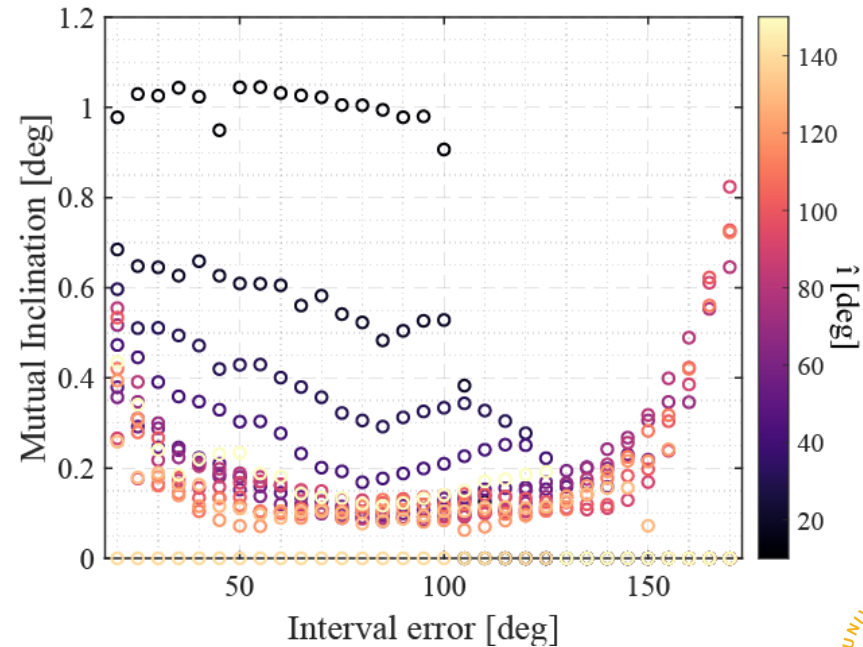
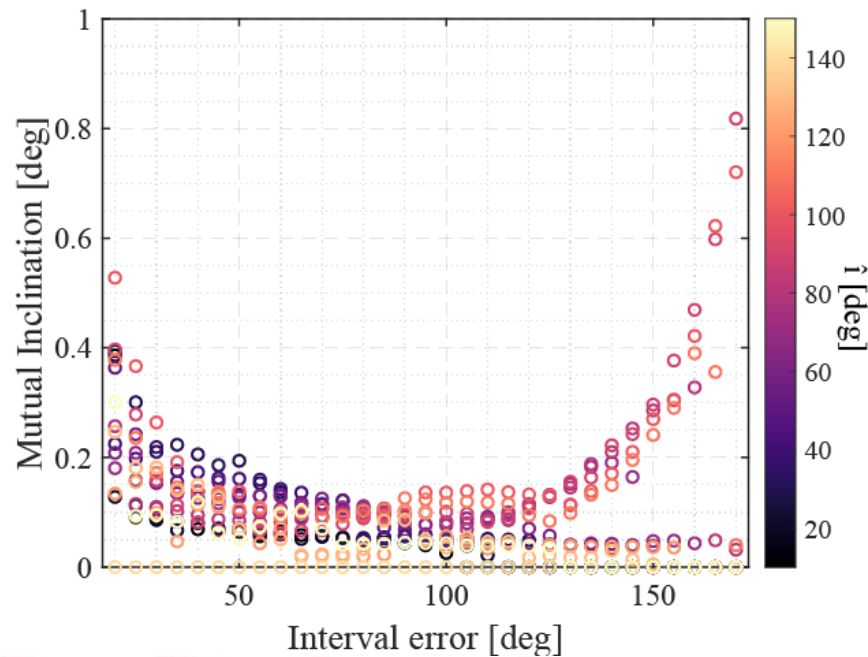
- Collaboration at the University of Pisa (Giulio Baù): Development of the second conjunction filter, classical **orbit path filter**, based on the **Space Occupancy concept** and **Cook's theory**.



# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- SO Orbit path filter: Analysis of the error in the computation of relative nodes, and its dependence on mutual inclination and orbital inclination.

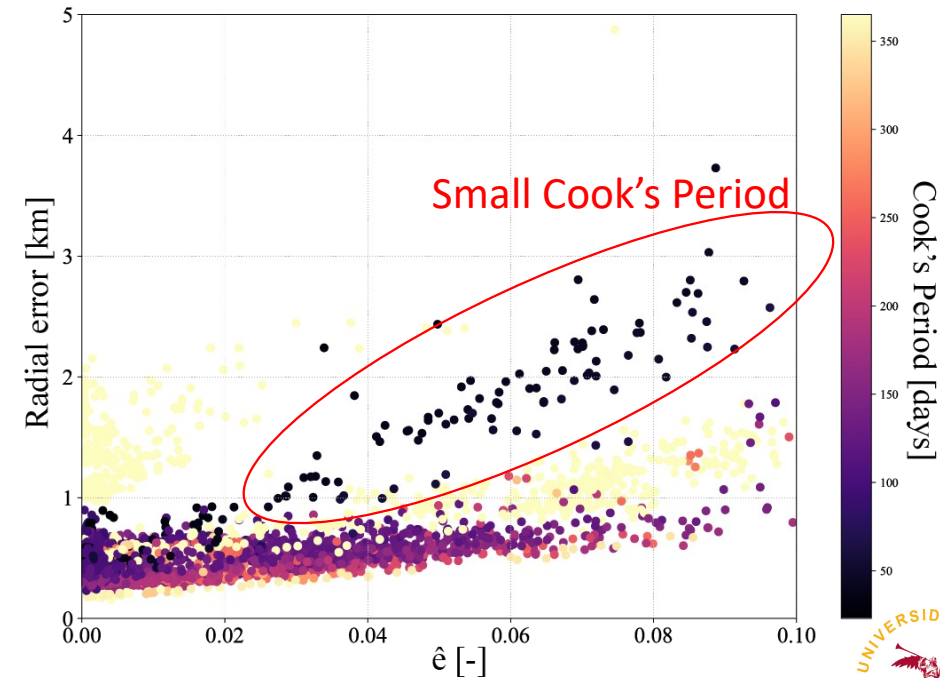
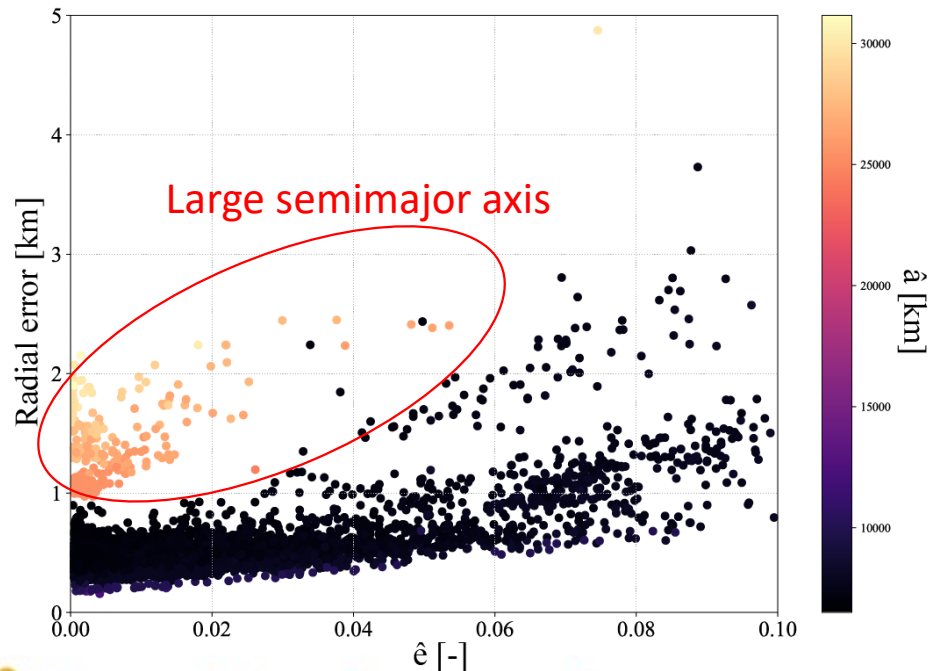




# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

- SO Orbit path filter: Analysis of the radius error and its dependence on eccentricity, semimajor axis, and Cook's period.





# SSA Activities at GIA (Grupo de Ingeniería Aeroespacial) at Univ. Sevilla

## Conjunction analysis & Conjunction screening (Ana S. Rivero PhD)

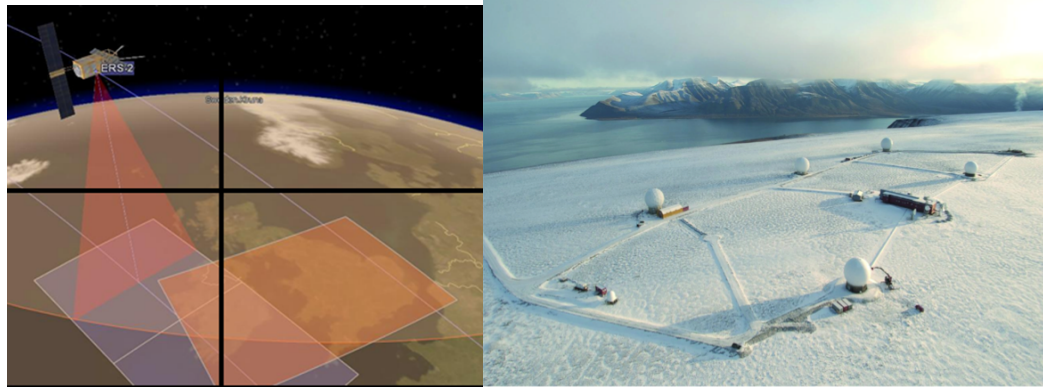
- SO Orbit path filter

- Pairs to check: **32 481 309 pairs**
- Coplanar: 1 650 186 pairs
- Pairs with one equatorial orbit: 9 212 pairs

Negatives	23 084 673
Positives	7 737 238
False Negatives	0
False Positives	1 290 591
$\eta$	71.07 %

- Pairs to check after the filter: **9 396 636 pairs**
- After applying the **two filters**, **only 6.52 % of the initial pairs** need further analysis

# Optimization in Space



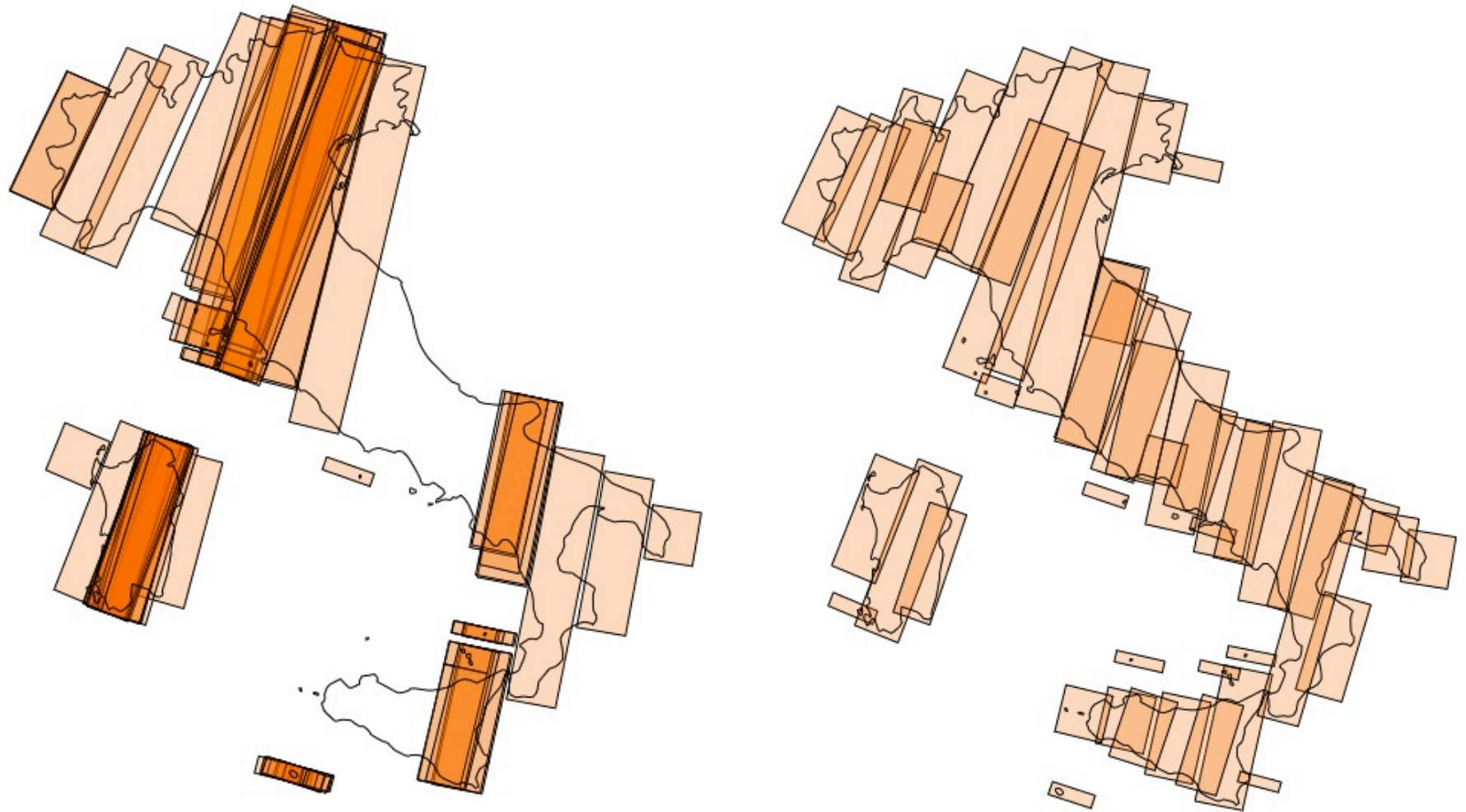
- ▶ Project 1: Design and integration of a real time optimal planner for swath acquisitions for **multiple** EOS satellites.

Swath acquisition planning in multiple-mission EOSs: exact and heuristic approaches. IEEE Trans. Aerospace Elec. Systems (51) 1717-1725. 2015.

- ▶ Project 2: Minimize the number of cancellations of satellite-antenna requests due to **conflicts** in **real time**.

Resolution of an Antenna-Satellite assignment problem by means of Integer Linear Programming. Aerospace Science and Technology (39) 567-574, 2014

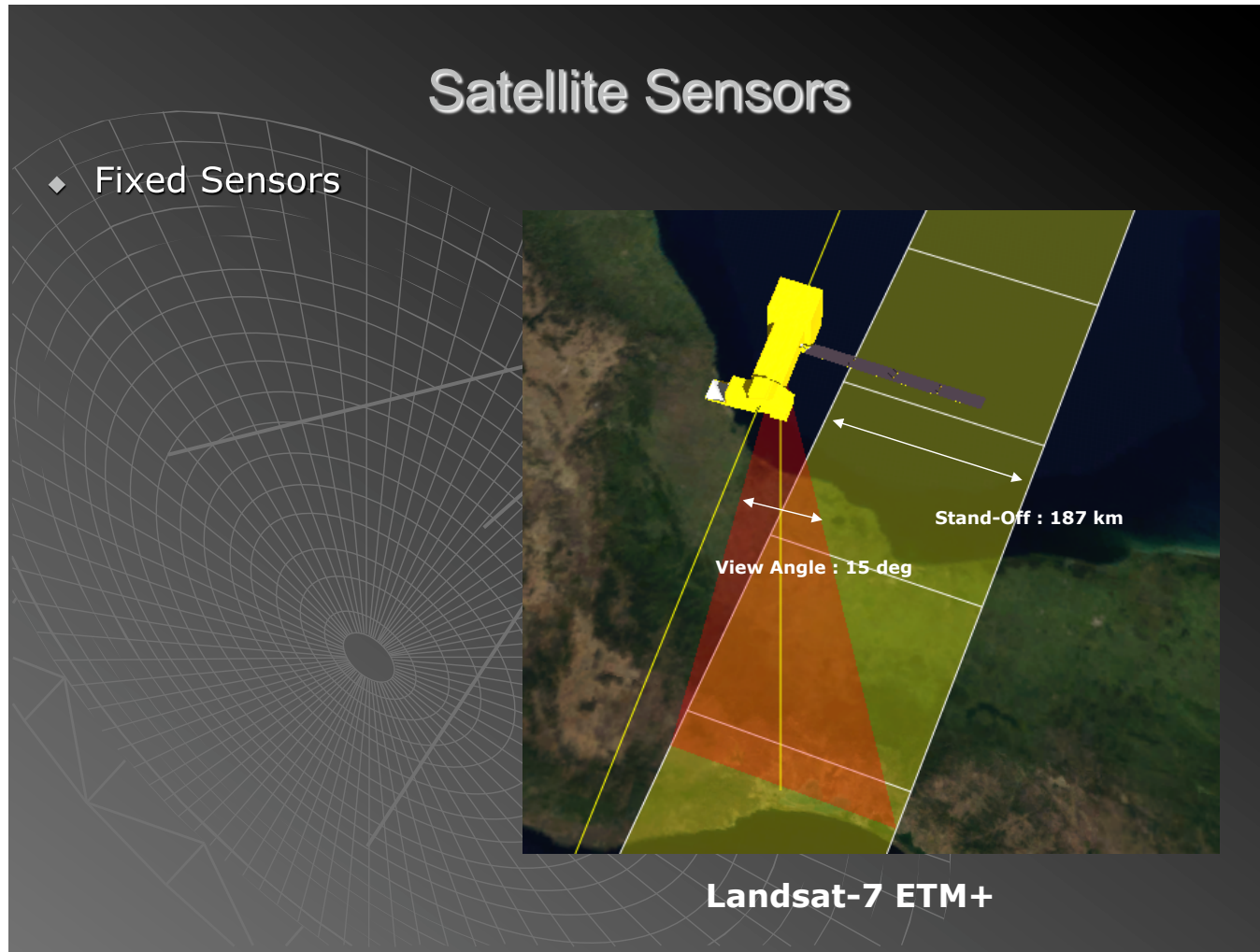
# Project 1: swath acquisitions to cover an area



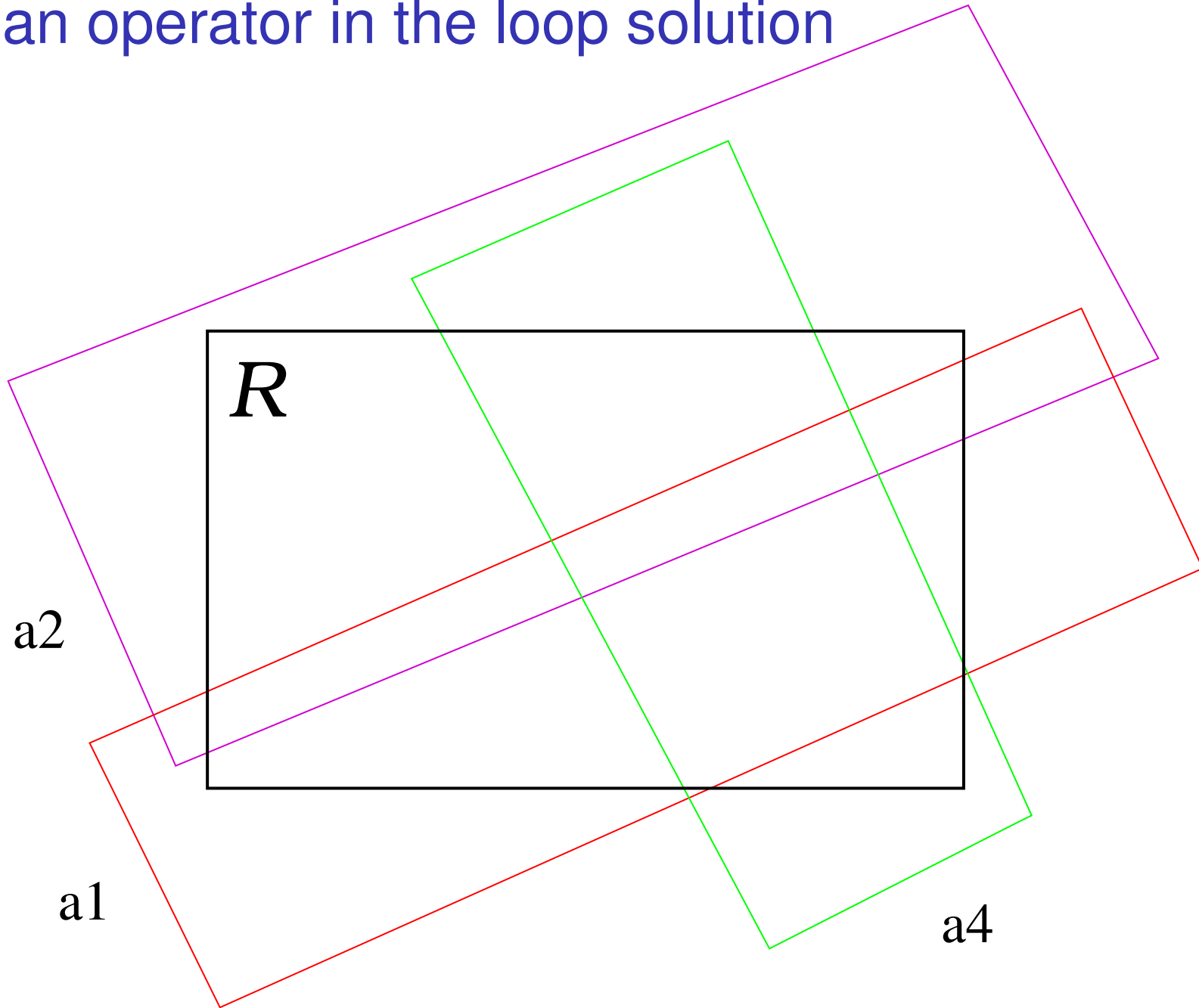
How do we select the images?

How close is a feasible solution to the optimal one?

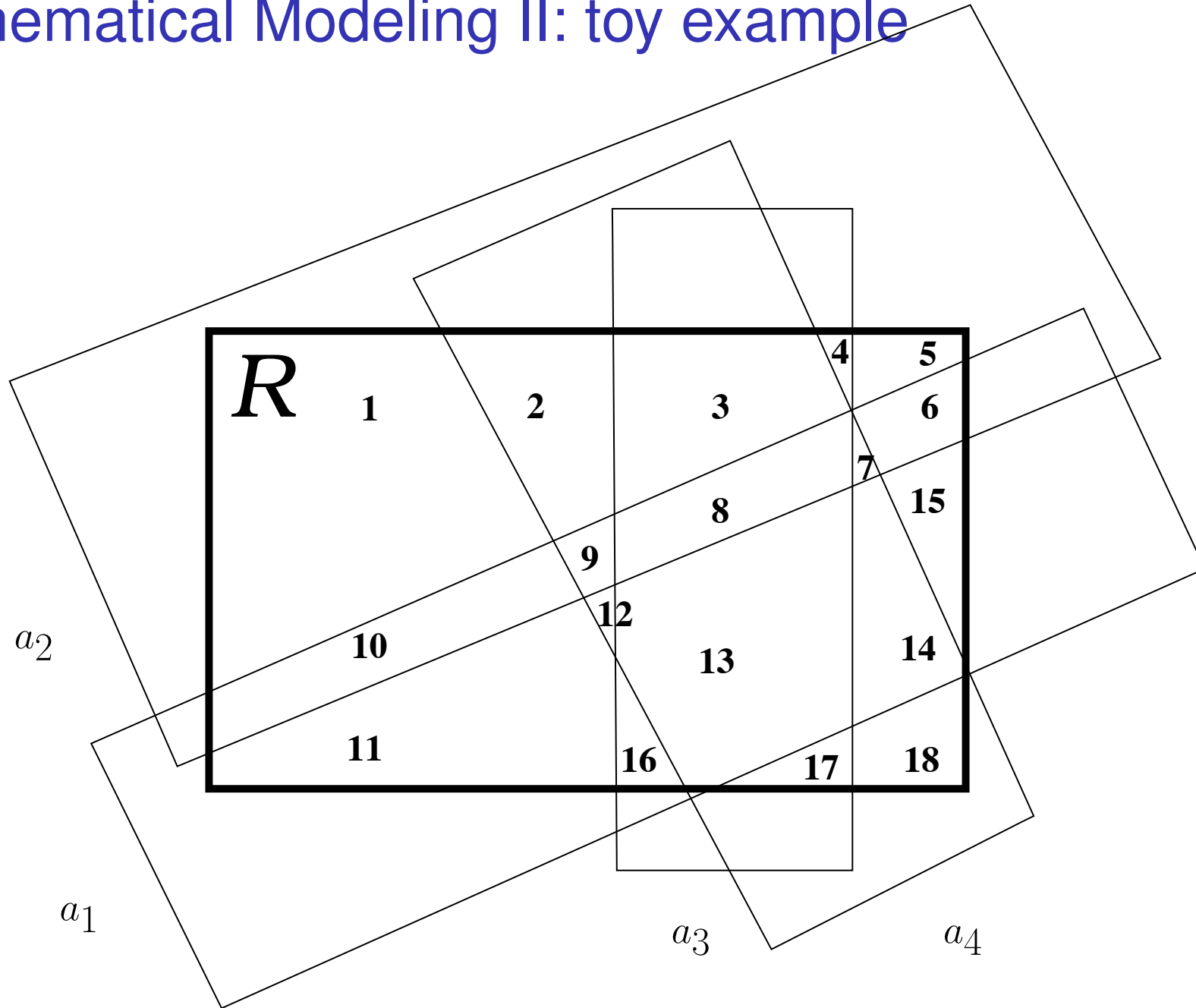
# Mathematical Modeling I:



# Human operator in the loop solution



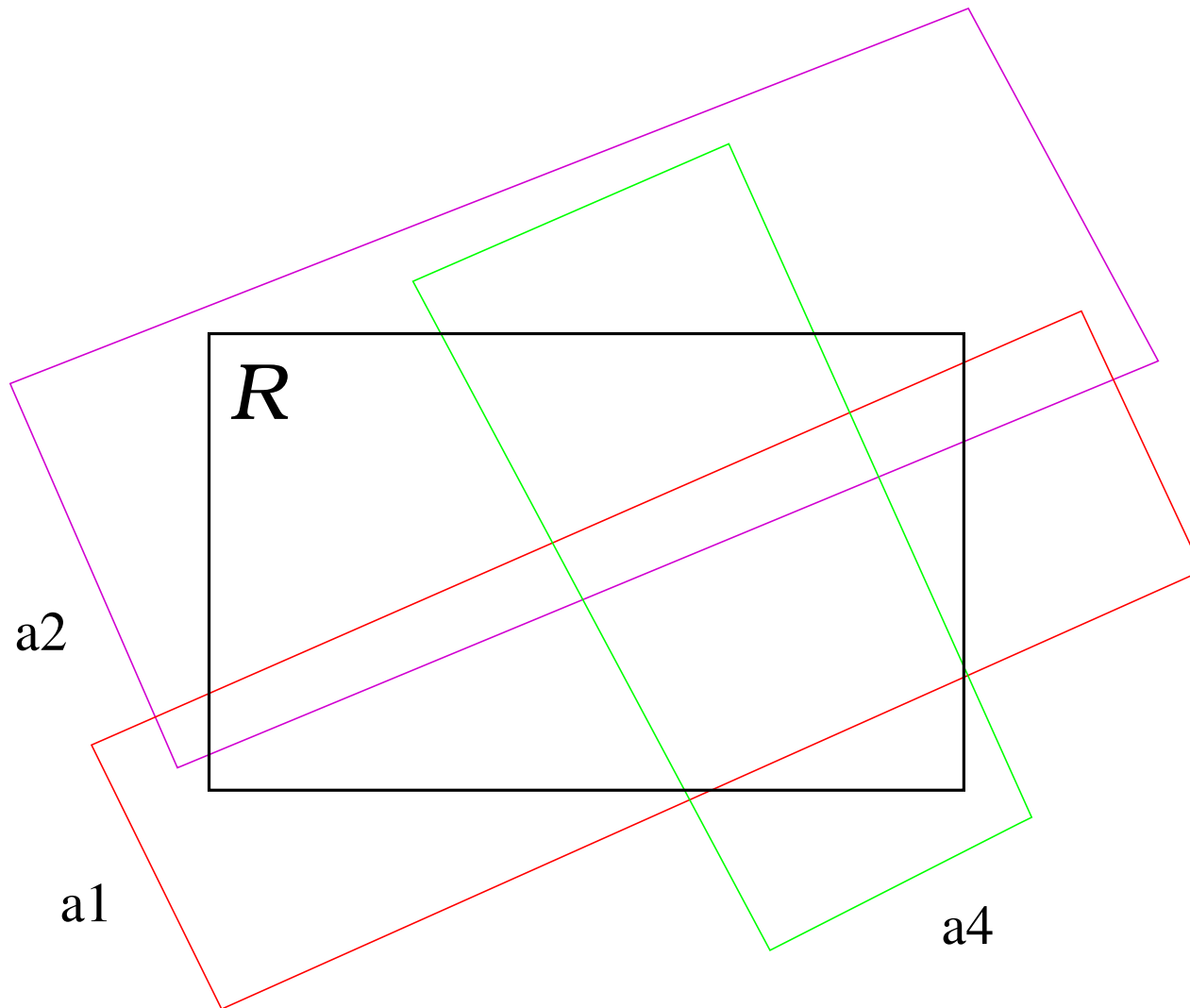
# Mathematical Modeling II: toy example

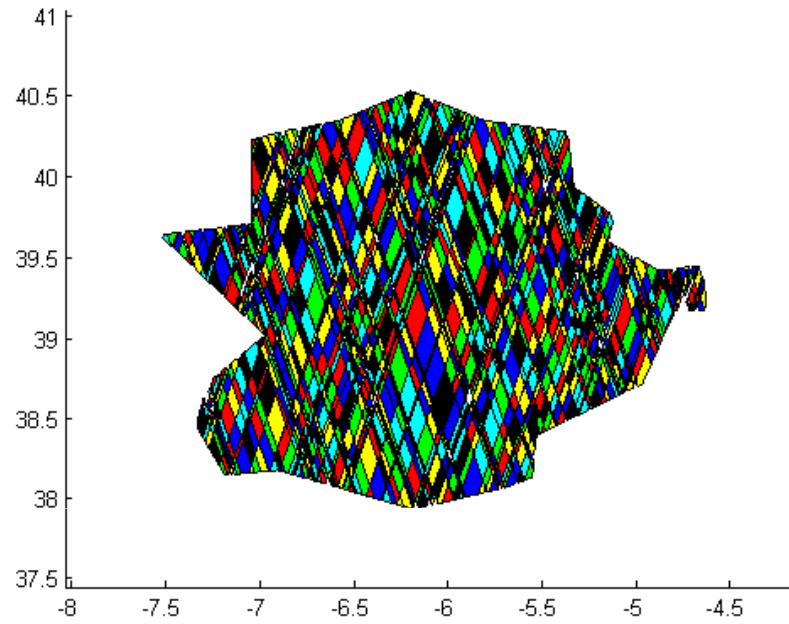


# Optimal solution

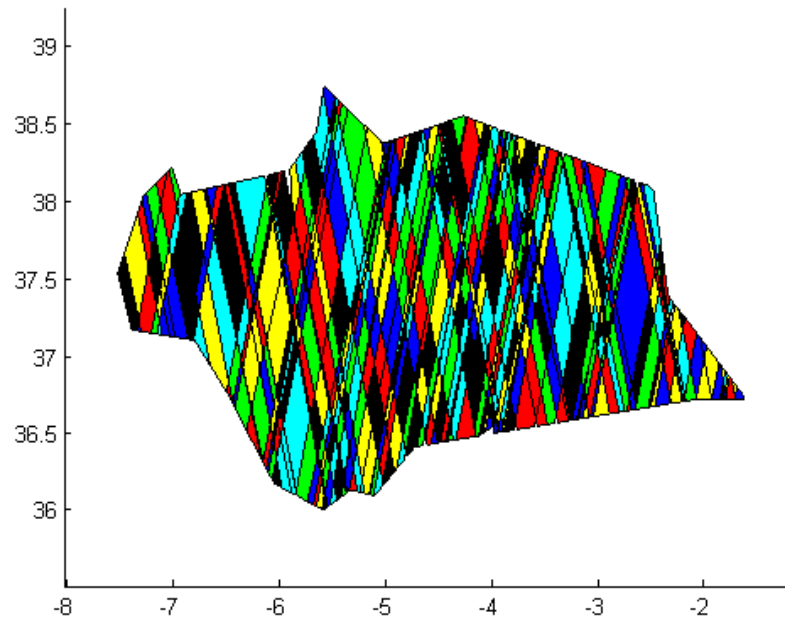
Given the region  $\mathcal{R}$  and the set of acquisitions  $\{a_1, a_2, a_3, a_4\}$

the optimal solution is  $\{1, 1, 0, 1\} = \{a_1, a_2, a_4\}$





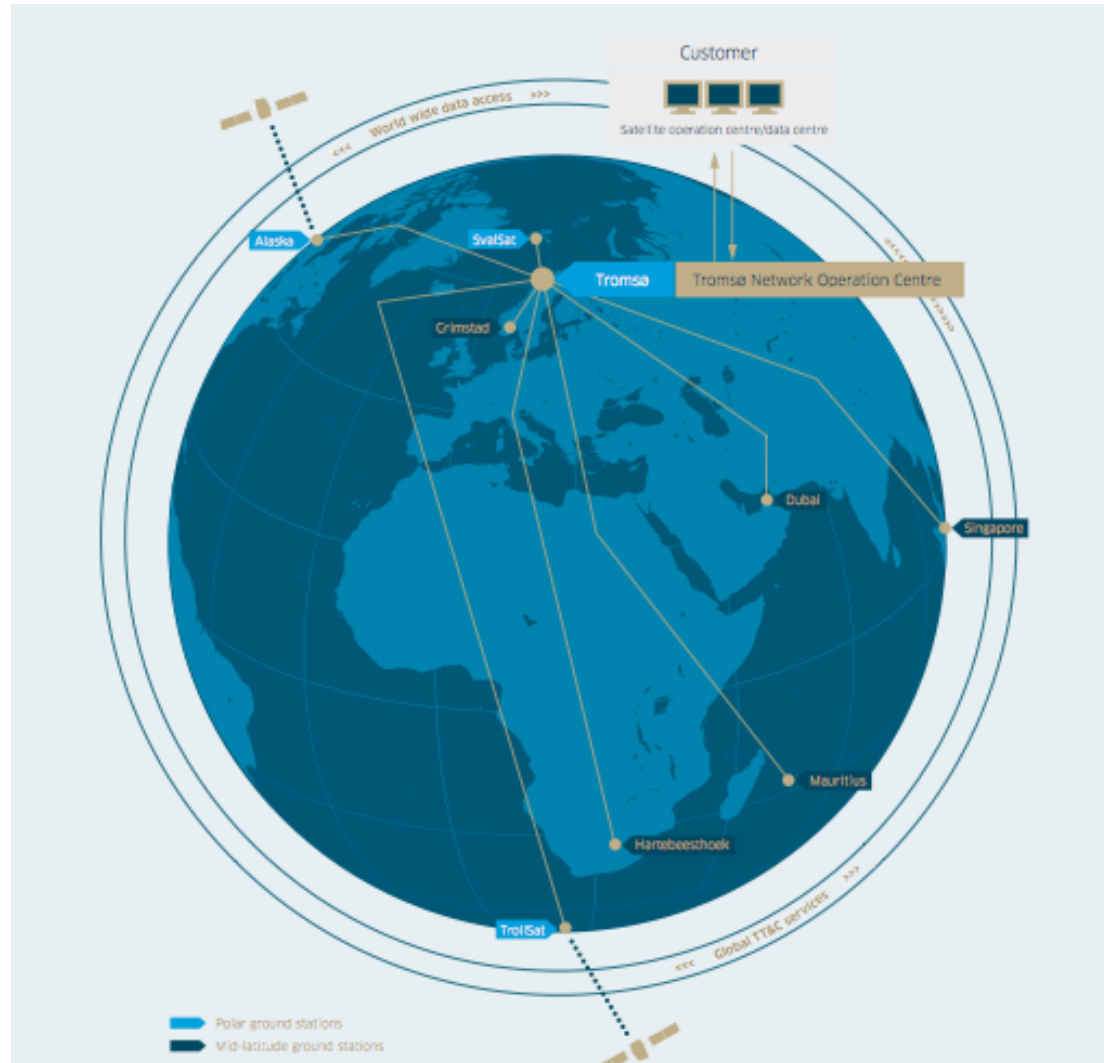
More complicated regions and acquisitions: Extremadura, Andalucia



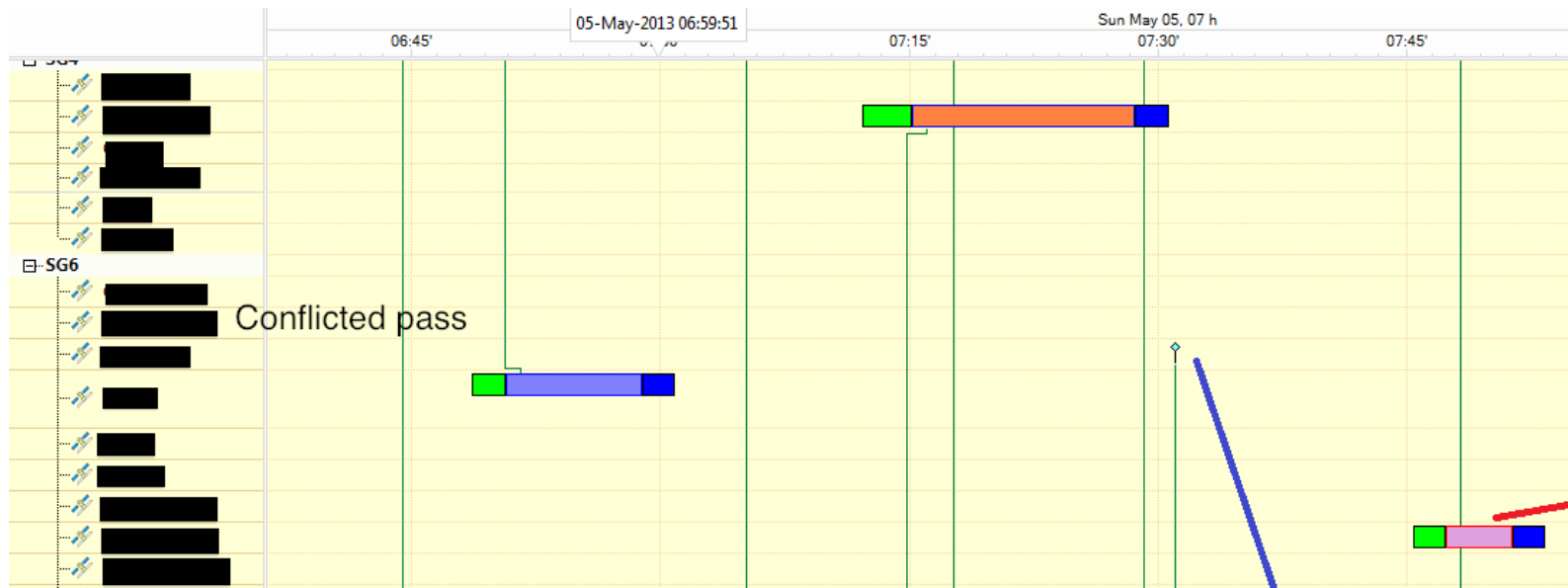


# Project 2: Satellite Range Problem

## Ksat Global Network



# Some orders of magnitude



- ▶ # antennas  $\sim$  50 (9 ground stations, pole to pole)
- ▶ # satellites  $\sim$  80.
- ▶ # revisits  $\rightarrow$  up to 14 a day (polar orbiting satellites).
- ▶ Optimization horizon  $\rightarrow$  one week  $\rightarrow$  3000 p/w ( $\sim$  45%).
- ▶ Typical pass duration  $\rightarrow$  10-20 minutes.
- ▶ operating restrictions (bands, up/down links, priorities)
- ▶ KSAT already had a deconflicting procedure (KNOS+WM).

# KSAT Facility at Tromso, Norway



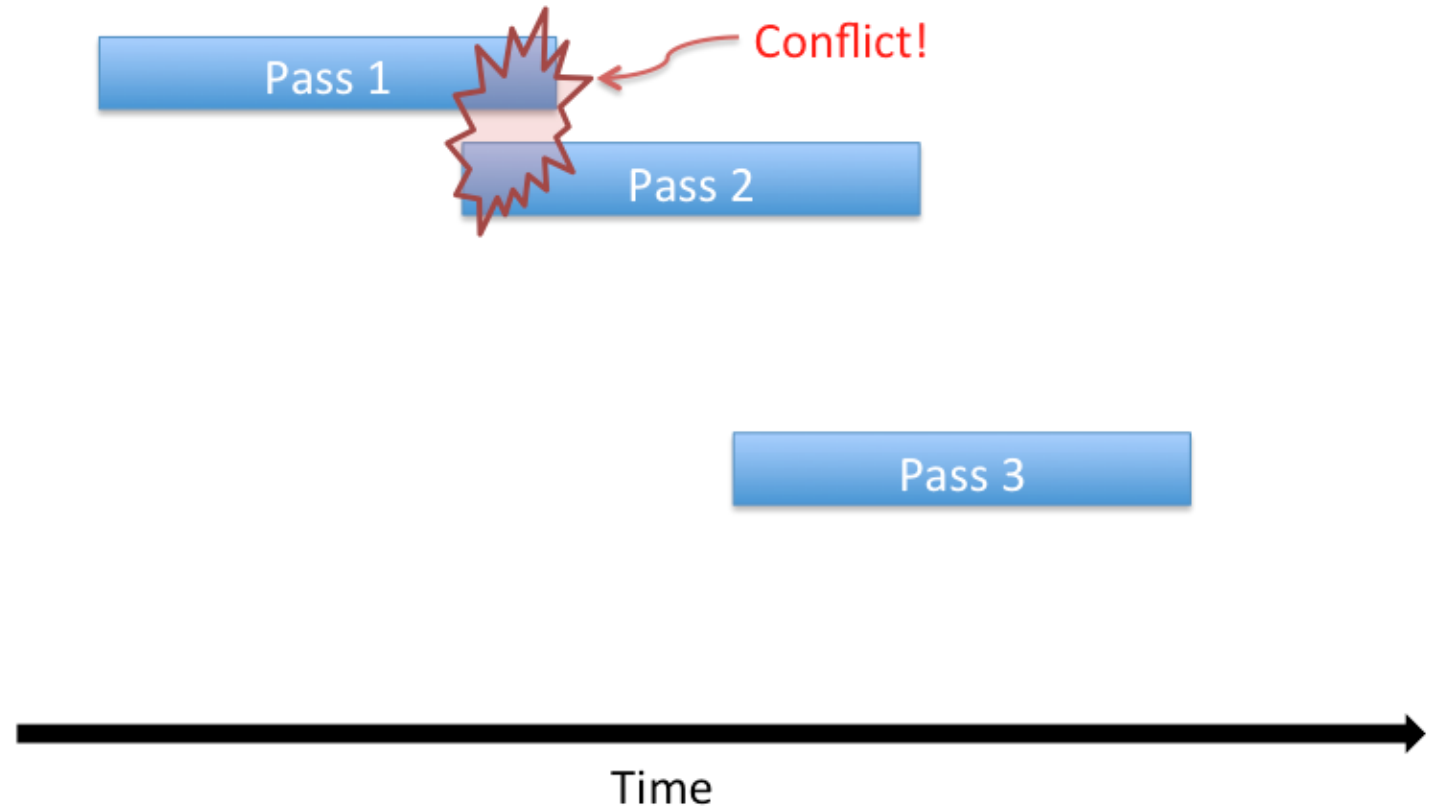
# Some definitions



Antenna 1

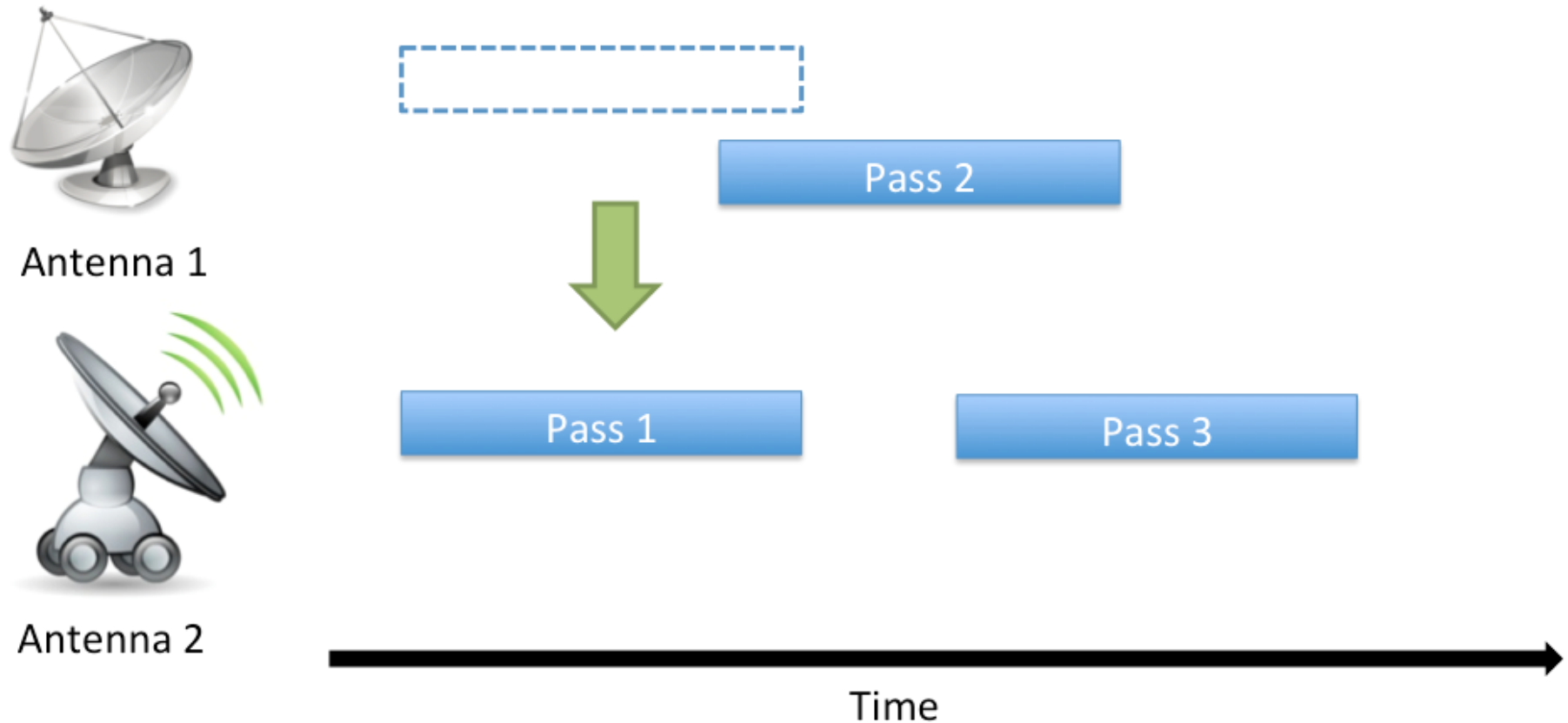


Antenna 2



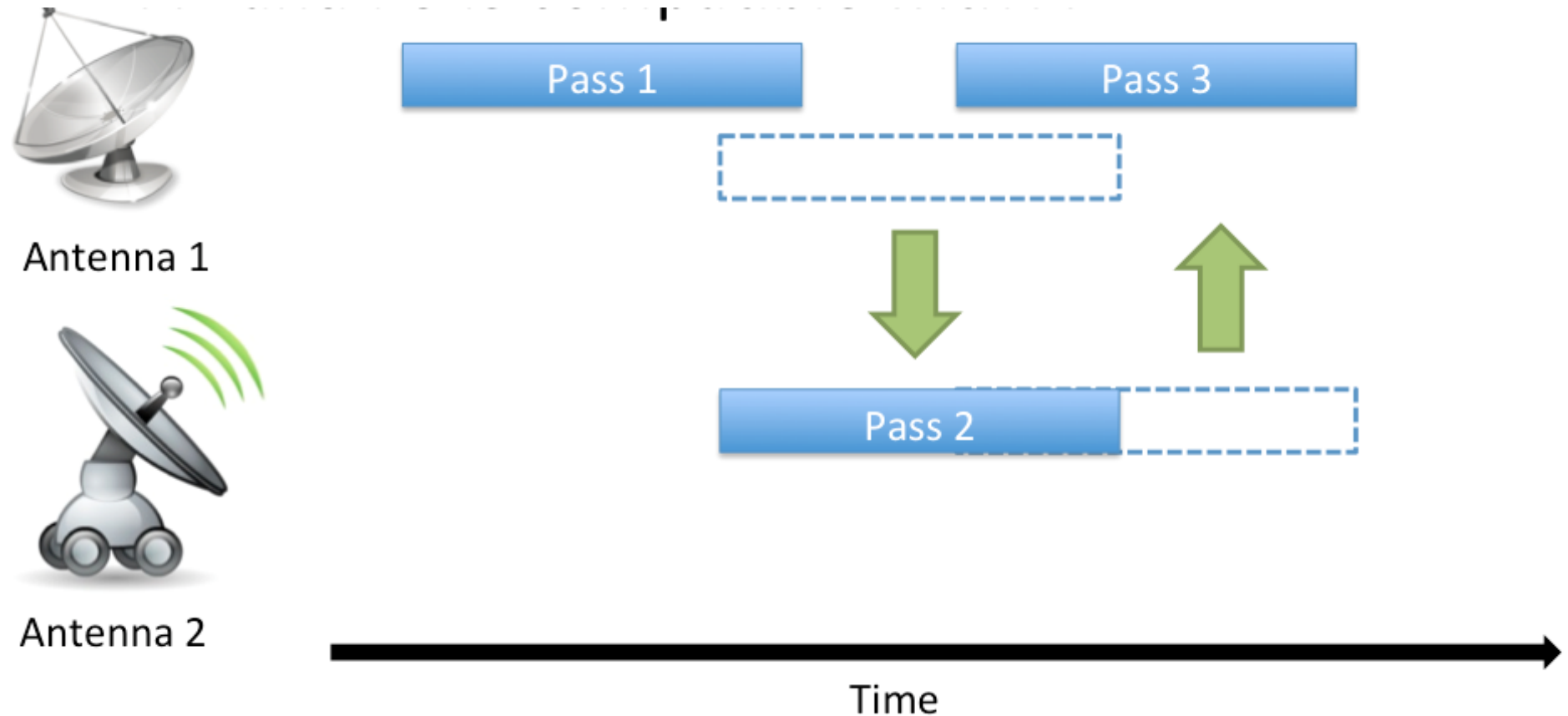
**Keywords:** timeline, passes, conflicts, ASAP=SRS, deconflicting tool, real time.

# Deconflicting I: **moving** passes



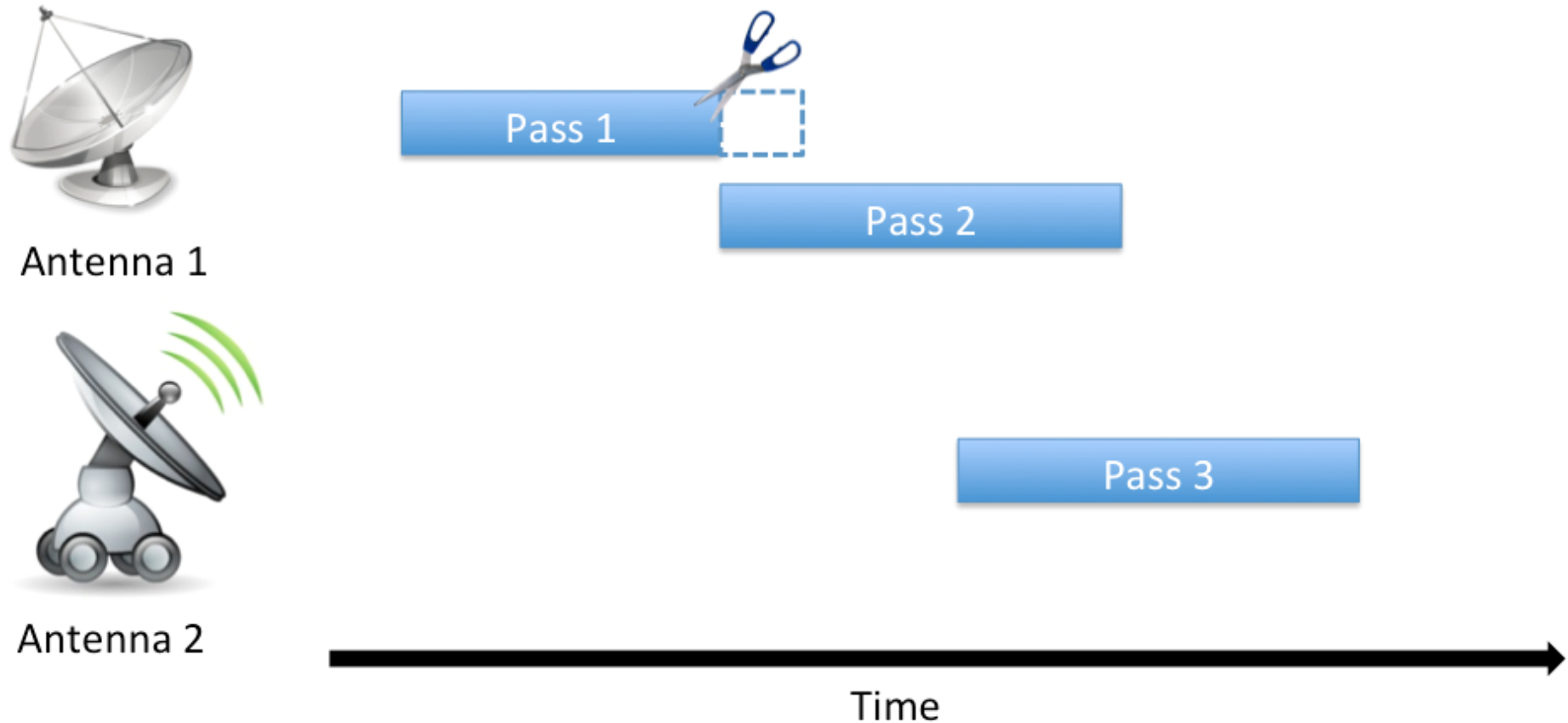
If pass 1 is also compatible with antenna 2 → move the pass!

# Deconflicting II: multiple moving passes



**moving** may involve more than one pass → domino effect.

# Deconflicting III: shortening of passes



If pass 1 is shortable  $\rightarrow$  allocated time slot  $>$  minimum duration

# Deconflicting III: **shortening** of passes



Antenna 1

Pass 1



Pass 2



Antenna 2

Pass 3

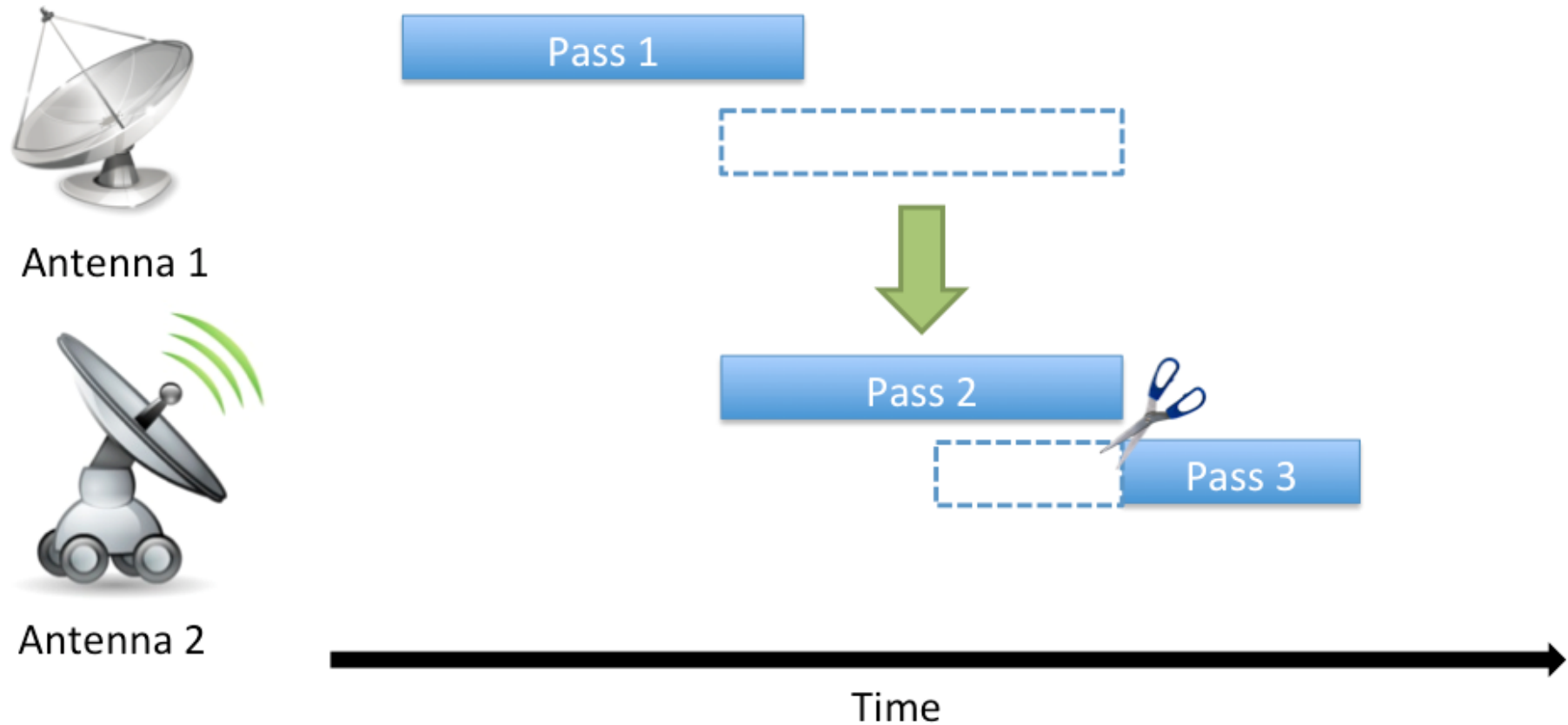


Time

If pass 2 is shortable  $\rightarrow$  allocated time slot  $>$  minimum duration



# Deconflicting IV: moving and shortening of passes

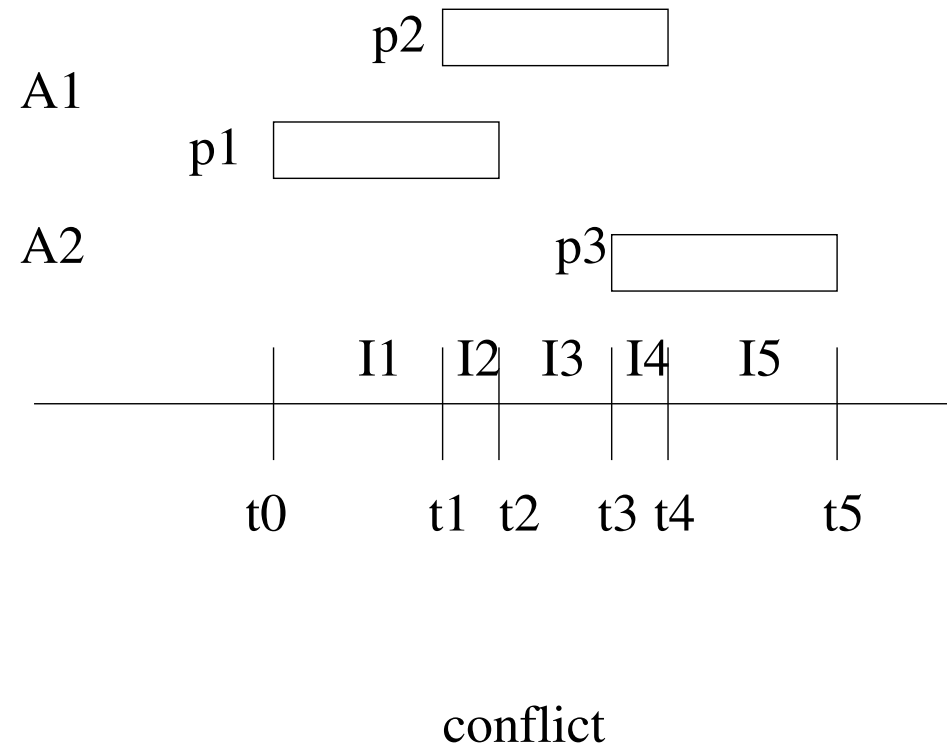


If pass 2 is movable and pass 3 shortable

# Can we build an automatic **Deconflicting tool**?

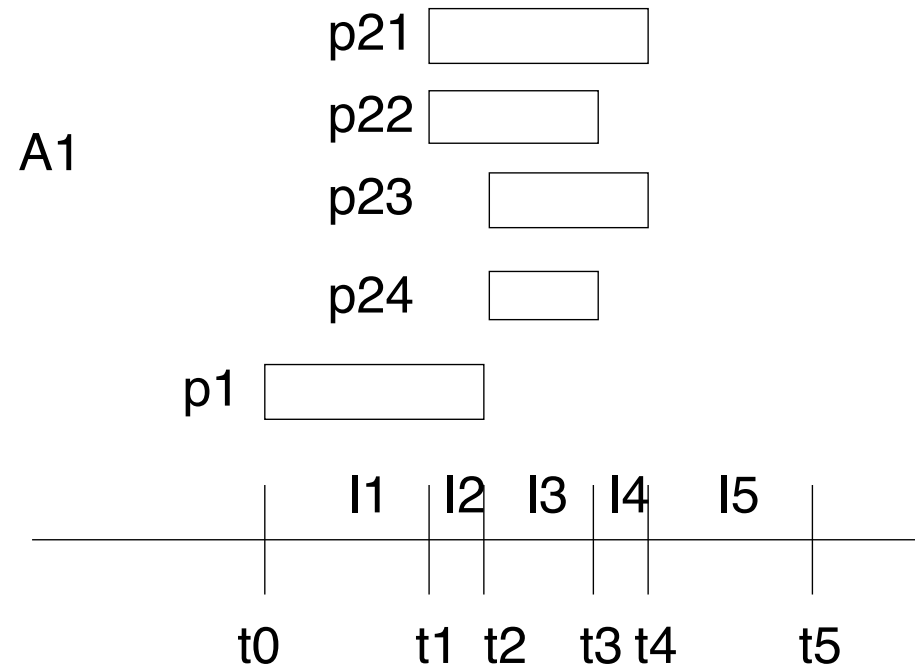
- ▶ Clearly define the hierarchy of allowed deconflicting mechanisms.
- ▶ define a decision variable (if possible binary).
- ▶ define a cost function.
- ▶ implement an optimization procedure.
- ▶ test against **experimental** data.
- ▶ use the tool to deconflict passes and (hopefully) make design decisions.

# Mathematical modeling of deconflicting I



- compute the time intervals along the time line

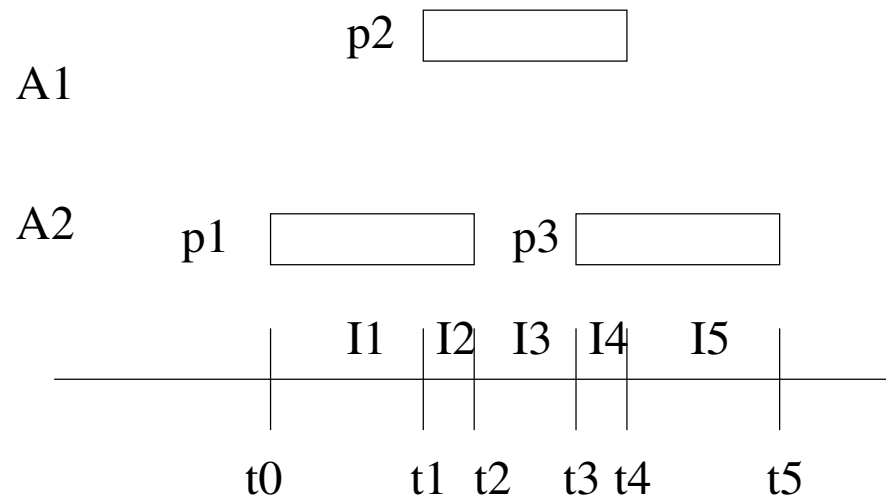
# Mathematical modeling of deconflicting II



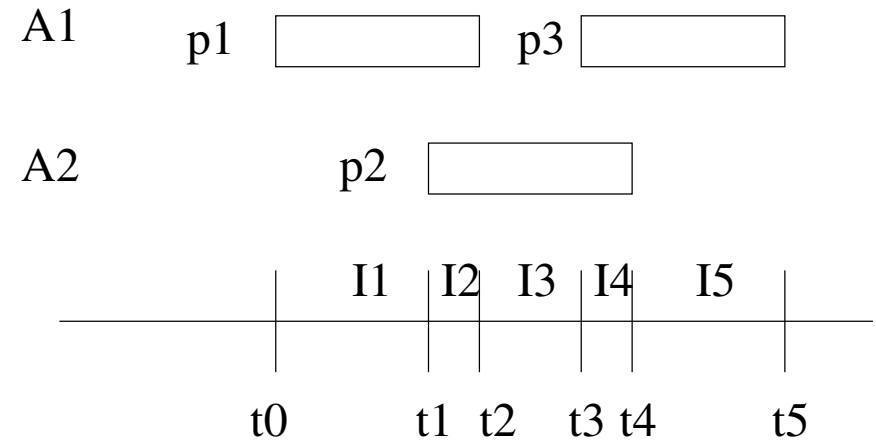
- ▶ for each pass and its alternatives generate all possible **compatible subpasses** in all the alternative antennas.

KEY INGREDIENT

# Mathematical modeling of deconflicting: solutions



solution 1



solution 2

- ▶ assign a **binary** value for each pass
- ▶ add **feasibility** conditions
- ▶ define an appropriate **linear cost function**
- ▶ optimize (ILP).

# Mathematical modeling: equations

Define the binary variable  $y_{ik}$  that is 1 if pass  $P_i$  is assigned to antenna  $A_k$  and 0 otherwise.

Constraints:

1. Every pass has to be assigned at most to one antenna.

$$\sum_{k \in C_i} y_{ik} \leq 1, \forall i \in F.$$

2. For a given antenna  $A_k$  and a time interval  $I_{jk}$  available for passes, there should be no conflict among the passes.

$$\sum_{i \in F: j \in S_{ik}, k \in C_i} y_{ik} \leq 1, \forall k, j: \cup_{i \in F} S_{ik} \neq \emptyset.$$

Maximize the linear cost function:

$$J = \sum_{i \in F} \sum_{k \in C_i} (p^* - p_{ik} + 1) \xi_{ik} y_{ik},$$

where  $p^* = \max_j p_j$  and  $\xi_{ik}$  is a weighting function

# Does it work?

Passes	Ant.	Sat.	Shortable passes (%)	Conflicts	Cancell.	Short.	Move. total (other site)	Vars.	Constr.	Time (s)
3356	22	49	22	537	116	1	839 (13)	8090	9322	64
3006	22	47	22	196	75	3	703 (5)	11219	12157	61
3356	22	49	22	231	94	0	517 (6)	12465	12245	73
3566	22	50	22	306	114	1	561 (11)	12360	12714	73
3470	22	52	22	253	100	0	557 (17)	12016	12788	80
3408	22	52	22	196	91	0	478 (5)	12289	12025	71
1573	14	47	24	68	0	0	487 (7)	7143	4467	64
384	20	46	22	33	21	0	101 (0)	1247	1075	22
1586	16	45	24	6	2	0	479 (2)	7892	4964	63
2287	18	55	26	1053	250	4	312 (35)	6820	8380	77

- ▶ Implemented in C++ within a Savoir tool
- ▶ The **bottleneck** is not the dimension or the ILP solver (LPSOLVE) but the preprocessing and the **complexity** of the conflicts (Gurobi).
- ▶ We perform in real time at least as good (slightly better) as the trained deconflicting department.

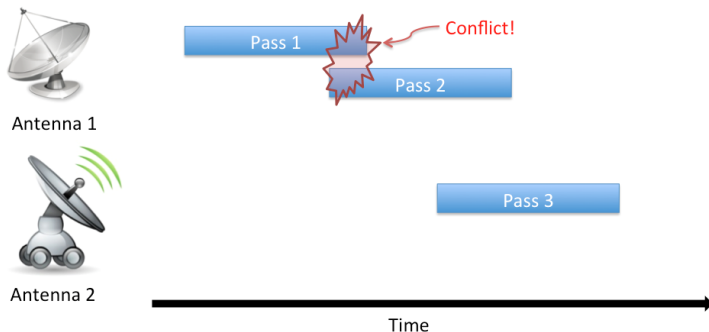
# Our NEW mathematical modeling

- ▶ allow “shaving” passes (no need to generate the subpasses)



# Shaving strategy

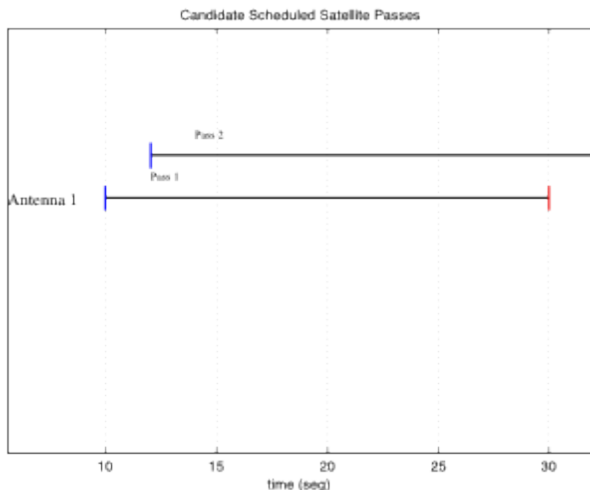
The **shaving** of passes.



After some "shaving", passes 1 and 2 they may fit in antenna 1!

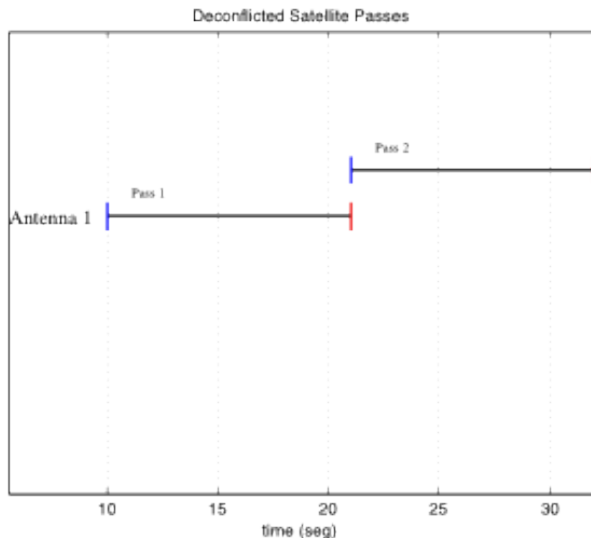
## Toy example of shaving

With almost overlapping passes and a minimum duration of more than 3 units, one pass has to be cancelled.



## Shaving strategy

Shave **both passes** an appropriate amount compatible with the minimum duration but not directly available in the timeline.



# Mathematical modeling: variables

Shorteinig Model (old)	Shaving Model (new)
$Y_{ik} \in \{0, 1\}$ (subpasses)	$Y_{ik} \in \{0, 1\}$ assignment (passes)
	$S_i \geq 0$ Start of connection
	$E_i \geq S_i$ End of connection
	$W_{ii'} \in \{0, 1\}$ $i$ is "left of" $i'$

More sets of variables in the new model!!

# Initial comparison

- ▶ The Shaving model (new model) has more variables and constraints than the Shortening model (old model)
- ▶ However, for the new model we do not need to compute all intersections between passes, and build all time intervals.
- ▶ In the experiments we will see that this increase in model complexity is compensated by an improvement in efficiency and quality

# Results

Next tables summarize the results obtained in the 32 tested scenarios (max. CPU time = 1 hour).

- ▶ Column “Scenario” identifies the tested scenario as a 3-tuple of the form  $D - |N| - |A|$
- ▶ Column “m” refers to the resulting number of passes.
- ▶ Column “ST” (shortened/shaved time) shows the total time (in hours) that visible satellites are not connected to any antenna.
- ▶ Column “CP” shows the number of passes that have been cancelled.

## Results: first 16 tested scenarios

Instance		Shaving Model		Shortening Model	
Scenario	m	ST	CP	ST	CP
1-06-2	85	0.50	1	0.50	1
1-12-2	173	1.45	4	1.66	2
1-18-2	253	5.95	3	6.31	9
1-24-2	336	12.63	9	13.11	28
1-30-2	421	20.73	9	21.28	61
1-36-2	502	32.73	11	33.22	92
1-42-2	582	41.90	19	42.35	84
1-48-2	660	50.75	23	51.34	124
1-06-4	85	0.00	0	0.00	0
1-12-4	173	0.00	0	0.11	2
1-18-4	253	0.42	1	0.73	5
1-24-4	336	2.39	1	2.74	9
1-30-4	421	4.95	4	5.59	21
1-36-4	502	9.79	1	10.34	18
1-42-4	582	14.42	2	15.20	25
1-48-4	660	20.24	2	20.95	36

## Results: last 16 tested scenarios

Instance		Shaving Model		Shortening Model	
Scenario	m	ST	CP	ST	CP
4-06-2	337	1.04	1	1.25	5
4-12-2	685	9.31	7	10.12	12
4-18-2	1004	26.21	19	27.56	37
4-24-2	1336	56.87	21	58.59	132
4-30-2	1660	85.67	30	87.22	226
4-36-2	1982	134.39	35	135.88	370
4-42-2	2300	171.26	75	172.86	525
4-48-2	2604	206.15	102	208.04	704
4-06-4	337	0.00	0	0.35	4
4-12-4	685	0.10	0	0.43	6
4-18-4	1004	1.31	1	2.46	20
4-24-4	1336	9.68	1	11.79	47
4-30-4	1660	18.50	4	20.76	55
4-36-4	1982	39.51	5	41.62	70
4-42-4	2300	57.34	7	-	-
4-48-4	2604	80.20	8	-	-



## Results: Average results

	Shaving Model		Shortening Model	
	ST	CP	ST	CP
Average	32.63	13.03	33.48	93.3

- ▶ Column “ST” (shortened/shaved time) shows the total time (in hours) that visible satellites are not connected to any antenna.
- ▶ Column “CP” shows the number of passes that have been cancelled.
- ▶ Only results on the first 30 scenarios are considered (the other two not solved by the Shortening model)

## Results: comments on average results

- ▶ The Shortening model is not able to find a feasible solution in the 3600 seconds given for the last two scenarios. The Shaving model found feasible solutions to all scenarios.
- ▶ The Shaving model reduces less time than the Shortening model (32.63 hours vs. 33.48).
- ▶ The Shaving model cancels less passes (13.03 vs. 93.3).
- ▶ The Shaving model cancels passes of 7.47 satellites, whereas the Shortening model cancels passes of 18.63 satellites.

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Thanks!



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Some recent publications (most of them downloadable at <http://aero.us.es/rvazquez> or ArXiv)

- J.M. Montilla, J. Siminski and R. Vazquez. "Single track orbit determination analysis for low Earth orbit with approximated J2 dynamics," submitted, 2024.
- J.M. Montilla, R. Vazquez and P. Di Lizia, "Maneuver detection with two mixture-based metrics for radar track data," submitted, 2024.
- A.S. Rivero, C. Bombardelli, and R. Vazquez "Space-Occupancy Conjunction Filter," submitted, 2024.
- J.M. Montilla, J.C. Sanchez, R. Vazquez, J. Galan-Vioque, J. Rey Benayas, J. Siminski, "Manoeuvre detection in Low Earth Orbit with Radar Data," Advances in Space Research, 2023.
- J.M. Montilla, R. Vazquez and P. Di Lizia, "Mixture-Based Cost Metrics for Maneuver Detection Using Radar Track Data," 33rd AIAA/AAS Space Flight Mechanics Meeting, 2023.
- A.S. Rivero, C. Bombardelli, and R. Vazquez, "Fast Orbit Propagation for Conjunction Screening," KEPASSA 2022.
- A.S. Rivero, C. Bombardelli, and R. Vazquez, "Space-Occupancy Conjunction Filter," 33rd AIAA/AAS Space Flight Mechanics Meeting, 2023.
- R. Vazquez, F. Perea, J. Galan-Vioque, "Resolution of an Antenna-Satellite assignment problem by means of Integer Linear Programming," Aerospace Science and Technology, vol. 39, pp. 567-574, 2014.
- L. Linares, R. Vazquez, F. Perea, J. Galan-Vioque, "The Shaving Algorithm: A Mixed Integer Linear Programming-based Model for Resolution of the Antenna-Satellite Scheduling Problem", IEEE Transactions on Aerospace and Electronic Systems, vol. 60, pp. 463-473, 2024.