APPENDIX C

Mean Aerodynamic Chord, Mean Aerodynamic Center, and $C_{m_{ac_w}}$

C.1 Basic Definitions

In the normal flight range, the resultant aerodynamic forces acting on any lifting surface can be represented as a lift and drag acting at the mean aerodynamic center ($\bar{x}$, $\bar{y}$, $\bar{z}$), together with a pitching couple $C_{m_{ac_w}}$ which is independent of angle of attack (see Fig. 2.8).

The pitching moment of a wing is nondimensionalized by the use of the mean aerodynamic chord $\bar{c}$.

Both the m.a. center and the m.a. chord lie in the plane of symmetry of the wing. However, in determining them it is convenient to work with the half-wing.

These quantities are defined by (see Fig. C.1)[1]

$$\bar{c} = \frac{2}{S}\int_0^{b/2} c^2\,dy \tag{C.1,1}$$

$$\bar{x} = \frac{2}{C_L S}\int_0^{b/2} C_{l_a} c x\,dy \tag{C.1,2}$$

$$\bar{y} = \frac{2}{C_L S}\int_0^{b/2} C_{l_a} c y\,dy = \eta_{cp}\frac{b}{2} \tag{C.1,3}$$

$$\bar{z} = \frac{2}{C_L S}\int_0^{b/2} C_{l_a} c z\,dy \tag{C.1,4}$$

where b = wing span

c = local chord

C_L = total lift coefficient

C_{l_a} = local additional lift coefficient, proportional to C_L

C_{l_b} = local basic lift coefficient, independent of C_L

$C_l = C_{l_b} + C_{l_a}$ = total local lift coefficient

[1]The coordinate system used applies only to this appendix.

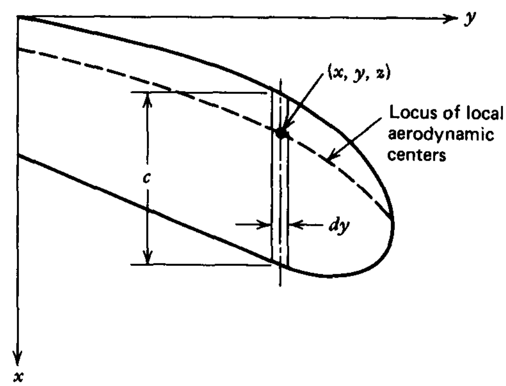

Figure C.1 Local aerodynamic center coordinates.

m_{ac} = pitching moment, per unit span, about aerodynamic center (Fig. C.4)

S = wing area

y = spanwise coordinate of local aerodynamic center measured from axis of symmetry

x = chordwise coordinate of local aerodynamic center measured aft of wing apex

z = vertical coordinate of local aerodynamic center measured from xy plane

η_{cp} = lateral position of the center of pressure of the additional load on the half-wing as a fraction of the semispan

The coordinates of the m.a. center depend on the additional load distribution; hence the position of the true m.a. center will vary with wing angle of attack if the form of the additional loading varies with angle of attack. For a wing that has no aerodynamic twist, the m.a. center of the half-wing is also the center of pressure of the half-wing. If there is a basic loading (i.e., at zero overall lift, due to wing twist), then $(\bar{x}, \bar{y}, \bar{z})$ is the center of pressure of the additional loading.

The height and spanwise position of the local aerodynamic centers may be assumed known, and hence $\bar{y}$ and $\bar{z}$ for the half-wing can be calculated once the additional spanwise loading distribution is known. However, in order to calculate $\bar{x}$, the fore-and-aft position of each local aerodynamic center must be known first. If all the local aerodynamic centers are assumed to lie on the nth-chord line (assumed to be straight), then

$$\bar{x} = nc_r + \bar{y} \tan \Lambda_n \qquad \text{(C.1,5)}$$

where c_r = wing root chord

Λ_n = sweepback of nth-chord line, degrees

Ideal two-dimensional flow theory gives $n = \frac{1}{4}$ for subsonic speeds and $n = \frac{1}{2}$ for supersonic speeds.

The m.a. chord is located relative to the wing by the following procedure:

1. In (C.1,2) replace C_{l_a} by C_L, and for x use the coordinates of the $\frac{1}{4}$-chord line.
2. The value of $\bar{x}$ so obtained (the mean quarter-chord point) is the $\frac{1}{4}$-point of the m.a. chord.

The above procedure and the definition of $\bar{c}$ (see C.1,1) are used for *all* wings.

C.2 Comparison of m.a. Chord and m.a. Center for Basic Planforms and Loading Distributions

In Table C.1 taken from (Yates, 1952), values of m.a. chord and $\bar{y}$ are given for some basic planforms and loading distributions.

In the general case the additional loading distribution and the spanwise center-of-pressure position can be obtained by methods such as those of De Young and Harper (1948), Weissinger (1947), and Stanton-Jones (1950). For a trapezoidal wing with the local aerodynamic centers on the nth-chord line, the chordwise location of the mean aerodynamic center from the leading edge of the m.a. chord expressed as a fraction of the m.a. chord h_{n_w} is given by

$$h_{n_w} = n + \frac{3(1+\lambda)^2}{8(1+\lambda+\lambda^2)}\left[\eta_{cp} - \frac{1+2\lambda}{3(1+\lambda)}\right] A \tan \Lambda_n \qquad \text{(C.2,1)}$$

Table C.1

Planform	*Additional Loading Distribution*	*M.A.C.* $\bar{c}$	$\bar{y}$
Constant taper and sweep (trapezoidal)	Any	$\frac{2c_r}{3}\frac{1+\lambda+\lambda^2}{1+\lambda}$	$\eta_{cp}\cdot\frac{b}{2}$
Constant taper and sweep (trapezoidal)	Proportional to wing chord (uniform C_{l_a})	$\frac{2c_r}{3}\frac{1+\lambda+\lambda^2}{1+\lambda}$	$\frac{b}{2}\cdot\frac{1+2\lambda}{3(1+\lambda)}$
Constant taper and sweep (trapezoidal)	Elliptic	$\frac{2c_r}{3}\frac{1+\lambda+\lambda^2}{1+\lambda}$	$\frac{b}{2}\cdot\frac{4}{3\pi}$
Elliptic (with straight sweep of line of local a.c.)	Any	$\frac{c_r}{3}\cdot\frac{8}{\pi}$	$\eta_{cp}\cdot\frac{b}{2}$
Elliptic (with straight sweep of line of local a.c.)	Elliptic (uniform C_{l_a})	$\frac{c_r}{3}\cdot\frac{8}{\pi}$	$\frac{b}{2}\cdot\frac{4}{3\pi}$
Any (with straight sweep of line of local a.c.)	Elliptic	$\frac{2}{S}\int_0^{b/2} c^2\,dy$	$\frac{b}{2}\cdot\frac{4}{3\pi}$

where A = aspect ratio, b^2/S

λ = taper ratio, c_t/c_r

c_t = wing-tip chord

The length of the chord through the centroid of area of a trapezoidal half-wing is equal to $\bar{c}$. For the same wing with uniform spanwise lift distribution (i.e., C_{l_a} = const) and local aerodynamic centers on the nth-chord line, the m.a. center also lies on the chord through the centroid of area. The chord through the centroid of area of a wing having an elliptic planform is not the same as $\bar{c}$, but the m.a. center for elliptic loading and the centroid of area both lie on the same chord (see Yates, 1952).

C.3 m.a. Chord and m.a. Center for Swept and Tapered Wings (Subsonic)

The ratio $\bar{c}/c_r$ is plotted against λ in Fig. C.2 for straight tapered wings with streamwise tips. The spanwise position of the m.a. center of the half-wing (or the center of pressure of the additional load) for uniform spanwise loading is also given in Fig. C.2. These functions are given in Table C.1.

The m.a. chord is located by means of the distance x of the leading edge of the m.a. chord aft of the wing apex:

$$x = \frac{b}{2} \cdot \frac{1}{3} \frac{1 + 2\lambda}{1 + \lambda} \tan \Lambda_0$$

$$= \frac{1 + 2\lambda}{12} c_r A \tan \Lambda_0 \tag{C.3,1}$$

where Λ_0 = sweepback of wing leading edge, degrees.

The sweepback of the leading edge is related to the sweep of the nth-chord line Λ_n by the relation

$$A \tan \Lambda_0 = A \tan \Lambda_n + 4n \frac{1 - \lambda}{1 + \lambda} \tag{C.3,2}$$

Using (C.3,2) and the expression for $\bar{c}/c_r$, x can be obtained in terms of $\bar{c}$ and Λ_n from

$$\frac{x}{\bar{c}} = \frac{(1 + 2\lambda)(1 + \lambda)}{8(1 + \lambda + \lambda^2)} \left[A \tan \Lambda_n + 4n \frac{1 - \lambda}{1 + \lambda}\right] \tag{C.3,3}$$

The fractional distance of the m.a. center aft of the leading edge of the m.a. chord, h_{n_w}, is given for swept and tapered wings at low speeds and small incidences in Fig. C.3. The dotted lines show the aerodynamic-center position for wings with unswept trailing edges. The curves have been obtained from theoretical and experimental data. The curves apply only within the linear range of the curve of wing lift against pitching moment, provided that the flow is subsonic over the entire wing. The probable error of h_{n_w} given by the curves is within 3%.

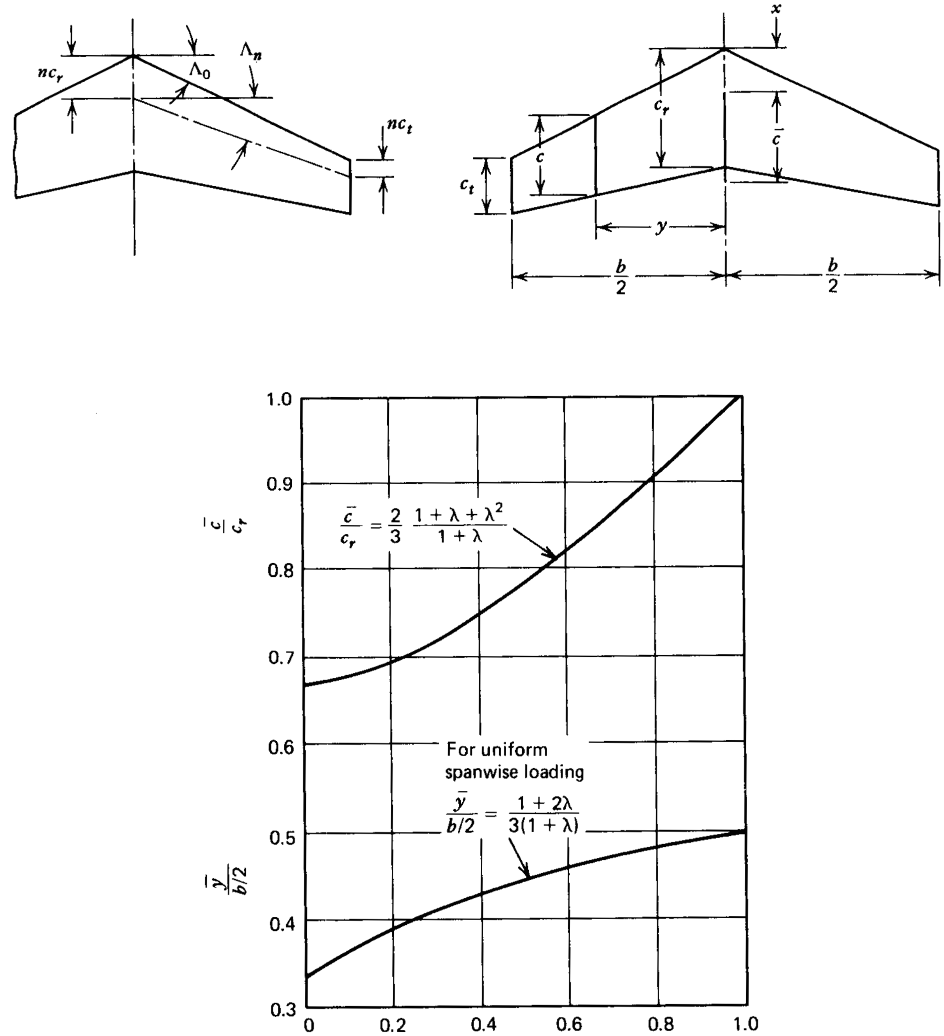

Figure C.2 Mean aerodynamic chord for straight tapered wings; and spanwise position of mean aerodynamic center for uniform spanwise loading (i.e., constant C_{L_α}). (From "Notes on the Mean Aerodynamic Chord and the Mean Aerodynamic Center of a Wing" by A. H. Yates, *J. Roy. Aero. Soc.*, June 1952.)

C.4 C_{mac_w}

The total load on each section of a wing has three parts as illustrated by Fig. C.4*a*. The resultant of the local additional lift l_a, is the lift L_a acting through the m.a. center (Fig. C.4*b*).

The resultant of the distribution of the local basic lift l_b is a pitching couple whenever the line of aerodynamic centers is not straight and perpendicular to x. This couple is given by

$$M_1 = 2\int_0^{b/2} (x - \bar{x}) l_b \, dy = 2\int_0^{b/2} x l_b \, dy$$

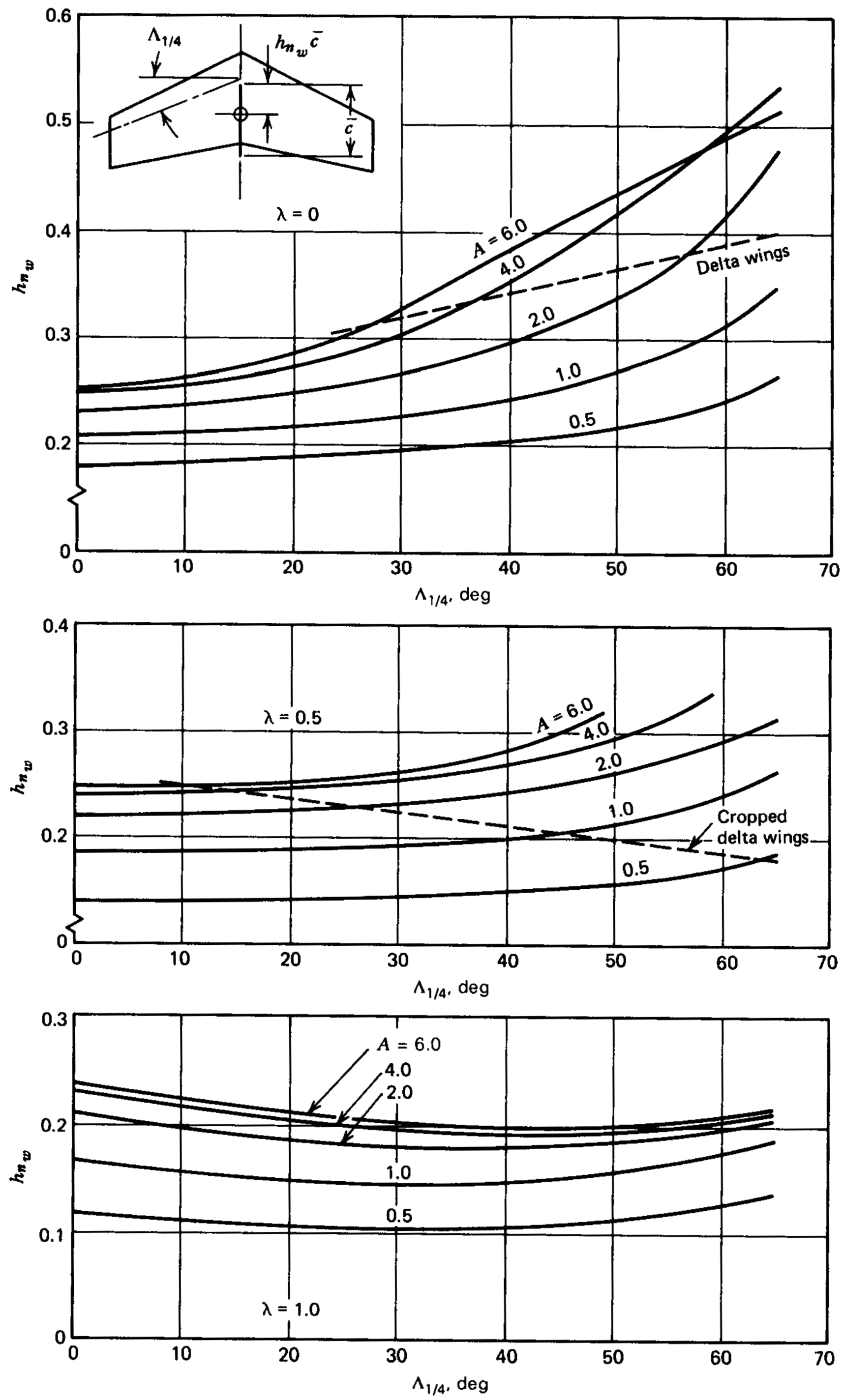

Figure C.3 Chordwise position of the mean aerodynamic center of swept and tapered wings at low speeds expressed as a fraction of the mean aerodynamic chord. (From Royal Aeronautical Data Sheet Wings 08.01.01.)

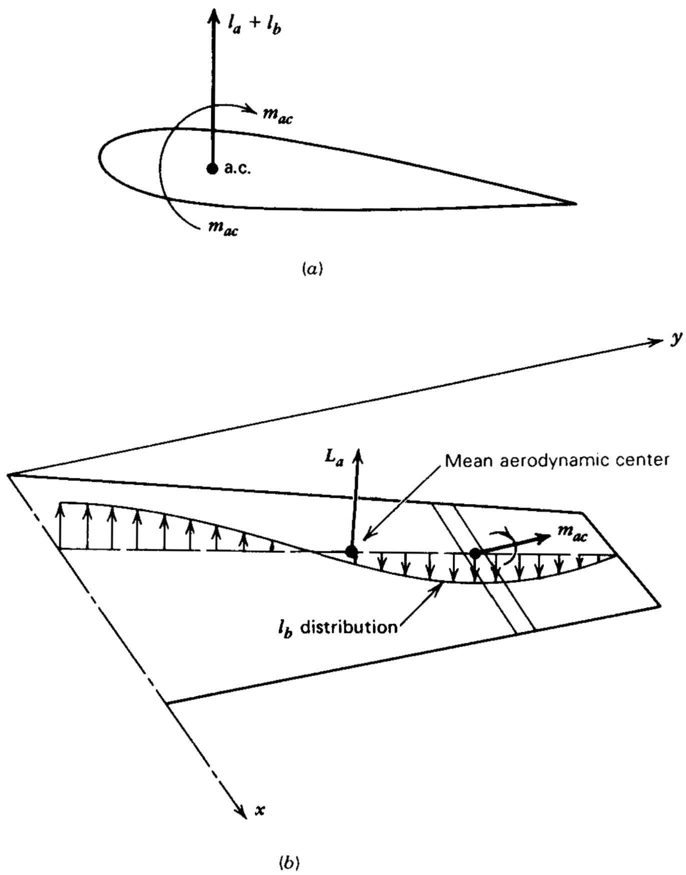

Figure C.4 (*a*) Section total load. (*b*) Wing loads.

since the resultant of $l_b = 0$.

Then

$$C_{m_1} = \frac{4}{\rho V^2 S\bar{c}} \int_0^{b/2} xC_{l_b} \frac{1}{2} \rho V^2 c \, dy$$

$$= \frac{2}{S\bar{c}} \int_0^{b/2} C_{l_b} xc \, dy \tag{C.4,1}$$

The resultant of the m_{ac} distribution is given by

$$C_{m_2} = \frac{2}{S\bar{c}} \int_0^{b/2} C_{m_{ac}} c^2 \, dy \tag{C.4,2}$$

The total pitching-moment coefficient about the m.a. center is then

$$C_{m_{ac_w}} = C_{m_1} + C_{m_2} = \text{const} \tag{C.4,3}$$

If $C_{m_{ac}}$ is constant across the span, and equals C_{m_2}, then (C.4,2) also becomes the defining equation for $\bar{c}$.