Spacecraft Dynamics

Lesson 7: Passive attitude control systems.

Rafael Vázquez Valenzuela

Departamento de Ingeniería Aeroespacial
Escuela Superior de Ingenieros, Universidad de Sevilla rvazquez1@us.es

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Attitude control

- The attitude control subsystem of satellites can be divided, in general, in two families:
  - Spin-stabilized satellites: using the gyroscopic effect/major axis rule to mantain an inertial direction (which would be the major axis). Cheap and simple but only the major axis can be stabilized (unless wheels are used).
  - Three-axis stabilized satellites: they use some kind of active control to maintain the attitude with some orientation w.r.t. some reference frame.
- Satellites can potentially use the two types of control, depending on the phase of the mission (for instance interplanetary probes).
- Another possible method is the use of gravity gradient, which does not require control (in principle), but it is not very accurate.

Attitude control

- Another classification of attitude control methods is in two kinds: active and passive.
  - We interpret active in the sense of requiring additional use of energy and some command logic (requiring some computational power).
  - Whereas a passive control system does rely on some natural/physical effect to achieve stabilization (e.g. the major axis rule).
- Nevertheless these two classes sometimes overlap as for instance to start the rotation of a spin-stabilized satellite (a passive kind of stabilization) some kind of command and energy contribution is required.
- Thus all satellites in the end should have some kind of active system.

Control of a spin stabilized satellite

- By the major axis rule, we know that a satellite spinning along its major axis is stable; in addition, we know that its response to external perturbations is a small nutation/precession movement that would end up dissipating.
- A spin-stabilized stallite can have a rather simple control system, with the following goals:
  1. Initiate or increase rotation.
  2. Increase the stability of the satellite.
  3. Modify the direction of the rotation axis.
  4. Slow down or completely stop.
- The first goal is trivial with thrusters or even considering the initial spin due to the launch.
- The second goal can be achieved with nutation dampers that increase energy dissipation and thus strengthen the major axis rule (see Lesson 5 and 8).
- In the rest of the lesson we study goals 3 and 4.

Modifying the direction of the rotation axis

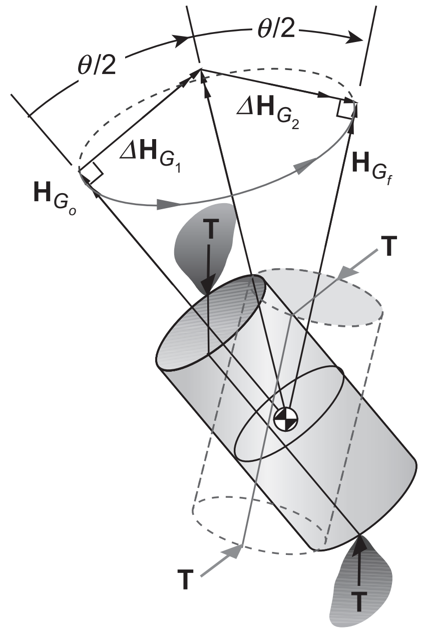
- A simple way to modify the direction of a rotation axis is to stop the rotation, modify the axis, and then start spinning again. However, this procedure would be expensive and slow. Another procedure, known as the "coning" manoeuvre, is explained next.
- To simplify, consider an axisymmetrical spacecraft ($I_1 = I_2 = I < I_3$) and consider we can perform impulsive manoeuvres that instantaneously modify the angular momentum, namely, apply an impulse $\Delta\Gamma$ by using thrusters.
- Let us consider the vehicle rotating only along axis 3 (major axis) with angular velocity n, so that $\vec{\omega}$ and $\vec{\Gamma}$ are aligned.
- Remember (Lesson 5) when we studied the gyroscopic effect, if we apply a perpendicular torque to the axis 3, we get a precession and nutation movement of the body axis 3.

Modifying the direction of the rotation axis

- To simplify, consider that we can directly apply an impulse in $\vec{\Gamma}$, so that $\vec{\Gamma}_f = \vec{\Gamma}_i + \Delta\vec{\Gamma}$. After that, the movement is free again.
- In Lesson 5 we studied that the free movement of an axisymmetrical satellite rotating around its simmetry axis was a precession movement with fixed nutation, so that $\vec{\omega}$ rotates describing a cone around the angular momentum $\vec{\Gamma}$.
- Thus, with this hypothesis of instantaneous change of angular momentum, we simplify the nutation which also changes instantaneously and stays constant, so we can use the exact solution of the free movement of an axisymmetrical body.

Coning

- Consider that we want to displace the rotation axis an angle θ.
- Apply a $\Delta\vec{\Gamma}$ so that $\vec{\Gamma}$ has an angle of $\theta/2$ with the angular velocity. This causes that the speed describes a cone around the new $\vec{\Gamma}$ with angle $\theta/2$, and when it has gone 180º around the cone it has rotated a total angle θ w.r.t. its former position. Then apply a $\Delta\vec{\Gamma}$ such that the final $\vec{\Gamma}$ is again coincident with the angular speed. Note that in the figure, $\mathbf{H_G} = \vec{\Gamma}$.

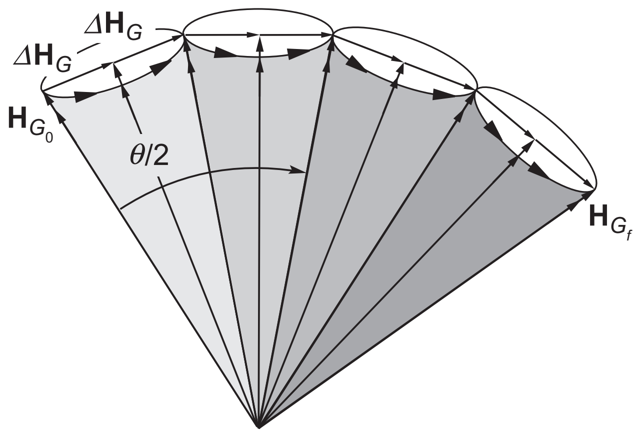


Coning

- From the figure: $\Delta\Gamma_1 = \Delta\Gamma_2 = \Gamma \tan\theta/2$, so the total $\Delta\Gamma_{coning} = 2\Gamma\tan\theta/2$. The final angular momentum es equal to the initial one (but in the intermediate position it is slightly larger: $\frac{\Gamma}{\cos\theta/2}$).
- The time one takes to perform the manoeuvre is π divided by the precession angular speed: $t = \frac{\pi}{\dot\phi}$.
- From Lesson 5 (free movement of axisymmetrical spacecraft) $\dot\phi = \frac{I_3 n}{I\cos\theta/2} = \frac{\Gamma}{I\cos\theta/2}$, thus $t = \frac{\pi I\cos\theta/2}{\Gamma}$.
- During that time, the body would rotate w.r.t. its symmetry axis (Lesson 5), an angle $\psi = t\lambda = \frac{\pi I\cos\theta/2}{\Gamma}\frac{n(I-I_3)}{I} = \frac{\pi(I-I_3)\cos\theta/2}{I_3}$.
- In general this angle is not 180 degrees (unless $\frac{(I-I_3)\cos\theta/2}{I_3} = 1$) thus one has to use a different set of thrusters to get to the final position.

Multiple coning

- An idea to reduce the fuel consumption (and break down large angles of rotation of the spin axis) is dividing the coning manoeuvre into m smaller manoeuvres, as seen in the figure.



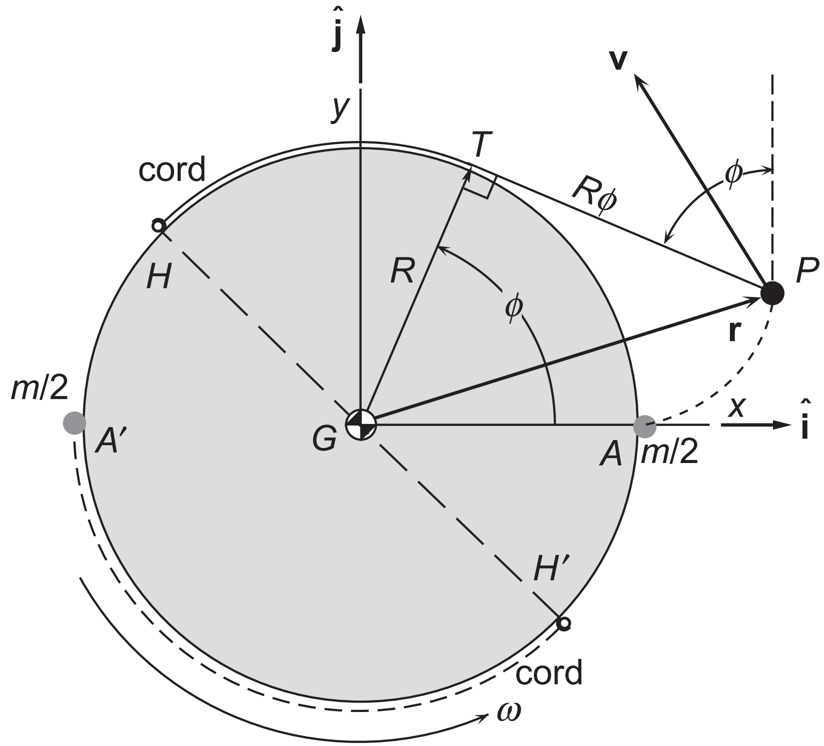
- In each manoeuvre one needs to displace Γ by an angle $\theta/2m$ and wait 180 degrees.
- The total manoeuvre is $\Delta\Gamma_{coning} = 2m\Gamma\tan\theta/2m$ (if m is large this tends to $\theta\Gamma$, and thus this is worthy for large angles).
- The total time of manoeuvre is $t = \frac{m\pi I\cos\theta/2m}{\Gamma}$ (if m is large this goes to infinity, so there is a tradeoff).

Slowing the rotation: yo-yo device

- A yo-yo device is a single-use device that can be used to totally or partially stop the satellite's spin. The mechanism consists on two symmetrical masses fixed to the vehicle by a joint that can be released. The masses are also attached to a wire that is winded around the vehicle with a single point of union, in a plane perpendicular to the rotation that has to be stopped.
- To slow down or stop the rotation, one releases the masses. The start to get away from the vehicle and the wire starts to unwind until the stress reaches the point at which the wire is fixed to the vehicle. Then the wire is also released. If the length of the wire is well designed, then the vehicle has stopped.
- Assumptions: Masses are considered points with mass $m/2$, the wire is massless and not flexible, axisymmetrical vehicle of radius R initially spinning around its symmetry axis with speed n_0.

Yo-yo device

- Initial kinetic energy is $T_0 = \frac{1}{2}\left(I_3 n_0^2 + mR^2 n_0^2\right)$. Initial angular momentum is $\Gamma_0 = I_3 n_0 + mR^2 n_0$. Defining $K = 1 + \frac{I_3}{mR^2}$, we can write $T_0 = \frac{1}{2} mR^2 K n_0^2$ and $\Gamma_0 = mR^2 K n_0$.
- At a given instant the situation is as in the figure:



Yo-yo device

- In the figure, the angle has already been unwound by an angle ϕ, and the vector $\vec{r}$ is the position vector of one of the masses (since they are symmetrical it is enough to study one of them). Given the assumptions, the wire should be tangent at the point T. Use body axes $\vec{i}$ and $\vec{j}$ as in the figure.
- In this frame, $\vec{r}$ is written as $\vec{r} = \vec{GT} + \vec{TP} = R(\cos\phi\vec{i} + \operatorname{sen}\phi\vec{j}) + R\phi(\operatorname{sen}\phi\vec{i} - \cos\phi\vec{j})$.
- To find the kinetic energy and the angular momentum we need the inertial speed. One has:

$$\vec{v} = \dot{\vec{r}}|_{IN} = \dot{\vec{r}}|_{ROT} + \vec{\omega} \times \vec{r}$$

where $\vec{\omega} = n\vec{k}$.

- Now, $\dot{\vec{r}}|_{ROT} = \dot{\phi}R\phi(\cos\phi\vec{i} + \operatorname{sen}\phi\vec{j})$ y $\vec{\omega} \times \vec{r} = nR(\cos\phi\vec{j} - \operatorname{sen}\phi\vec{i}) + nR\phi(\operatorname{sen}\phi\vec{j} + \cos\phi\vec{i})$.

Yo-yo device

- Therefore
  $\vec{v} = R\left((\dot{\phi}+n)\phi\cos\phi - n\,\mathrm{sen}\,\phi\right)\vec{i} + R\left((\dot{\phi}+n)\phi\,\mathrm{sen}\,\phi + n\cos\phi\right)\vec{j}.$
- Computing the norm of the speed:
  $v = R\sqrt{\left((\dot{\phi}+n)\phi\cos\phi - n\,\mathrm{sen}\,\phi\right)^2 + \left((\dot{\phi}+n)\phi\,\mathrm{sen}\,\phi + n\cos\phi\right)^2}.$
- Therefore: $v = R\sqrt{(\dot{\phi}+n)^2\phi^2 + n^2}.$
- Thus, $T = \frac{1}{2}\left(I_3 n^2 + mR^2((\dot{\phi}+n)^2\phi^2 + n^2)\right)$ and using K,
  $T = \frac{mR^2}{2}\left(Kn^2 + (\dot{\phi}+n)^2\phi^2\right).$
- On the other hand the angular momentum of the masses is $\Gamma_m = |\vec{r}\times m\vec{v}|$. Computing the product we get
  $\Gamma_m = mR^2(n + (n+\dot{\phi})\phi^2).$
- Therefore
  $\Gamma = I_3 n + mR^2(n + (n+\dot{\phi})\phi^2) = mR^2(Kn + (n+\dot{\phi})\phi^2).$

Yo-yo device

- By conservation of kinetic energy and angular momentum $T = T_0$, $\Gamma = \Gamma_0$, thus reaching two equations
$$K(n_0^2 - n^2) = (\dot{\phi} + n)^2 \phi^2, \quad K(n_0 - n) = (n + \dot{\phi})\phi^2$$
- Dividing the first equation by the second, we find $n_0 + n = n + \dot{\phi}$, thus $\dot{\phi} = n_0$, this is, the unwinding rate of the wire is equal to the initial angular velocity of the vehicle. Substituting this value in the second equation and solving for ϕ, one can find the angle of unwound wire as a function of the instantaneous angular velocity:
$$\phi = \sqrt{K \frac{n_0 - n}{n_0 + n}}$$
- If one wants that at the end $n = 0$, replacing this value, we find $\phi = \sqrt{K}$, and since the length of wire is $l = R\phi$, we find $l = R\sqrt{K}$, which does not depend on the initial speed!
- One can find an adequate length of wire for any value $n \in (-n_0, n_0)$.