Chapter 12

Design of Control Surfaces

12.6.1. Introduction to Rudder Design

Rudder is a primary control surface and is responsible for the aircraft directional control. The rudder is a movable surface located on the trailing edge of the vertical tail. The rudder is the vertical counterpart to the elevator. When the rudder is rotated (i.e. deflected; δ_R), a lift force (i.e. side force, L_V) is created (Figure 12.24) by the rudder-vertical tail combination. Consequently, a yawing moment (N) about aircraft center of gravity (about aircraft z-axis) is generated. Thus, control of the yawing moment about the center of gravity is primarily provided by means of the rudder. The third unintended production of the rudder is a rolling moment. This is due to the fact that the vertical tail (i.e. rudder) is usually placed above the aircraft cg. Two fundamental roles of rudder are directional control and directional trim. Therefore, parameters of the rudder are determined by the directional trim and control requirements. The rudder control power must be sufficient to accomplish these two requirements in various flight conditions. The aircraft heading angle (ψ) is mainly determined through a directional control process.

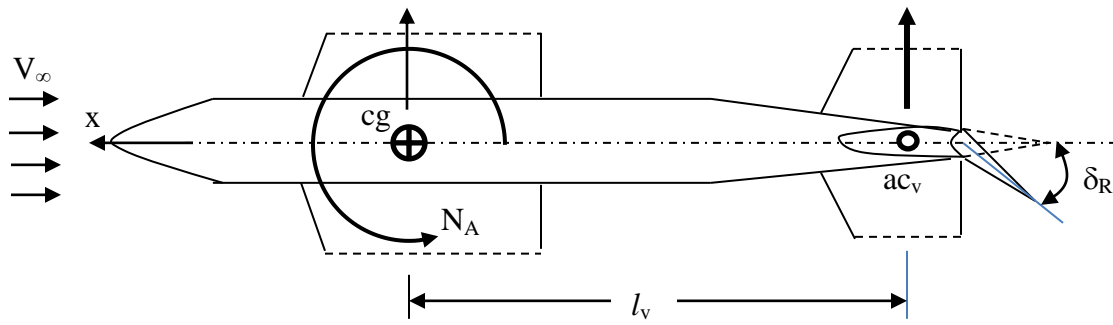y   1 L_V

Figure 12.1. Directional control via rudder deflection (Top view)

There are interferences between rudder and aileron, and they are often applied simultaneously. Thus, the lateral and directional dynamics are frequently coupled. Thus, it is a good practice to design aileron and rudder concurrently. Rudder, similar to elevator, is a displacement control device, while the aileron is a rate control device. The fundamentals of design of elevator and rudder are similar, but since their applications are different, the design of rudder is generally more complicated. However rudder deflections to the right and to the left are the same, but up and down elevator deflections are different.
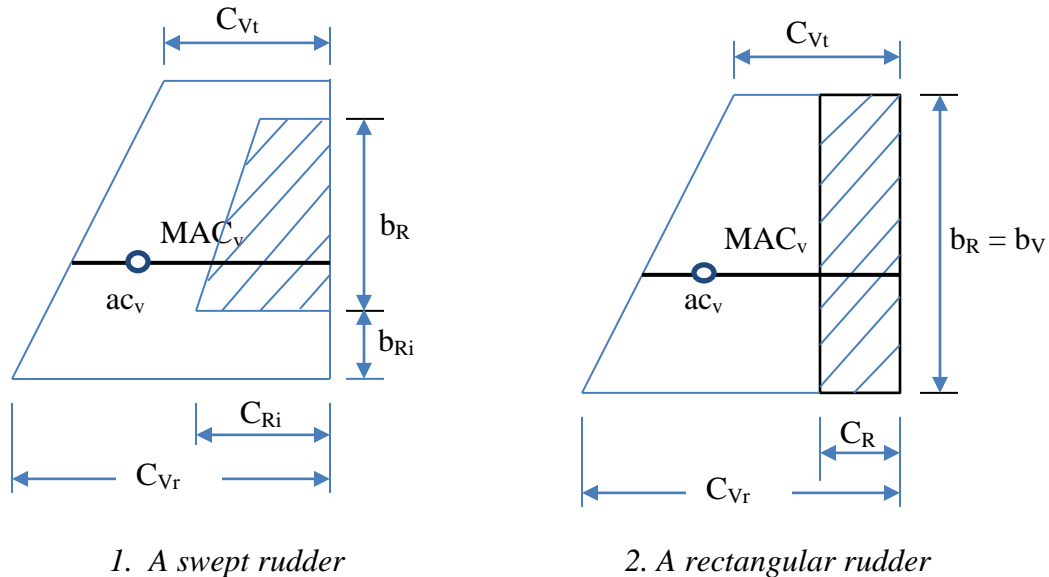

1. A swept rudder   2. A rectangular rudder

Figure 12.2. Vertical tail and rudder geometry

In the design of the rudder, four parameters must be determined. They are: 1. rudder area (S_R), 2. rudder chord (C_R), 3. rudder span (b_R), 4. maximum rudder deflection ($\pm \delta_{Rmax}$), and 5. location of inboard edge of the rudder (b_{Ri}). Figure 12.25 shows the vertical tail geometry and rudder parameters. Table 12.20 illustrates *c*haracteristics of

rudder for several aircraft. Table 12.3 shows typical values for geometry of rudder (ratio between rudder chord, span, and area to vertical tail chord, span, and area) from which one can select preliminary data.

The convention for the positive rudder deflection is defined as the deflection to the left (of the pilot). As Figure 12.24 demonstrates, a positive ruder deflection creates a positive side-force (i.e. in the positive y direction), but results in a negative yawing moment (i.e. counterclockwise). In a symmetric aircraft with a zero sideslip angle, and a zero aileron deflection, the yawing moment is determined by multiplying the vertical tail lift by the vertical tail arm:

$$N_A = l_v \cdot L_V \tag{12.94}$$

where l_v is the vertical tail arm and is the distance; along x axis; between aircraft cg and vertical tail aerodynamic center (ac_v). The vertical tail aerodynamic center is usually located at the quarter chord of the vertical tail mean aerodynamic chord.

No	Aircraft	Type	m_{TO} (kg)	S_R / S_V	C_R/C_V	δ_{Rmax} (deg)	Max cross wind speed (knot)
1	Cessna 182	Light GA	1,406	0.38	0.42	±24	
2	Cessna 650	Business jet	9,979	0.26	0.27	±25	
3	Gulfstream 200	Business jet	16,080	0.3	0.32	±20	
4	Air Tractor AT-802	Regional airliner	18,600	0.61	0.62	±24	
5	Lockheed C-130E Hercules	Military cargo	70,305	0.239	0.25	35	-
6	DC-8	Transport	140,600	0.269	35	±32.5	34
7	DC-10	Transport	251,700	0.145	38	±23/±46 [1]	30
8	Boeing 737-100	Transport	50,300	0.25	0.26		
9	Boeing 777-200	Transport	247,200	0.26	0.28	±27.3	
10	Boeing 747-200	Transport	377,842	0.173	0.22	±25	30
11	Lockheed C-5A	Cargo	381,000	0.191	0.2	-	43
12	Fokker 100A	Airliner	44,450	0.23	0.28	±20	30
13	Embraer ERJ145	Regional jet	22,000	0.29	0.31	±15	
14	Airbus A340-600	Airliner	368,000	0.31	0.32	±31.6	

Table 12.1. Characteristics of rudder for several aircraft

The aircraft side-force is primarily a function of dynamic pressure, vertical tail area (S_V), and in the direction of the vertical tail lift (L_V).

$$L_V = \bar{q} S_V C_{L_V} \tag{12.95}$$

Tandem rudder[1]

where C_{L_V} is the vertical tail lift coefficient and is a function of vertical tail airfoil section, sideslip angle, and rudder deflection. The yawing moment coefficient is linearly modeled as:

$$C_{L_V} = C_{L_{Vo}} + C_{L_{V_\beta}}\beta + C_{Lv_{\delta_R}}\delta_R \tag{12.96}$$

The aircraft aerodynamic yawing moment is a function of dynamic pressure, wing area (S), and wing span (b); and is defined as:

$$N_A = \bar{q}SC_n b \tag{12.97}$$

where C_n is the yawing moment coefficient and is a function of aircraft configuration, sideslip angle, rudder deflection and aileron deflection. The yawing moment coefficient is linearly modeled as:

$$C_n = C_{n_o} + C_{n_\beta}\beta + C_{n_{\delta_A}}\delta_A + C_{n_{\delta_R}}\delta_R \tag{12.98}$$

The parameter $C_{n_{\delta_R}}$ is referred to as the aircraft yawing moment-coefficient-due-to-rudder-deflection derivative and is also called the rudder yaw control power. The rudder yaw control effectiveness is mainly measured by the rate of change of yawing moment with respect to rudder deflection angle. In a non-dimensional form:

$$C_{n_{\delta_R}} = \frac{\partial C_n}{\partial \delta_R} \tag{12.99}$$

The directional control derivative ($C_{n_{\delta_R}}$) depends strongly on the vertical tail size, vertical tail moment arm, and is determined by:

$$C_{n_{\delta_R}} = -C_{L_{\alpha V}} \bar{V}_V \eta_V \tau_r \frac{b_R}{b_V} \tag{12.100}$$

where $C_{L_{\alpha V}}$ denotes vertical tail lift curve slope, $\bar{V}_V$ is the vertical tail volume coefficient, η_V is the vertical tail dynamic pressure ratio (q_v/q_∞). The parameter τ_r is referred to as the rudder angle-of-attack-effectiveness parameter and is a function of rudder-chord-to-vertical-tail-chord ratio (C_R/C_V). It is determined through Figure 12.12. The contribution of the rudder size to the rudder control effectiveness is reflected by the rudder angle-of-attack-effectiveness τ_r. The vertical tail volume coefficient is defined in Chapter 6 as the equation 6.72, and is repeated here for convenience:

$$\bar{V}_V = \frac{l_V S_V}{bS} \tag{12.101}$$

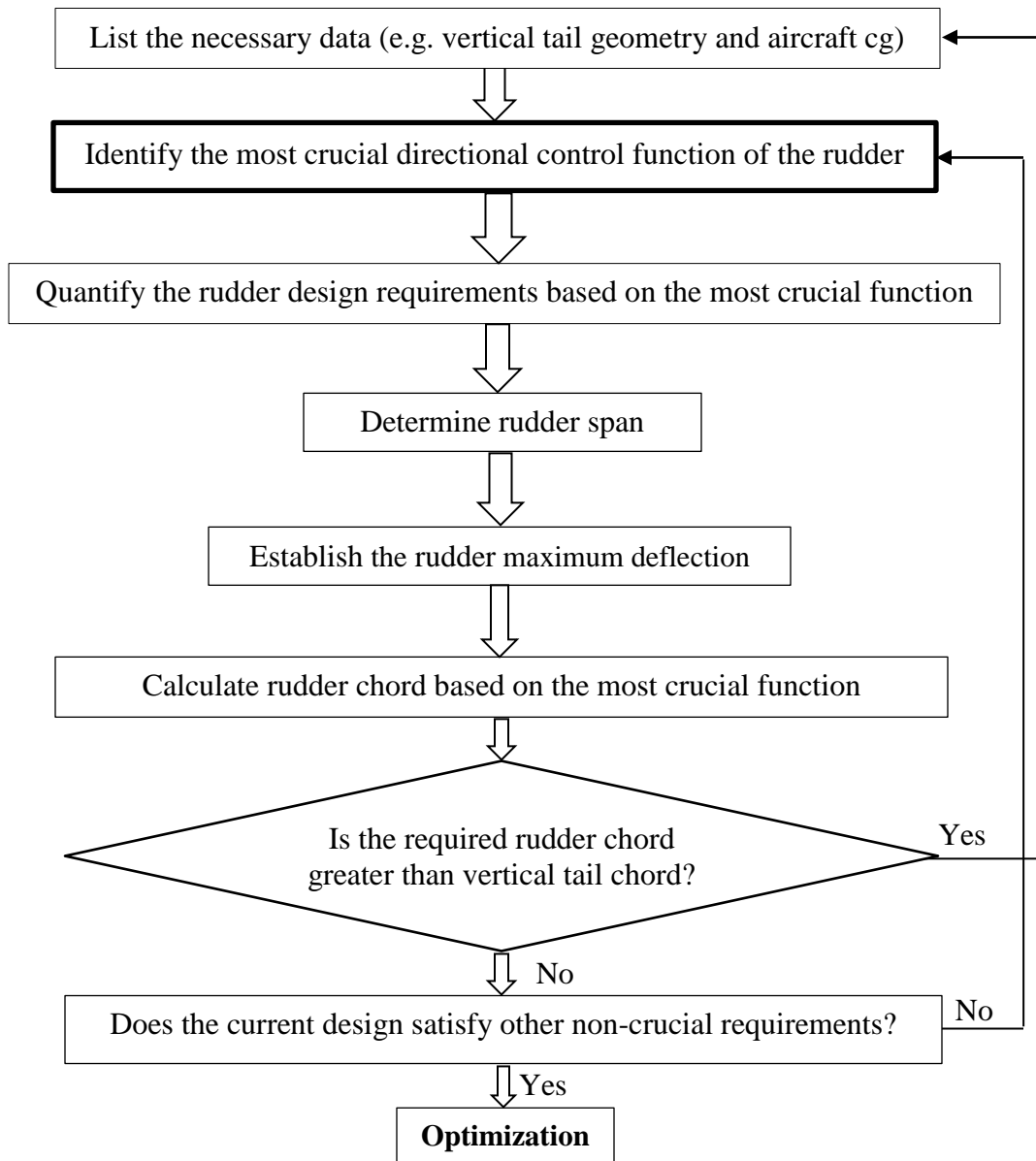Identify/define the directional control/trim requirements
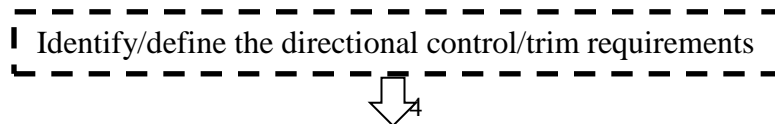
Figure 12.3. Rudder design flowchart

Table 6.4 (Chapter 6) shows typical values for vertical tail volume coefficients. In large high subsonic transport aircraft, directional control is provided by two in-tandem rudders; one for high speed flights; but both are employed in low speed operations such as take-off and landing. For the purpose of reliability, rudders could be split into upper and lower halves, with independent signals and actuators plus redundant processors.

Rudder design flowchart is presented in Figure 12.26. As it is observed, there are two checks which generate two feedbacks for the design procedure. In the first one, if the required rudder chord is greater than the vertical tail chord, the vertical tail must be

redesigned; or the aircraft cg must be relocated. In the second feedback, it is investigated to make sure that other rudder requirements are met; otherwise the designer has make a mistake in recognizing the critical role of the ruder. Both feedbacks demonstrate the iterative nature of the rudder design process. Various rudder design requirements are introduced in Section 12.6.2.

12.6.2. Fundamentals of Rudder Design
12.6.2.1. Rudder Design Requirements

The design requirements of rudder are primarily driven by directional control and directional trim. Directional control is governed mainly through yaw rate (R); while directional trim is often governed by maximum rudder deflection (δ_{Rmax}). FAA has a number of regulations for directional control; all of which must be addressed by a rudder designer.

FAR Part 25 Section 25.147 requires the following: It must be possible, with the wings level, to yaw into the operative engine and to safely make a reasonably sudden change in heading of up to 15 degrees in the direction of the critical inoperative engine. This must be shown at 1.3 V_S for heading changes up to 15 degrees, and with 1. The critical engine inoperative and its propeller in the minimum drag position; 2. The power required for level flight at 1.3 V_S, but not more than maximum continuous power; 3. The most unfavorable center of gravity; 4. Landing gear retracted; 5. Flaps in the approach position; and 6. Maximum landing weight. There is a similar regulation for GA aircraft by FAR 23; and for military aircraft by MIL-STD. These requirements must be addressed in the design of the rudder.

No	Requirements	Brief description	Aircraft
1	Asymmetric thrust	When one engine fails, the aircraft must be able to overcome the asymmetric thrust.	Multi-engine aircraft
2	Crosswind landing	An aircraft must maintain alignment with the runway during a crosswind landing.	All
3	Spin recovery	An aircraft must be able to oppose the spin rotation and to recover from a spin.	Spinnable aircraft
4	Coordinated turn	The aircraft must be able to coordinate a turn.	All
5	Adverse yaw	The rudder must be able to overcome the adverse yaw that is produced by the ailerons.	All
6	Glide slope adjustment	Aircraft must be able to adjust the glide slope by increasing aircraft drag using a rudder deflection.	Glider aircraft

Table 12.2. Rudder design requirements

The rudder plays different roles for different phases of flight in various aircraft. Six major functions of a rudder are: 1) crosswind landing, 2) directional control for

balancing asymmetric thrust on multi-engine aircraft, 3) turn coordination, 4) spin recovery, 5) adverse yaw, and 6) glide slope adjustment for a glider. Table 12.21 tabulates these cases that impose different requirements for various aircraft. In this section, these design requirements are introduced, formulated and a technique to design the rudder to satisfy these requirements is developed.

Among these functions, one of them is usually the most critical one depending upon the aircraft mission and configuration. From six duties of rudder mentioned above, the first three ones are simple but last three ones are more important. For instance, multi-engine aircraft often have directional trim in case of asymmetric thrust as the most critical case for a rudder. Single engine aircraft often have the maximum crosswind landing as the critical condition. In a spinnable aircraft, the spin recovery imposes the most critical rudder design requirement. The design of rudder is performed with regard to the most critical role of rudder. In some aircraft, spin recovery is the critical one, but in some, asymmetric power condition is critical role of rudder. In un-spinnable normal aircraft, cross wind landing is often the most critical condition for rudder by which its design proceed. Therefore, one of the first tasks of the rudder designer is to identify the most crucial case for rudder to function within aircraft flight envelope.

12.6.2.2. Asymmetric Thrust

In a multi-engine aircraft; when all engines are not located along fuselage center line; directional trim must be achieved when the critical engine(s) fails (e.g. one or more engines are inoperative). The critical engine failure must represent the most critical mode of powerplant failure with respect to controllability expected in service. Is such a case, operative engine(s) creates an undesirable yawing moment that must be nullified by the rudder. This design requirement is not applicable to a single engine aircraft where the engine thrust is aligned with fuselage centerline (in fact, when thrust-line is passing through the aircraft center of gravity). The same is true for a twin engine aircraft when both engines are placed along fuselage center-line (such as Voyager aircraft where one prop-engine is located at the fuselage nose and the other one is at the fuselage rear section). The critical asymmetric power/thrust condition frequently occurs when all engines of one side of the aircraft fail at low speeds. The rudder must be powerful enough to overcome the yawing moment produced by the asymmetric thrust arrangement. Figure 12.27-1 shows a Boeing 777 with right engine inoperative.

Since one engine may suddenly fail during flight, thrust of other engine(s) may impose a yawing moment about aircraft center of gravity, so it may disturb aircraft directional trim, then deviate direction of flight. In this condition which is called asymmetric power condition, rudder deflection must produce a side force, then a moment in order to counter yawing moment of working engine(s). The critical asymmetric power

condition occurs for a multi-engine airplane when engine(s) of one side fail at low flight speeds. FAR regulations section 25.149 requires a multi-engine transport aircraft to be directionally controllable at a critical speed referred to as the minimum controllable speed (V_{MC}). This speed may not exceed 1.13 stall speed at the most unfavorable cg location and the most critical takeoff configuration. The rudder must be able to overcome the yawing moment produced by the asymmetric thrust arrangement. Furthermore; lateral control must be sufficient to roll the airplane, from an initial condition of steady flight, through an angle of 20 degrees in the direction necessary to initiate a turn away from the inoperative engine(s), in not more than 5 seconds.

In a single engine aircraft; when the engine thrust line is passing through the aircraft cg; this condition would not be conceivable; thus the rudder does not have such a role. However the vertical tail and rudder are expected to offset the moment produced by rotating propeller. To consider a safety margin, the author recommends the rudder designers to consider the minimum controllable speed (V_{MC}) to be 80 percent of the stall speed (i.e. $V_{MC} = 0.8V_s$). This suggested speed clearly requires that the aircraft be directionally trimmable during take-off ground roll (i.e. low altitude). The minimum controllable speed for a number of aircraft is as follows: Lockheed C-130 Hercules (Figure 5.4): 93.5 KEAS; Fokker 28: 71 KEAS; Airbus A-300B: 103 KEAS; Boeing 707-320B: 122 KEAS; and Boeing 747-200 (Figures 3.7, 3.12, 9.4): 138 KEAS.

Suppose the aircraft pictured in Figure 12.27-3 lost power in its right engine (i.e. $T_R = 0$). Both engines are located a distance of y_T from the fuselage centerline. The resulting asymmetric thrust would produce a yawing moment about the aircraft cg equal to $T_L \times y_T$. In a steady-state trimmed flight, the thrust of the operative engine (i.e. T_L) must equal the aircraft drag; as well as the summation of yawing moments must be zero. Therefore:

$$\sum N_{cg} = 0 \Rightarrow T_L y_T + L_V l_v = 0 \Rightarrow N_A = -T_L y_T \tag{12.102}$$

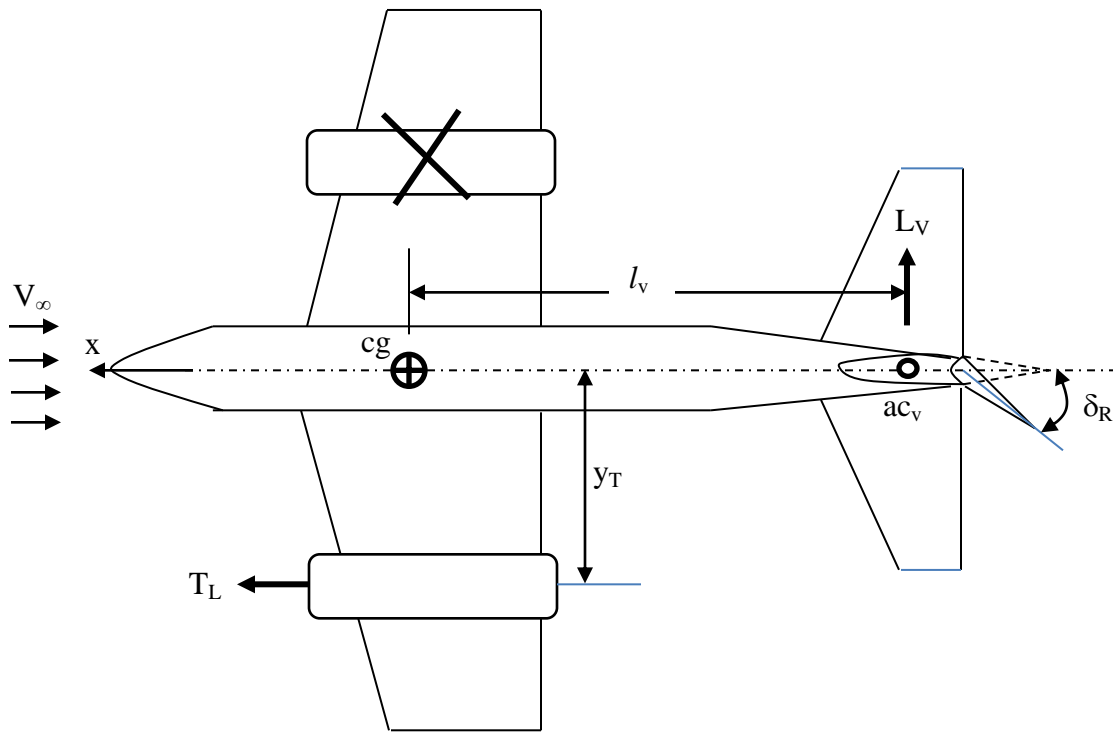Inserting equations 12.94, 12.97, and 12.98 into equation 12.102 yields:

$$N_A = \bar{q} S b \left(C_{n_o} + C_{n_\beta} \beta + C_{n_{\delta_A}} \delta_A + C_{n_{\delta_R}} \delta_R \right) \tag{12.103}$$

1. A Boeing 777 with right engine inoperative (Courtesy of Hideki Nakamura)

2. Applications of aileron, rudder, and elevator in a maneuver of a McDonnell Douglas EF-18 Hornet (Courtesy of Antony Osborne)

3. The balance of moments in a twin-engine aircraft when right engine is inoperative

Figure 12.4. Directional control and trim

You may assume that the aircraft is symmetric about xz plane (i.e. $Cn_o = 0$), the aileron is not deflected (i.e. $\delta_A = 0$), and there is no sideslip angle (i.e. $\beta = 0$). Therefore, the required rudder deflection to trim directional the aircraft in an asymmetric thrust condition is:

$$\delta_R = \frac{T_L y_T}{-\bar{q} S b C_{n_{\delta_R}}}$$

(12.104a)

For other aircraft configurations such as an aircraft with three or more engines, a similar technique is employed to determine the required rudder deflection. If the aircraft possesses more than one engine in one side of the aircraft, the total yawing moment of the operative engines in one side is considered:

$$\delta_R = \frac{\sum_{i=1}^{n} T_{L_i} y_{T_i}}{-\bar{q}SbC_{n_{\delta_R}}}$$

(12.104b)

where n denotes number of engine in one side of the aircraft. Given the aircraft geometry and engine thrust, one can calculate the rudder deflection to keep aircraft directionally trimmed. The maximum rudder deflection is required when aircraft has the lowest airspeed, and the operative engine is generating its maximum thrust. In Figure 12.27, the rudder is deflected positively due to the right inoperative engine. In case the left engine is inoperative, a negative rudder deflection must be utilized; and equations 12.103 and 12.104 must be revised accordingly.

Example 12.1

A large transport aircraft with a maximum take-off mass of 65,000 kg is equipped with two turbofan engines each generating 116 kN of thrust. The lateral distance between two engines is 12 m, and the maximum allowable rudder deflection is ±30 degrees. Other characteristics of the aircraft are as follows:

$C_{L_{\alpha_V}} = 4.5 \ \frac{1}{rad}$; S = 125 m²; b = 34 m; S_V = 26 m²; b_v= 7.6 m; b_R = b_v; l_v = 18 m, C_R/C_V = 0.3; η_v = 0.97; V_s = 110 knot

Is the rudder acceptable for maintaining directional trim in an asymmetric thrust flight condition?

Solution:

It is desired to directionally trim the aircraft in an asymmetric thrust flight condition when the aircraft minimum controllable speed is 80 percent of stall speed; so:

$$V_{MC} = 0.8 \cdot V_S = 0.8 \times 110 = 88 \ knot = 45.27 \ \frac{m}{\sec}$$

The vertical tail volume coefficient is:

$$\bar{V}_V = \frac{l_V S_V}{bS} = \frac{18 \times 26}{34 \times 125} = 0.114$$

(12.101)

The rudder angle of attack effectiveness (τ_r) is a function of rudder-chord-to-vertical-tail-chord ratio (C_R/C_V). The parameter τ_r is given to be 0.3; so from figure 12.12, rudder angle of attack effectiveness is determined to be 0.52. The rudder control derivative is:

$$C_{n_{\delta_R}} = -C_{L_{\alpha_V}} \bar{V}_V \eta_V \tau_r \frac{b_R}{b_V} = -4.5 \times 0.114 \times 0.97 \times 0.52 \times 1 = -0.266 \; \frac{1}{rad} \qquad (12.100)$$

The rudder deflection to balance the asymmetric thrust at sea level is:

$$\delta_R = \frac{T_L y_T}{-\bar{q} S b C_{n_{\delta_R}}} = \frac{116{,}000 \times \dfrac{12}{2}}{-\dfrac{1}{2} \times 1.225 \times (45.27)^2 \times 125 \times 34 \times (-0.266)} \qquad (12.104a)$$

or

$$\delta_R = 0.49 \; rad = 28.06 \; \deg$$

The required rudder deflection is less the maximum allowable rudder deflection (i.e. 28 < 30). Therefore this rudder geometry is acceptable and can satisfy the asymmetric thrust balance requirement.

12.6.2.3. Crosswind Landing

One of the most important functions of a rudder in all types of aircraft is to maintain safe landing while a crosswind is blowing. When a cross wind blows during landing operation; and if the pilot does not react, the aircraft will exit out of the runway. The pilot is required to employ a special technique to maintain alignment with the runway during cross-wind landing. In general, the final approach under crosswind conditions may be conducted in two ways: 1. with wings-level (i.e., applying a drift correction in order to track the runway centerline, this type of approach is called a crabbing; 2. with a steady sideslip (i.e., with the aircraft fuselage aligned with the runway centerline, using a combination of into-wind aileron and opposite rudder to correct the drift). Most airlines recommend the first technique.

During the crosswind landing, rudder is applied to align the aircraft with the runway heading. The rudder must be powerful enough to permit the pilot to trim for the specified cross-winds. The reason why aircraft deviate toward cross wind (and then changing the right direction of landing) is directional (weathercock) stability of aircraft. In such situation, the rudder produces a sideslip angle to maintain alignment with the runway. In this section, the first technique is addressed and governing equations are developed. When touching down with some crab angle on a dry runway, the aircraft automatically realigns with the direction of travel down the runway. However, if prevailing runway conditions and crosswind component are considered inadequate for a safe landing, the pilot may request the assignment of a more favorable runway.
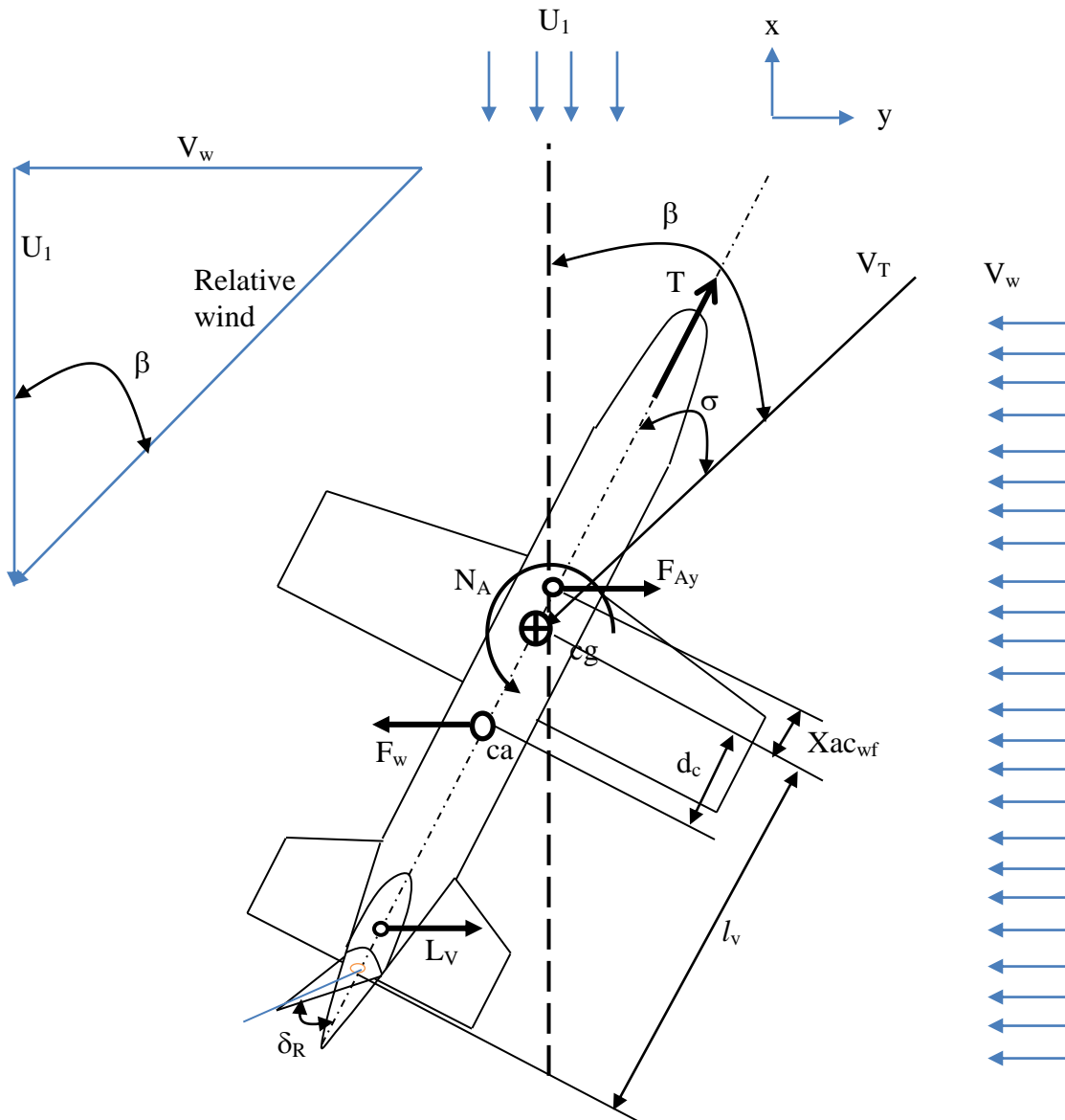
Figure 12.5. Forces and angles in crosswind crabbed landing

According to airworthiness standards, aircraft must be able to land safely during cross-wind with specified speed. For instance, according to JAR-VLA article 233, in every very light aircraft, landing may be carried out for 90 degrees cross-winds of up to 10 knots. Federal Aviation Regulations (FAR) Part 23 section 233 requires that each GA aircraft must be able to carry out landing for 90 degrees cross-winds of up to a wind velocity of 25 knots. There may be no uncontrollable ground-looping tendency in cross-wind landing. It is evident that the critical aircraft speed at cross-wind landing is the minimum speed ($1.1\ V_S$) which is a good criterion in the design of rudder for single engine GA aircraft. Operations in crosswind conditions require strict adherence to

applicable crosswind limitations and operational recommendations. About 85 % of crosswind incidents and accidents occur at landing.

To evaluate the rudder power in a crabbed landing, consider the aircraft in Figure 12.28 which is approaching with a forward airspeed of U_1 along runway. There is a crosswind of V_W from the right that is creating a positive sideslip angle. The sideslip angle is defined as the angle between flight direction and the relative wind:

$$\beta = \tan^{-1}\left(\frac{V_W}{U_1}\right) \tag{12.105}$$

The sideslip angle generates a yawing moment (N_A) and an aerodynamic side-force (F_{Ay}) by the aircraft. The weathercock characteristic of the aircraft tends initially to rotate the aircraft about cg (z-axis) and to yaw the aircraft toward the relative wind. The relative wind or aircraft total speed is the vector summation of the aircraft forward speed and wind speed:

$$V_T = \sqrt{U_1^2 + V_W^2} \tag{12.106}$$

In order to keep the aircraft landing direction along the runway, the rudder is employed to counteract the yawing moment created by the wind. The rudder produces a vertical tail lift along y axis (L_V) which consequently contributes the aircraft yawing moment and aerodynamic side force. The application or rudder is to create a crab angle (σ) in order to prevent aircraft from yawing to the relative wind and avoid drifting away from the runway. The rudder must be powerful enough to create the desired crab angle. The crab angle is defined as the angle between fuselage centerline and the runway (i.e. heading direction). Figure 12.28 shows all the forces and moments affecting the final approach operation while the aircraft is in a crabbed landing. The aircraft is in directional trim during a crabbed landing; so the following three force and moment equilibrium equations govern the flight condition.

$$\sum N_{cg} = 0 \Rightarrow N_A + F_w \cdot d_c \cos\sigma = 0 \tag{12.107}$$

$$\sum F_x = 0 \Rightarrow T\cos\sigma = D \tag{12.108}$$

$$\sum F_y = 0 \Rightarrow F_w = F_{A_y} \tag{12.109}$$

where d_c is the distance between the aircraft cg and center of the projected side area of the aircraft, and Xac_v is the distance between the aircraft cg and vertical tail aerodynamic center, T denotes engine thrust, and D is aircraft drag. The aircraft aerodynamic side force (F_{Ay}) and yawing moment (N_A) are determined as follows:

$$N_A = \bar{q}Sb\left(C_{n_o} + C_{n_\beta}(\beta - \sigma) + C_{n_{\delta_R}}\delta_R\right) \tag{12.110}$$

$$F_{A_y} = \bar{q}S\left(C_{y_o} + C_{y_\beta}(\beta - \sigma) + C_{y_{\delta_R}}\delta_R\right) \tag{12.111}$$

where S denotes the wing area and $\bar{q}$ is the dynamic pressure which is a function aircraft total speed.

$$\bar{q} = \frac{1}{2} \rho V_T^2 \qquad (12.112)$$

The force of the crosswind (F_w) acts on the center of the side area (ca) of aircraft (See Figure 12.29). In a conventional aircraft, the center of side area is always behind aircraft center of gravity. This is mainly due to the fact that the vertical tail is located at the rear section of the fuselage. The crosswind force is drag-like and shall be determined in a similar fashion. The force generated by the crosswind (F_w) is a function of wind speed, aircraft side area, and side given by:

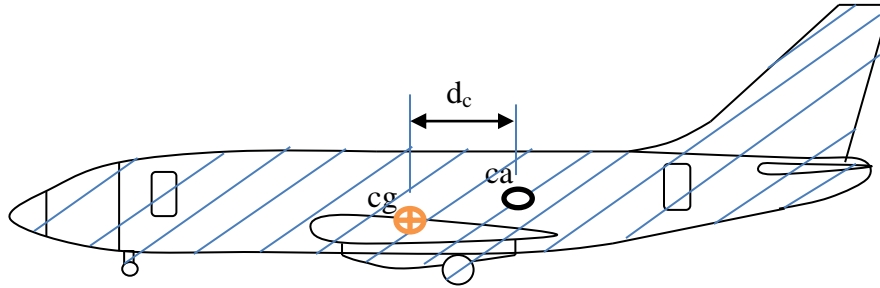$$F_w = \frac{1}{2} \rho V_W^2 S_S C_{D_y} \qquad (12.113)$$

Figure 12.6. Aircraft projected side area and center of side area

where S_S represents the aircraft projected side area; and C_{Dy} is aircraft side drag coefficient. Typical values for aircraft side drag coefficient for conventional aircraft are 0.5 to 0.8. Only the aircraft aerodynamic side force (F_{Ay}) and yawing moment (N_A) are functions of rudder deflections, so equations 12.107 and 12.109 suffices for rudder design process. Inserting equations 12.110, 12.111, 12.112, and 12.113 into equations 12.107, and 12.109 results in:

$$\frac{1}{2} \rho V_T^2 Sb\left(C_{n_o} + C_{n_\beta}(\beta - \sigma) + C_{n_{\delta_R}} \delta_R\right) + F_w \cdot d_c \cos\sigma = 0 \qquad (12.114)$$

$$\frac{1}{2} \rho V_W^2 S_S C_{D_y} - \frac{1}{2} \rho V_T^2 S\left(C_{y_o} + C_{y_\beta}(\beta - \sigma) + C_{y_{\delta_R}} \delta_R\right) = 0 \qquad (12.115)$$

In this set of equations, there are two unknowns: 1. rudder deflection (δ_R), 2. crab angle (σ). Solving these equations simultaneously yields two unknowns. Other flight parameters such as sideslip angle, engine thrust, aircraft drag, and wind force would be calculated separately. In designing rudder, the designer should make certain that the rudder is powerful enough to allow for the aircraft to safely land in a crosswind situation.

Four directional stability and control derivatives of C_{n_β}, $C_{n_{\delta_R}}$, C_{y_β}, and $C_{y_{\delta_R}}$ are affecting Equations 12.114 and 12.115. The directional control derivative $C_{n_{\delta_R}}$ is already introduced by equation 12.100. The static stability derivatives C_{n_β} is defined in Chapter 6 as equation 6.73, but is repeated here for the convenience.

$$C_{n_\beta} = K_{f1} C_{L_{\alpha_V}} \left(1 - \frac{d\sigma}{d\beta}\right) \eta_V \frac{l_{V_t} S_V}{bS} \tag{12.116}$$

The other two derivatives of C_{y_β} and $C_{y_{\delta_R}}$ must be calculated using wind tunnel testing or references such as [7]. These derivatives may be determined as following:

$$C_{y_\beta} = \frac{\partial C_y}{\partial \beta} \approx C_{y_{\beta_V}} = -K_{f2} C_{L_{\alpha_V}} \left(1 - \frac{d\sigma}{d\beta}\right) \eta_V \frac{S_V}{S} \tag{12.117}$$

$$C_{y_{\delta_R}} = \frac{\partial C_y}{\partial \delta_R} = C_{L_{\alpha_V}} \eta_V \tau_R \frac{b_R}{b_V} \frac{S_V}{S} \tag{12.118}$$

The parameter K_{f2} represents the contribution of fuselage to derivative C_{y_β} and depends strongly on the shape of the fuselage and its projected side area. The fuselage contribution to derivative C_{y_β} tends to be positive. The typical value of K_{f2} for a conventional aircraft is about 1.3 to 1.4. The parameter $d\sigma/d\beta$ is referred to as the vertical tail sidewash gradient. Please note that, in Figure 12.28, the rudder is deflected positively due to a positive sideslip angle. In case, there is a crosswind from the left (i.e. a negative sideslip angle), a negative rudder deflection must be utilized; and equations 12.105 through 12.118 must be revised accordingly.

In order to determine the center of the projected area of an aircraft (ca), the aircraft side-view must be divided into several standard geometric shapes (segments) such as rectangle, triangle, and circle. By selecting a reference line (say fuselage nose), the distance between "ca" and the reference line (Figure 12.29) is obtained through the following mathematical relationship:

$$x_{ca} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} \tag{12.119}$$

where n represents the number of segments, A_i the projected side area of ith segment, and x_i is the distance between the center of projected side area of ith segment to the reference line. The center of standard geometric shapes such as triangle and rectangle is known and may be readily obtained from standard mathematical handbooks such as [22]. Examples 12.2 and 12.6 will demonstrate the application of the technique.

Example 12.2
Problem Statement: Consider a light transport aircraft with a take-off mass of 7,400 kg, wing area of 32 m² and wing span of 8 m. The aircraft projected side area is 34 m² and the aircraft center of projected side area is 1.8 m behind aircraft cg. The aircraft approach speed is 82 knot; and maximum allowable rudder deflection is ±30 degrees. Other aircraft characteristics including two rudder related derivatives are as follows:

$$C_{n_\beta} = 0.1 \ \frac{1}{rad}; \ C_{n_{\delta_R}} = -0.08 \ \frac{1}{rad}; \ C_{y_\beta} = -0.6 \ \frac{1}{rad}; \ C_{y_{\delta_R}} = 0.15 \ \frac{1}{rad}; \ C_{D_y} = 0.6$$

$$C_{n_o} = 0; \ C_{y_o} = 0$$

Is the aircraft rudder powerful enough to allow for a safe crabbed landing when a perpendicular crosswind of 30 knots is blowing? What about 25 knots?

Solution:

a. Crosswind of 30 knot

Due to simplicity, a crosswind from the right is assumed; which is generating a positive sideslip angle. The aircraft total speed, wind force, and sideslip angle are:

$$V_T = \sqrt{U_1^2 + V_W^2} = \sqrt{(82)^2 + (30)^2} = 87.316 \ knot = 44.92 \ \frac{m}{sec} \tag{12.106}$$

$$F_w = \frac{1}{2}\rho V_W^2 S_S C_{D_y} = \frac{1}{2}\times 1.225 \times (30 \times 0.514)^2 \times 34 \times 0.6 = 2976 \ N \tag{12.113}$$

$$\beta = \tan^{-1}\left(\frac{V_W}{U_1}\right) = \tan^{-1}\left(\frac{30}{82}\right) = 0.351 \ rad = 20.1 \ deg \tag{12.105}$$

Now, we have two equations and two unknowns:

$$\frac{1}{2}\rho V_T^2 Sb\left(C_{n_o} + C_{n_\beta}(\beta-\sigma) + C_{n_{\delta_R}}\delta_R\right) + F_w \cdot d_c \cos\sigma = 0 \tag{12.114}$$

$$\frac{1}{2}\rho V_W^2 S_S C_{D_y} = \frac{1}{2}\rho V_T^2 S\left(C_{y_o} + C_{y_\beta}(\beta-\sigma) + C_{y_{\delta_R}}\delta_R\right) \tag{12.115}$$

or

$$\frac{1}{2}\times 1.225 \times (44.92 \times 0.514)^2 \times 32 \times 8[0.1(0.351-\sigma) - 0.08\delta_R] + 2976 \times 1.8\cos\sigma = 0 \tag{12.114}$$

$$2976 = \frac{1}{2}\times 1.225 \times (44.92 \times 0.514)^2 \times 32(-0.6(0.351-\sigma) + 0.15\delta_R) \tag{12.115}$$

Simultaneous solution of these equations results in:
$\delta_R = 0.64$ rad $= 36.6$ deg
$\sigma = 0.316$ rad $= 18.12$ deg

The required rudder deflection (35.6 deg) exceeds the maximum allowable deflection (30 deg). Thus the aircraft is not able to handle a crosswind of 30 knots.

b. Crosswind of 25 knot

The same technique is employed; only the wind speed is 25 knot. The calculation results in a sideslip angle of +16.9 degrees and crab angle of 14.7 degrees. In addition, the required rudder deflection is +29.64 degrees which is slightly less than maximum allowable limit. Therefore; this aircraft is able to safely carb land with a crosswind of 25 knot.

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12.6.2.4. Spin Recovery

One of the most important roles of a rudder in majority of airplanes is spin recovery. The most significant instrument to recover aircraft from a spin is a powerful rudder. Spin is a self-sustaining (auto-rotational) spiral motion of an airplane about vertical (z) axis, during which the mean angle of attack of the wings is beyond the stall. Almost since man first flew, spinning has caused many fatal accidents, so that most accidents were due to spin. During years 1965 to 1972, US Navy has lost an average of 2 aircraft per month and total of 169 aircraft due to spin, the list of which is headed by 44 fighter aircraft F-4s (Phantom). This statistics show the crucial role of the rudder in a spin.

Spin is a high angle of attack/low airspeed situation; the airspeed will be hovering somewhere down in the stall area. Spin has two particular specifications: 1. Fast rotation around vertical axis, 2. Fully stalled wing. Spin is usually starts after wing stalls. One of the reasons why aircraft enter into spin is that inboard of the wing stalls before outboard of the wing, in other word, lift distribution over the wing is not elliptic. Spin is recovered by a procedure which all control surfaces (elevator, aileron, and rudder) contribute; particularly the rudder in an apparently unnatural way. The rudder is the most significant element is spin recovery to stop rotation. The primary control for spin recovery in many airplanes is a powerful rudder.

The rudder must be powerful enough to oppose the spin rotation in the first place. A spin follows departures in roll, yaw and pitch from the condition of trim between the predominantly pro-spin moment due to the wings and the generally anti-spin moments due to other parts of the aircraft. If spin is not recovered, aircraft will eventually crash. The criterion for rudder design in a spinnable aircraft may be spin recovery. Acrobatic and fighter airplanes are usually spinnable, but there are some airplanes such as some transport aircraft that are spin-proof or un-spinnable.

In un-spinnable aircraft, spin recovery is not a criterion for design of rudder, i.e. rudder does not have to recover aircraft from spin. According to airworthiness standards, in a spinnable aircraft, rudder must have enough power to recover spin in a limited time. For instance, JAR-VLA article 221 requires that any very light aircraft must be able to recover from spin in maximum period of one turn. FAR Part 23 Section 23.221 regulate the spinning for Normal category airplanes as follows: A single-engine, normal category airplane must be able to recover from a one-turn spin or a three-second spin, whichever takes longer, in not more than one additional turn after initiation of the first control action for recovery, or demonstrate compliance with the optional spin resistant requirements of this section.
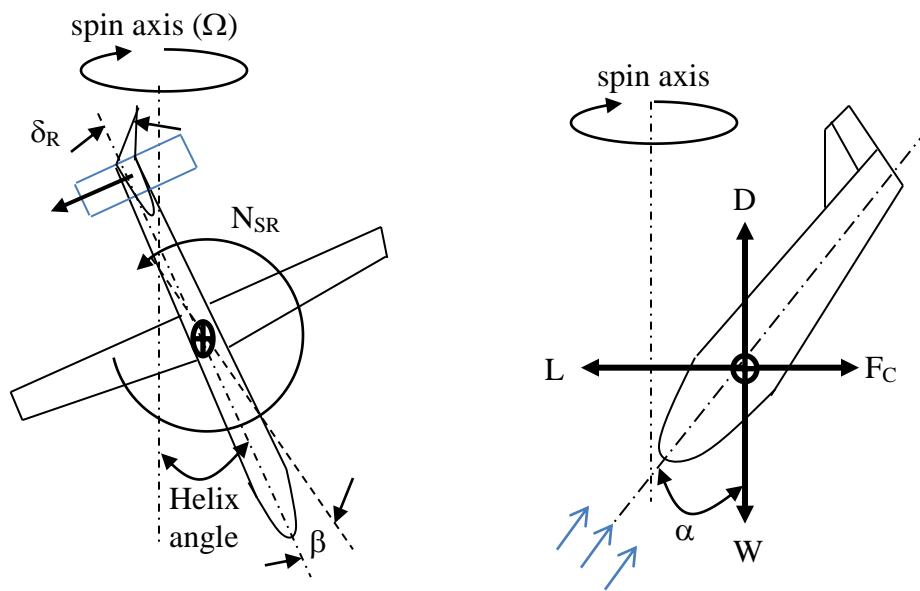
1. Rudder counteracts the yawing motion (top-view) 2. Equilibrium of forces in spin (side-view)

Figure 12.7. Forces and moments contributing to spin

Although the wing is fully stalled; left and right wing sections produce different lift. So, aircraft begins to roll around x-axis. Furthermore, since drags of right and left wing sections are different, aircraft yaws towards down-going wing (i.e. autorotation). In addition, since the aircraft is stalled, it loses lift and starts to dive, when in a normal spin entry the spin develops to an equilibrium state. Thus there is a mixture of stall, roll, yaw and dive in a spin. Although the autorotation property of a wing, when a large part of it has an angle of attack beyond the stall, is the primary cause of a spin, this does not necessarily mean that a spin will occur.

There are damping moments provided by the fuselage, and the anti-spin moment from the vertical tail which together counter the propelling moments from the wings. The

result is that for a given combination of control settings there is one equilibrium rate of rotation at each angle of attack. A spin can only follow from autorotation if the equilibrium of pitching moments, and inertial moments can be sustained. Aircraft which have such characteristics are called spinnable. But if equilibrium of the pitching moments and the inertial moments cannot be obtained simultaneously, the aircraft will recover itself.

Typical range of some spin parameters is as follows: angle of attack (α): 30 to 60 degrees; rate of descent (ROD): 20 to 100 m/sec; rate of spin (Ω): 20 to 40 rpm; helix angle (γ): 3 to 6 degrees; and helix radius (R): half of wing span. As angle of attack increases; rate of rotation increases; and helix radius decreases.

Basically, the rudder is not the only factor to feature an acceptable spin recovery. Two other significant factors are as follows: 1. aircraft mass distribution and aircraft moments of inertia, 2. fuselage side area and cross section. It is very important that the inertia term be made anti-spin (negative for right spin) for recovery. When the magnitudes of pitch (I_{yy}) and roll (I_{xx}) inertia are close, the effect of inertia term is little; and hence the rudder, will be the primary control for spin recovery. But whenever the inertia term becomes quite significant, they have a considerable impact on the spin motion, and thus, the size of rudder. The application of aileron to aid recovery in generally not recommended due to its nuisance impact. In some cases, the use of ailerons while stopping a spin may suddenly cause a spin in the reverse direction.

Prior to 1940's, due to placing fuel, stores and engines on the wing, the changes in the inertia term were small (i.e. $I_{xx} - I_{yy} \approx 0$). Thus, the rudder was the only effective control to prevent spin and spin recovery. However, today the changes in the inertia term is much more significant, because of distribution of mass along the fuselage, so pitch to roll inertia ratio (I_{yy} / I_{xx}) has a considerable value; and the term ($I_{xx} - I_{yy}$) has a large negative value. For this reason; although in modern aircraft; the control surfaces are no larger similar to those of older aircraft, their spin recovery characteristics are much less important.

When the aircraft mass concentration in the wing is greater than the mass concentration in the fuselage (e.g. sailplane); the aircraft moment of inertial will induce a pro-spin behavior. The net result is that, the aircraft inertia produces an in-spin yawing moment which increases the outward sideslip. However; when the aircraft mass concentration in the wing is smaller than the mass concentration in the fuselage (e.g. fighter); the aircraft moment of inertial will induce an anti-spin behavior. Hence, the aircraft inertia creates an out-of-spin yawing moment which decreases the outward sideslip.

Damping provided by various parts of aircraft such as fuselage and rudder can counter the yawing moment of the wings during spin. So provision of a large amount of damping in yaw for fuselage and vertical will be two most effective means for the prevention of a spin. The aerodynamic yawing moment due to rotation of the fuselage about spin axes is largely dependent on the fuselage shape and its cross section. In addition, the provision of strakes on the fuselage, in front of tail, will increase the fuselage damping. Therefore; the aircraft designer can reduce the spin recovery load on the rudder by a careful design of the fuselage and a proper aircraft weight distribution.

When a steady-state spin is developed, the equilibrium of forces implies that the lift is equal to centrifugal force and aircraft weight is equal to aircraft drag (See Figure 12.30). In order to stop the spin, a yawing moment is needed. In this situation, three moments of inertia of rolling moment of inertia (I_{xx}), yawing moment of inertia (I_{yy}), and product of inertia (I_{zz}) influence the recovery process. Newton's second law governs the aircraft rotation about z-axis (i.e. yaw rate; R) in a spin recovery operation as follows:

$$N_{SR} = \left(\frac{I_{xx} I_{zz} - I_{xz}^2}{I_{xx}} \right)_w \dot{R}_{SR}$$

(12.120)

The aircraft rolling moment of inertia (I_{xx}), the yawing moment of inertia (I_{yy}), and the product of inertia (I_{zz}) are generally calculated in the body-fixed axis system. Table 11.12 (Chapter 11) illustrates body-axis mass moments of inertia for several aircraft. During a spin where the aircraft possesses a high angle of attack; the aircraft is not yawing about body z-axis; rather it is rotation about wind z-axis. The subscript w is the equation 12.120 indicates that all three moments of inertia (I_{xx}, I_{zz}, and I_{xz}) must be computed in the wind-axis system. Thus a transformation involving aircraft angle of attack is necessary. This transformation is mathematically performed by the following matrix equation:

$$\begin{bmatrix} I_{xx_w} \\ I_{zz_w} \\ I_{xz_w} \end{bmatrix} = \begin{bmatrix} \cos^2 \alpha & \sin^2 \alpha & -\sin 2\alpha \\ \sin^2 \alpha & \cos^2 \alpha & \sin 2\alpha \\ \frac{1}{2}\sin 2\alpha & -\frac{1}{2}\sin 2\alpha & \cos 2\alpha \end{bmatrix} \begin{bmatrix} I_{xx_B} \\ I_{zz_B} \\ I_{xz_B} \end{bmatrix}$$

(12.121)

It is interesting to note that, the effect of a high angle of attack on I_{xz} is quite large. The desired rate of yaw rate (i.e. $\dot{R}$) is generally given by references such as FAR regulations. For instance, FAR Part 23 Section 23.221 requires that a GA aircraft airplane must be able to recover from a one-turn spin, in not more than three seconds. In addition, a typical value for the rate of spin (Ω) is about 20 to 40 rpm or 120 degrees/sec to 240

degrees/sec; where the higher rate is the most critical one. Hence the rate of spin recovery ($\dot{R}_{SR}$) will be desired to be:

$$\dot{R}_{SR} = \frac{\Omega}{t} = \frac{240 \text{ deg/sec}}{3 \text{ sec}} = 80 \frac{\text{deg}}{\text{sec}^2} = 1.396 \frac{rad}{\text{sec}^2} \approx 1.4 \frac{rad}{\text{sec}^2} \qquad (12.122)$$
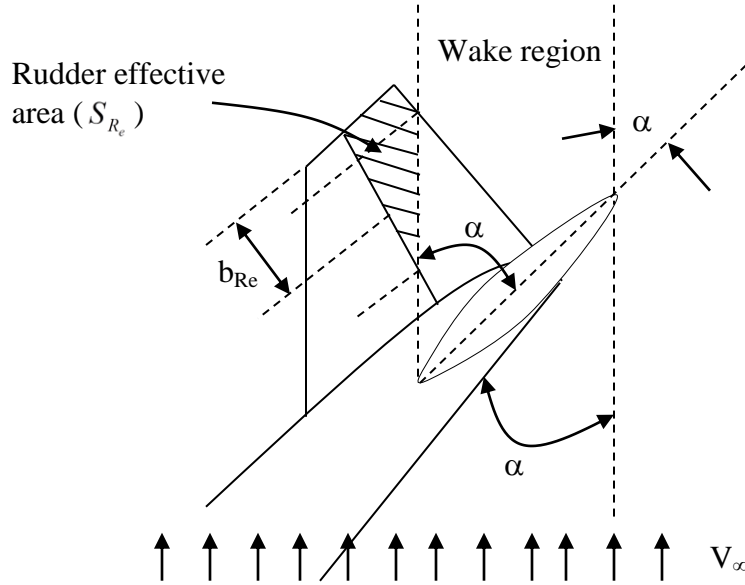

Figure 12.8. The influence of horizontal tail on the effectiveness of vertical tail and rudder

Therefore it is suggested to design the rudder such that the aircraft be able to recover from a spin by the rate of yaw rate of 80 deg/sec^2 or 1.4 rad/sec^2. The desired aircraft counteracting yawing moment (N_{SR}); created by the rudder deflection; is already defined by equation 12.97; where the aircraft airspeed is assumed to be equal to the stall speed. Hence,

$$N_{SR} = \frac{1}{2} \rho V_s^2 SbC_{n_{\delta_R}} \delta_R \qquad (12.123)$$

Furthermore, the rudder control derivate $C_{n_{\delta_R}}$ is already defined by equation 12.100 for a clean flight condition. However; during a spin; due to the shielding of the rudder by the horizontal tail (See Figure 12.31), parts of the vertical tail and rudder are in the wake region. The shielding effect will negatively impact the effectiveness of the rudder and vertical tail. Thus, only unshielded areas of vertical tail and rudder may contribute to the yawing moment generation and to derivative $C_{n_{\delta_R}}$. Therefore the equations 12.100 and 12.101 should be modified to include the effective vertical tail area

(S_{V_e}), effective ruder area (S_{R_e}), effective rudder span (b_{R_e}), and effective rudder chord (C_{R_e}). The effective rudder and vertical span, chord, and area are determined by measurement of the rudder and vertical tails areas out of wake region (See Figure 12.31). Thus, the rudder control derivative $C_{n_{\delta_R}}$ during a spin is redefined as follows:

$$C_{n_{\delta_R}} = -C_{L_{\alpha_V}} \overline{V}_{V_e} \eta_V \tau_R \frac{b_{R_e}}{b_V} \tag{12.124}$$

Accordingly, the effective vertical tail volume ratio is given by:

$$\overline{V}_{V_e} = \frac{l_V S_{V_e}}{bS} \tag{12.125}$$

The horizontal tail wake region is the area above a stalled horizontal tail where the tail has a high angle of attack of α. Since the aircraft and tail angles of attack during spin vary and are functions of a number of parameters, a rule of thumb is established. As a rough rule, the horizontal tail wake region is considered to lie between two lines. The first line is drawn at the horizontal tail trailing edge by the orientation of 30 degrees. The second line is drawn at the horizontal tail leading edge by the orientation of 60 degrees. Therefore, the influence of horizontal tail on the effectiveness of the vertical tail and rudder during a spin shall be applied using equations 12.124 and 12.125.

Example 12.3
Consider the single engine utility aircraft; shown in Figure 12.32; with a maximum take-off mass of 1,400 kg and a cruciform tail configuration.
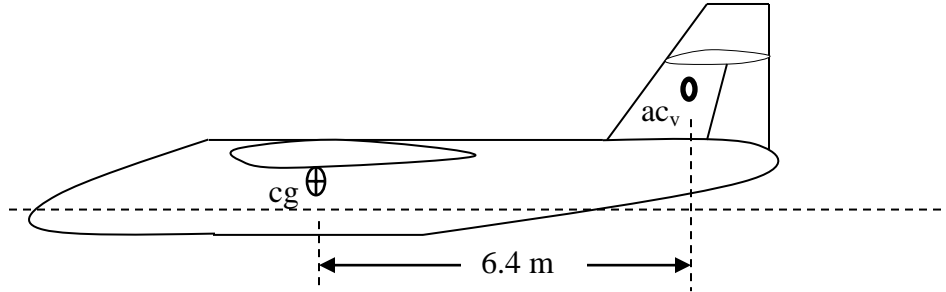

Figure 12.9. The aircraft in Example 12.3

$C_{L_{\alpha_V}} = 4.4 \ \dfrac{1}{rad}$; S = 15 m²; b = 12 m; S$_V$ = 2 m²; b$_v$ = 2.3 m; b$_R$ = 0.7 b$_v$; C$_R$/C$_V$ = 0.4;

η_v = 0.96; V$_s$ = 55 knot; $I_{xx_B} = 1{,}150 \ kgm^2$; $I_{zz_B} = 2{,}400 \ kgm^2$; $I_{xz_B} = 120 \ kgm^2$

The maximum allowable rudder deflection is ±25 degrees. Is this rudder able to satisfy the spin recovery requirement at 15,000 ft altitude? Assume the aircraft will spin at an angle of attack of 40 degrees.

Solution:

The aircraft is spinning with a 40 degrees of angle of attack, so the moments of inertia must be transformed to the wind axes.

$$\begin{bmatrix} I_{xx_w} \\ I_{zz_w} \\ I_{xz_w} \end{bmatrix} = \begin{bmatrix} \cos^2\alpha & \sin^2\alpha & -\sin 2\alpha \\ \sin^2\alpha & \cos^2\alpha & \sin 2\alpha \\ \dfrac{1}{2}\sin 2\alpha & -\dfrac{1}{2}\sin 2\alpha & \cos 2\alpha \end{bmatrix} \begin{bmatrix} I_{xx_B} \\ I_{zz_B} \\ I_{xz_B} \end{bmatrix} \tag{12.121}$$

$$\begin{bmatrix} I_{xx_w} \\ I_{zz_w} \\ I_{xz_w} \end{bmatrix} = \begin{bmatrix} \cos^2(40) & \sin^2(40) & -\sin(80) \\ \sin^2(40) & \cos^2(40) & \sin(80) \\ \dfrac{1}{2}\sin(80) & -\dfrac{1}{2}\sin(80) & \cos(80) \end{bmatrix} \begin{bmatrix} 1,150 \\ 2,400 \\ 120 \end{bmatrix} \tag{12.121}$$

This transformation results in:

$I_{xx_w} = 1,548.3 \ kgm^2 ; I_{zz_w} = 2,001.7 \ kgm^2 ; I_{xz_w} = -594.7 \ kgm^2$

It is desired to recover from a spin with a rate of 1.4 rad/s^2 (i.e. $\dot{R}_{SR} = 1.4\dfrac{rad}{\sec^2}$) when the aircraft spin with stall speed; so the required yawing moment to stop the spin with the desired rate will be:

$$N_{SR} = \left(\frac{I_{xx}I_{zz} - I_{xz}^2}{I_{xx}}\right)_w \dot{R}_{SR} = \left[\frac{1,548.3 \times 2,001.7 - (-594.7)^2}{1,548.3}\right] \times 1.4 \tag{12.120}$$

or

$N_{SR} = 2,482.6 \ Nm$

The horizontal tail will shield part of the vertical tail, but due to the cruciform configuration, no part of the rudder shall be in the wake region of the horizontal tail (i.e. $b_{Re} = b_R$). Hence, the effective vertical tail area is determined as follows:

$$C_V = \frac{S_V}{b_V} = \frac{2}{2.3} = 0.87 \ m \tag{6.80}$$

$S_{V_e} = S_V - (0.3 b_V C_V) = 2 - (0.3 \times 2.3 \times 0.87) = 1.4 \ m^2$

The effective vertical tail volume coefficient is:

$$\overline{V}_{V_e} = \frac{l_V S_{V_e}}{bS} = \frac{6.4 \times 1.4}{12 \times 15} = 0.05 \tag{12.125}$$

The rudder angle of attack effectiveness (τ_r) is a function of rudder-chord-to-vertical-tail-chord ratio (C_R/C_V). Given that the parameter C_R/C_V is 0.4; from Figure 12.12, the rudder angle of attack effectiveness (τ_r) is determined to be 0.6. The rudder control derivative is:

$$C_{n_{\delta R}} = -C_{L_{\alpha V}} \overline{V}_{V_e} \eta_V \tau_r \frac{b_{R_e}}{b_V} = -4.4 \times 0.05 \times 0.96 \times 0.6 \times 0.7 = -0.088 \ \frac{1}{rad} \tag{12.124}$$

The rudder deflection to balance the asymmetric thrust at sea level is:

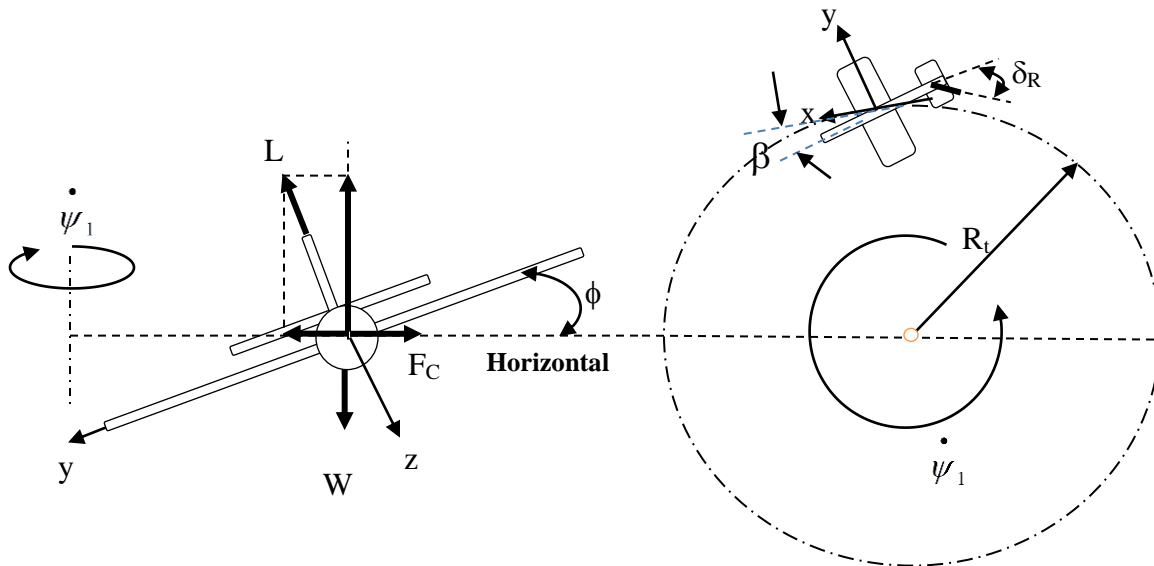$$N_{SR} = \frac{1}{2}\rho V_s^2 SbC_{n_{\delta_R}}\delta_R \tag{12.123}$$

where the air density at 15,000 ft altitude is 0.768 kg/m^3. Thus,

$$\delta_R = \frac{2N_{SR}}{\rho V_s^2 SbC_{n_{\delta_R}}} = \frac{2\times 2{,}482.6}{0.768\times(55\times0.541)^2\times15\times12\times(-0.088)} \tag{12.123}$$

or

$$\delta_R = -0.508 \; rad = -29.11 \; deg$$

The required rudder deflection is less the maximum allowable rudder deflection (i.e. 28.11 < 30). Therefore this rudder geometry is acceptable and can satisfy the spin recovery requirement.

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a. Front view   *b. Top-view*
Figure 12.10. An aircraft in a turning flight

12.6.2.5. Coordinated Turn

A simple and very essential function of a rudder is carried out during a turning flight. A turn operation is basically performed by banking the aircraft using ailerons. The rudder role is a turn is generally to coordinate the turn. A coordinated level turn is defined as a turn where the components of forces along the aircraft body-fixed y-axis sum to zero. In addition, it is desired that the aerodynamic side force (F_{Ay}) is equal to zero. This type of turn is desirable since it possesses a number of favorable features such as: 1. No net

lateral acceleration (i.e. no skidding/ no slipping), 2. Constant turn radius, 3. Constant turn rate, 4. Evenly fuel distribution between left and right fuel tanks, and 5. Passenger comfort. All of these are beneficial characteristics obtained by deflecting rudder during a turning flight.

Simultaneous deflections of aileron and rudder will create a coordinated turn, although the aircraft may have a non-zero sideslip angle (β). The coordinated turn governing equations may be derived using Newton's second law to lateral-directional axes (Figure 12.33):

$$F_{A_{y_t}} = 0 = F_C - W\sin\phi \tag{12.126}$$

$$L_{A_t} = (I_{zz} - I_{yy})R_1Q_1 \tag{12.127}$$

$$N_{A_t} = I_{xz}R_1Q_1 \tag{12.128}$$

where W represents aircraft weight; ϕ bank angle; R_1 yaw rate, Q_1 pitch rate. Furthermore, F_C denotes centrifugal force; $F_{A_{y_t}}$ represents the aerodynamic side force during turn; L_{A_t} denotes aerodynamic rolling moment during turn, and N_{A_t} denotes aerodynamic rolling moment during turn. These three lateral-directional forces and moments during a turn are given by:

$$F_C = m\frac{U_1^2}{R_t} \tag{12.129}$$

$$F_{A_{y_t}} = \frac{1}{2}\rho U_1^2 S\left(C_{y_\beta}\beta + C_{y_r}\frac{R_1 b}{2U_1} + C_{y_{\delta_A}}\delta_A + C_{y_{\delta_R}}\delta_R\right) \tag{12.130}$$

$$L_{A_t} = \frac{1}{2}\rho U_1^2 Sb\left(C_{l_\beta}\beta + C_{l_r}\frac{R_1 b}{2U_1} + C_{l_{\delta_A}}\delta_A + C_{l_{\delta_R}}\delta_R\right) \tag{12.131}$$

$$N_{A_t} = \frac{1}{2}\rho U_1^2 Sb\left(C_{n_\beta}\beta + C_{n_r}\frac{R_1 b}{2U_1} + C_{n_{\delta_A}}\delta_A + C_{n_{\delta_R}}\delta_R\right) \tag{12.132}$$

where R_t is turn radius, S wing area, b wing span, U_1 aircraft forward speed. The parameters $C_{y_\beta}; C_{y_r}; C_{y_{\delta_A}}; C_{y_{\delta_R}}; C_{l_\beta}; C_{l_r}; C_{l_{\delta_A}}; C_{l_{\delta_R}}; C_{n_\beta}; C_{n_r}; C_{n_{\delta_A}}; C_{n_{\delta_R}}$ are all aircraft stability and control derivatives. Two variables of steady-state yaw rate (R_1) and pitch rate (Q_1) are determined as follows:

$$Q_1 = \dot{\psi}_1\sin\phi = \frac{g\sin^2\phi}{U_1\cos\phi} \tag{12.133}$$

$$R_1 = \dot{\psi}_1\cos\phi = \frac{g\sin\phi}{U_1} \tag{12.134}$$

where $\dot{\psi}_1$ is the rate of turn. The ratio between the lift (L) and the aircraft weight (W) is an important parameter in turning performance. It is called load factor and represented by the symbol n:

$$n = \frac{L}{W} \tag{12.135}$$

Referring to Figure 12.33, the load factor will be equal to:

$$n = \frac{1}{\cos \phi} \tag{12.136}$$

It implies that as the aircraft bank angle (ϕ) increases, the load factor (n) will increase too. In order to determine the rudder deflection (δ_R) required to make turn coordinated, three equations of 12.130 through 12.132 must be solved simultaneously. Other two unknowns are this set of equations are usually aileron deflection (δ_A) and sideslip angle (β).

A special case for the rudder deflection during a turn is when the aircraft is experiencing an inoperative engine(s). The critical condition clearly would be at low speeds and the most unfavorable aircraft cg location. In such turning flight, an asymmetric thrust yawing moment term will be added to equation 12.132 as follows:

$$N_{A_t} + N_T = \frac{1}{2} \rho U_1^2 Sb \left(C_{n_\beta} \beta + C_{n_r} \frac{R_1 b}{2U_1} + C_{n_{\delta_A}} \delta_A + C_{n_{\delta_R}} \delta_R \right) \tag{12.137}$$

where N_T denotes the yawing moment produced by the operative (engines) about aircraft cg (z-axis).

$$N_T = \sum_{i=1}^{n_e} T_i Y_i \tag{12.138}$$

It is important to realize that the amount of rudder deflection required to hold an engine-out condition can be significantly reduced by allowing the aircraft to *bank into the operating engine*(s). In majority of aircraft, the aircraft is directionally uncontrollable if the aircraft banks into the in-operating engine(s) at low speeds (e.g. approach); since the required rudder deflection will be beyond maximum allowable deflection.

12.6.2.6. Adverse Yaw

When an aircraft is banked to execute a turn operation, conventional ailerons tend to create a yawing moment that opposes the turn (i.e. adverse yaw). The yaw is generated by the difference between induced drag ($K\ C_L^2$) of the downward-moving wing and upward-moving wing. Recall that the aileron is locally varying the wing lift when deflected. A down deflected aileron will increase the local lift; while an up deflected aileron will decrease the local lift. Hence, a downward-moving wing section has a decreased local lift, and consequently decreased the induced drag. Similarly, an upward-moving wing

section has an increased local lift, and consequently increased induced drag. Thus, a positive rolling moment (clockwise) will produce a negative yawing moment (counterclockwise) and a negative rolling moment (counterclockwise) will produce a positive yawing moment (clockwise). In this condition, the rudder must be able to overcome the adverse yaw so that a coordinated turn can be achieved.

The critical condition for rudder to overcome adverse yaw is when the airplane is flying slow. Compared with other rudder functions (e.g. crosswind landing and asymmetric thrust), eliminating an adverse yaw is not a critical role for a rudder. On the other hand, there are techniques to eliminating adverse yaw without employing rudder. Two convenient techniques to avoid adverse yaw are Frise aileron and differential deflection. Most Cessna aircraft are equipped with Frise ailerons, but most Piper aircraft employ differentially deflected ailerons.

12.6.2.7. Glide Slope Adjustment

Another role of a rudder is frequently applied during a gliding operation. One way of increasing glide slope and glide angle; in addition to pulling up the nose; is to use rudder. When a glider; or an engine-powered aircraft with all engines inoperative; is approaching a runway, the only way to adjust the glide slope is to increase the drag while keeping the path. An effective technique to deliberately increase the aircraft drag is to increasing the sideslip angle. During a glide, if the glide angle (γ) is less than a specific value, the glider or aircraft will passes over the runway and will land on an unsuitable ground. On such a flight condition, pilot will usually deflect the rudder to increase aircraft drag in order to increase glide angle, and then land safely. As the rudder deflection is increased, the glide slope gets steeper; thus a slow rate glide is converted to a fast rate glide. The rudder of a glider might be designed solely based on the glide slope requirements.

For a steady unaccelerated glide, the governing equations of motion are:
$$L - W\cos\gamma = 0 \tag{12.139}$$
$$D - W\sin\gamma = 0 \tag{12.140}$$
So the glide angle will be derived as:
$$\gamma = \tan^{-1}\left(\frac{1}{L/D}\right) = \tan^{-1}\left[2\sqrt{K \cdot C_{D_o}}\right] \tag{12.141}$$

For any sideslip angle (β), the drag force is increased, and the forward speed is reduced. Thus, the aircraft drag during a glide when involves a sideslip angle is obtained as:
$$D_{GL} = \frac{1}{2}\rho U_1^2 S C_{D_{GL}} \tag{12.142}$$
where aircraft drag coefficient due to a sideslip angle is given by:
$$C_{D_{GL}} = C_{D_o} + K C_L^2 + C_{D_\beta}\beta \tag{12.143}$$

The derivative is C_{D_β} the rate of change of aircraft drag with respect to sideslip angle (β):

$$C_{D_\beta} = \frac{\partial C_D}{\partial \beta}$$ (12.144)

Ref. [23] has introduced two non-dimensional coupling derivatives; one of which relates the sideslip angle to aircraft drag coefficient (i.e. C_{D_β}). This derivative is given by:

$$C_{D_\beta} = [sign(\beta)]\frac{S_V}{S}\frac{2}{\pi}$$ (12.145)

The relationship between rudder deflection and the sideslip angle is given by equation 12.110. The rate of sink (ROS) in a gliding operation is obtained by the following equations:

$$ROS = U_1 \sin(\gamma)$$ (12.146)

The rudder deflection is adjusted to produce the desired rate of sink in a gliding flight.

12.6.3. Rudder Design Steps

In Sections 12.6.1 and 12.6.2; rudder primary functions, parameters, governing rules and equations, design objectives, design criteria, formulation, as well as design requirements have been presented in details. In addition, Figure 12.26 illustrates the design flow chart of the rudder. In this section, rudder design procedure in terms of design steps is introduced. It must be noted that there is no unique solution to satisfy the customer requirements in designing a rudder. Several rudder designs may satisfy the requirements, but each will have a unique advantages and disadvantages. It must be noted, that there is a possibility that no rudder can satisfy the requirements due to the limits/constraints imposed by vertical tail design and aircraft cg location. In such a situation, the designer must return to vertical tail design and/or aircraft weight distribution; and redesign/redistribute those components.

No	Aircraft	The most critical flight condition
1	Glider/sailplane	Glide slope adjustment
2	Single engine Normal GA	Crosswind landing
3	Single engine Utility/Acrobatic GA	Spin recovery
4	Multi-engine Normal GA	Asymmetric thrust
5	Multi-engine Utility/Acrobatic GA	Asymmetric thrust/ Spin recovery
6	Multiengine transport (fuselage-installed engines)	Crosswind landing
7	Multiengine transport (wing-installed engines)	Asymmetric thrust/ Crosswind landing
8	Military fighter	Directional maneuverability/Spin recovery
9	Remote controlled/ model	Coordinated turn

Table 12.3. The most critical flight condition for a rudder

Based on the systems engineering approach, the rudder detail design process starts with identifying and defining design requirements and ends with optimization. Since there are a number of directional control/trim requirements, a separate procedure are set for each rudder design requirements. If you can evaluate/identify the most crucial directional control function of the rudder (See Section 12.6.2), begin the design process with the requirements to satisfy the most crucial directional control function. In the case where it is very hard to recognize which of the directional control requirements are the most critical one; follow the suggested ones as in the Table 12.22.

12.6.3.1. Rudder Design Steps to Satisfy Asymmetric Thrust Requirements
The following is the rudder design steps for a conventional aircraft to satisfy asymmetric thrust requirements:

1. List the available/given data related to rudder design (e.g. Vertical tail geometry, aircraft cg locations).
2. Identify the most unfavorable cg location and aircraft weight combination; the most unfavorable engines inoperative conditions; the most unfavorable altitude for directional control. This will be set as the most critical condition.
3. Select rudder-span-to-vertical-tail-span ratio; b_R/b_V (See Table 12.3)
4. Establish maximum rudder deflection to prevent flow separation (See Table 12.3)
5. Determine/Select the aircraft minimum controllable speed. FAR regulations provide some requirements for such speed. It is recommended to select a value equivalent to 80 percent of the stall speed to consider a safety factor.
6. Determine the required maximum yawing moment to directionally control/trim the aircraft in the most critical condition utilizing equation 12.102.
7. Compute the rudder control derivative $C_{n_{\delta_R}}$ utilizing equation 12.103 assuming that maximum rudder deflection is employed.
8. Calculate the rudder angle of attack effectiveness (τ_R) employing Equation 12.100.
9. Determine the corresponding rudder-to-vertical-tail-chord ratio (C_R/C_V) from Figure 12.12.
10. If the rudder-to-vertical-tail-chord ratio (C_R/C_V) is more than 0.5, it is suggested to select an all moving vertical tail (i.e. $C_R/C_V = 1$).
11. If the angle of attack effectiveness of the rudder (τ_r) is greater than 1, there is no rudder which can satisfy the most critical directional control/trim requirement by the current vertical tail/aircraft cg combination. In such a case, vertical tail must be redesigned and/or aircraft cg must be relocated. Then, return to step 1.
12. Evaluate the rudder design to make certain that other rudder design requirements (e.g. crosswind landing and spin recovery) are met. Otherwise redesign the ruder based on the new most critical directional control requirement.

13. Check whether or not the rudder deflection causes the vertical tail to stall during directional control by using the technique introduced by equation 12.92 and Table 12.19.
14. If vertical tail stall will be occurred during yawing motion, the rudder must be redesigned by reducing rudder deflection and/or rudder chord. Return to step 4.
15. If vertical tail stall will be occurred during yawing motion, and none of the two rudder parameters (i.e. rudder deflection and chord) may be reduced to prevent vertical tail stall; other aircraft components such as vertical tail, engine location, or aircraft center of gravity must be redesigned/relocated.
16. Aerodynamic balance/mass balance; if necessary (Section 12.7)
17. Optimize the rudder.
18. Calculate rudder span, rudder tip and root chords and rudder area; and then draw the top-view and side-view of the vertical tail (including rudder) with dimensions.

12.6.3.2. Rudder Design Steps to Satisfy Crosswind Landing Requirements

The following is the rudder design steps for a conventional aircraft to satisfy crosswind landing requirements:

1. List the available/given data related to rudder design (e.g. Vertical tail geometry, aircraft cg locations).
2. Identify the most unfavorable cg location and aircraft weight combination; and the most unfavorable altitude for directional control. This will be set as the most critical condition.
3. Determine/Select the maximum crosswind speed (V_W) which the aircraft must be able to land safely. FAR regulations provide some requirements for such wind speed.
4. Determine/Select the aircraft approach speed. FAR regulations provide some requirements for such speed.
5. Determine the aircraft total airspeed (V_T) when a crosswind is present using equation 12.106. Assume the worst wind condition; i.e. a perpendicular crosswind to the runway.
6. Calculate the projected side area of the aircraft (S_S).
7. Determine the center of the projected side area of the aircraft (S_S) and its distance to the aircraft cg (d_c) using equation 12.119.
8. Determine the aircraft side force produced by the crosswind (F_w) using equation 12.113.
9. Select rudder-span-to-vertical-tail-span ratio; b_R/b_V (See Table 12.3)
10. Select the rudder-to-vertical-tail-chord ratio; C_R/C_V (See Table 12.3)
11. Determine the aircraft sideslip angle (β) using equation 12.105.
12. Calculate aircraft sideslip derivatives C_{n_β} and C_{y_β} using equations 12.116, 12.117.

13. Calculate the rudder angle of attack effectiveness (τ_r) employing equation from Figure 12.12.
14. Calculate aircraft control derivative $C_{y_{\delta_R}}$ and $C_{n_{\delta_R}}$ using equations 12.118. 12.100
15. Compute the rudder control derivative by simultaneous solution of equations 12.114 and 12.115. Another unknown variable is side-wash angle (σ) which is beneficial in preparing the aircraft flight instruction manual.
16. If the rudder deflection is more than 30 degrees, it is suggested to increase the rudder-to-vertical-tail-chord ratio up to an all moving vertical tail (i.e. $C_R/C_V = 1$).
17. If the angle of attack effectiveness of the rudder (τ_R) is greater than 1, there is no rudder which can satisfy the most critical directional control/trim requirement by the current vertical tail/aircraft cg combination. In such a case, vertical tail must be redesigned and/or aircraft cg must be relocated. Then, return to step 1.
18. Evaluate the rudder design to make certain that other rudder design requirements (e.g. asymmetric thrust and spin recovery) are met. Otherwise redesign the ruder based on the new most critical directional control requirement.
19. Check whether or not the rudder deflection causes the vertical tail to stall during the directional control by using the technique introduced by equation 12.92 and Table 12.19.
20. If vertical tail stall will be occurred during yawing motion, the rudder must be redesigned by reducing rudder deflection and/or rudder chord. Return to step 8.
21. If vertical tail stall will be occurred during yawing motion, and none of the two rudder parameters (i.e. rudder deflection and chord) may be reduced to prevent vertical tail stall; other aircraft components such as vertical tail, engine location, or aircraft center of gravity must be redesigned/relocated.
22. Aerodynamic balance/mass balance if necessary (Section 12.7)
23. Optimize the rudder.
24. Calculate rudder span, rudder tip and root chords, and rudder area; and then draw the top-view and side-view of the vertical tail (including rudder) with dimensions.

12.6.3.3. Rudder Design Steps to Satisfy Spin Recovery Requirements
The following is the rudder design steps for a conventional aircraft to satisfy spin recovery requirements:

1. List the available/given data related to rudder design (e.g. Vertical tail geometry, aircraft cg locations).
2. Identify the most unfavorable cg location and aircraft weight combination; and the most unfavorable altitude for spin recovery. This will be set as the most critical condition.
3. Determine the aircraft angle of attack during a spin maneuver.

4. Calculate the aircraft mass moments of inertia I_{xx}, I_{zz}, and I_{xz} in the body axis coordinate system. The technique to determine aircraft mass moment of inertia is presented in Chapter 11, Section 11.7.

5. Determine the aircraft mass moments of inertia I_{xx}, I_{zz}, and I_{xz} in the wind axis coordinate system using equation 12.120.

6. Determine the desirable rate of spin recovery. A typical value for the rate is given is given in Section 12.6.2.4.

7. Calculate the required yawing moment to stop the spin (N_{SR}) using equation 12.120.

8. Compute the effective vertical tail area during spin operation. A graphical technique is depicted in Figure 12.31.

9. Calculate the effective vertical tail volume ratio using equation 12.125.

10. Select rudder-span-to-vertical-tail-span ratio; b_R/b_V (See Table 12.3)

11. Compute the effective rudder span during spin operation. A graphical technique is depicted in Figure 12.31.

12. Establish maximum rudder deflection to prevent flow separation (See Table 12.3).

13. Compute the rudder control derivative $C_{n_{\delta_R}}$ utilizing equation 12.123 assuming that maximum rudder deflection is employed.

14. Calculate the rudder angle of attack effectiveness (τ_R) employing Equation 12.124.

15. Determine the corresponding rudder-to-vertical-tail-chord ratio (C_R/C_V) from Figure 12.12.

16. If the rudder-to-vertical-tail-chord ratio (C_R/C_V) is more than 0.5, it is suggested to select an all moving vertical tail (i.e. $C_R/C_V = 1$).

17. If the angle of attack effectiveness of the rudder (τ_R) is greater than 1, there is no rudder which can satisfy the most critical directional control/trim requirement by the current vertical tail/aircraft cg combination. In such a case, vertical tail must be redesigned and/or aircraft cg must be relocated. Then, return to step 1.

18. Evaluate the rudder design to make certain that other rudder design requirements (e.g. crosswind landing and spin recovery) are met. Otherwise redesign the ruder based on the new most critical directional control requirement.

19. Investigate whether or not the rudder deflection causes the vertical tail to stall during directional control by using the technique introduced by equation 12.92 and Table 12.19.

20. If vertical tail stall will be occurred during yawing motion, the rudder must be redesigned by reducing rudder deflection and/or rudder chord. Return to step 4.

21. If vertical tail stall will be occurred during yawing motion, and none of the two rudder parameters (i.e. rudder deflection and chord) may be reduced to prevent vertical tail stall; other aircraft components such as vertical tail, engine location, or aircraft center of gravity must be redesigned/relocated.

22. Apply aerodynamic balance and/or mass balance the rudder; if necessary (Section 12.7)

23. Optimize the rudder.
24. Calculate rudder span, rudder tip and root chords and rudder area; and then draw the top-view and side-view of the vertical tail (including rudder) with dimensions.

12.8.3. Rudder Design Example

Example 12.6

Problem Statement: A large transport aircraft with a maximum take-off mass of 260,000 kg is equipped with four turbofan engines each generating 140 kN of thrust. The distance between the most aft and the most forward cg is 1.5 m. The top-view and side-view of the aircraft are shown in Figure 12.51; fuselage has a cylindrical shape. Other characteristics of the aircraft are as follows:

$S = 365$ m^2; b = 60 m; $S_V = 50$ m^2; $V_s = 120$ knot; $C_{L_{\alpha_V}} = 4.5 \ \dfrac{1}{rad}$; $\eta_V = 0.97$; $L_f = 63$ m;

$D_f = 5.5$ m; $\dfrac{d\sigma}{d\beta} = 0$; $C_{n_o} = 0$; $C_{y_o} = 0$

The aircraft is not spinnable, and is required to be able to land safely when there is a crosswind of 40 knots. Design a rudder for this aircraft.

Solution:
Step 0:
The first step is to layout the design requirements and to identify the most critical one. Since the aircraft is not spinnable, so the design requirements are: 1. Asymmetric thrust directional trim; 2. Crosswind landing; 3. coordinated turn; 4. Adverse yaw. According to table 12.22, the most critical rudder design requirements for a multiengine wing-installed engines transport aircraft are: either of asymmetric thrust or crosswind landing. At this moment, it is not evident which one is the most crucial one. Since the crosswind is 40 knot (a rrelatively high value), it is assumed that crosswind landing is the most critical design requirement. Nonethelesss, the asyymetric thrust will be also investigated. Therefore the steps of ruddr design based on crosswind landing (Section 12.6.3.2) will be followed.

Step 1:
List the available/given data related to rudder design.
The vertical tail is rectangular with a planform area of 50 m^2. From figure 12.51, the vertical tail span is 8 m, and the distance between most forward cg and the vertical tail aerodynamic center is 27 m.
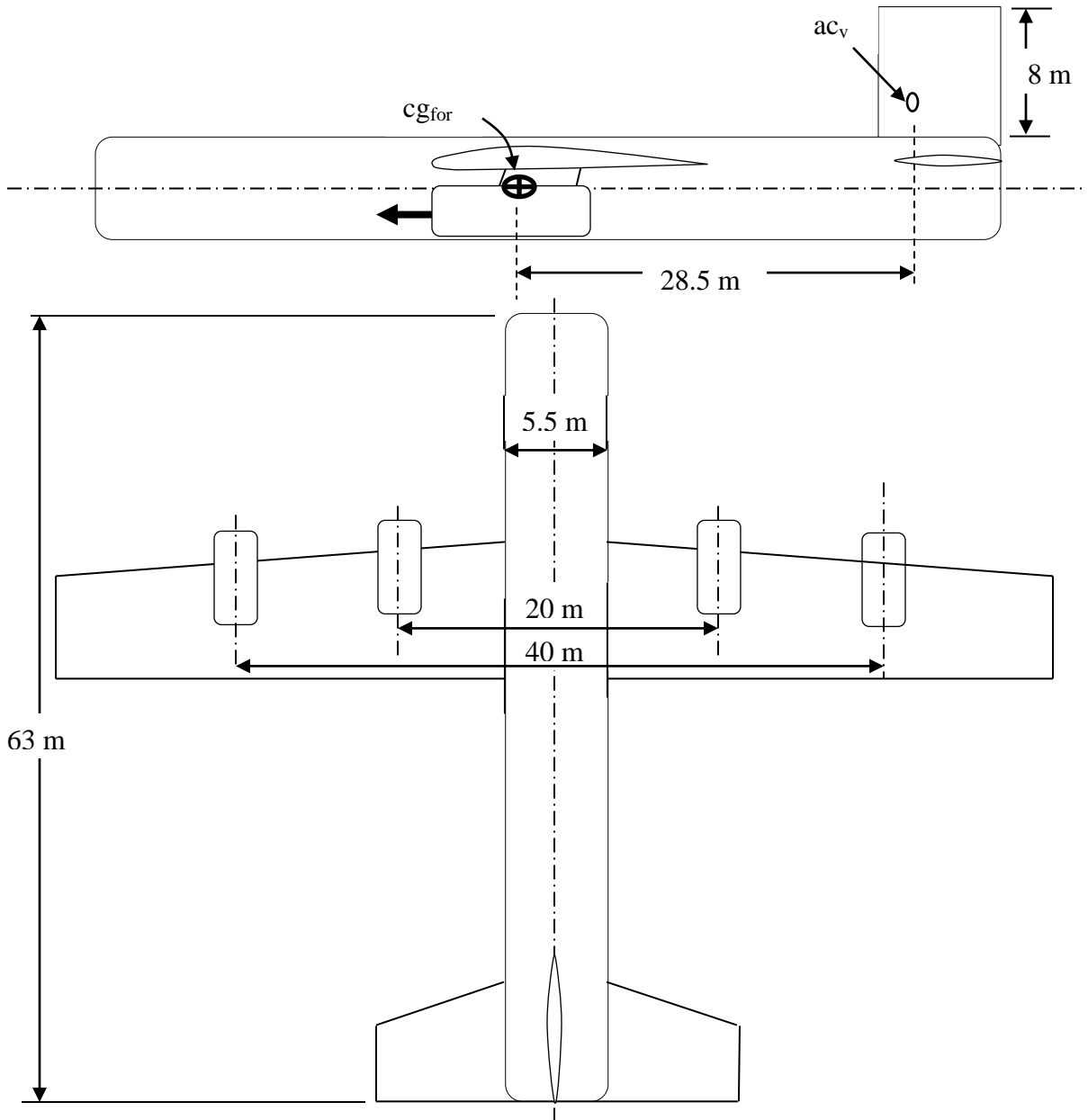
33

Figure 12.11. Top-view and side-view of the aircraft in Example 12.6

Step 2:
Identify the most unfavorable cg location and aircraft weight combination; and the most unfavorable altitude for directional control. The distance between the most aft and the most forward cg is given to be 1.5 m. The most critical condition would be when the cg is at its most aft location, so:

$l_v = 27 - 1.5 = 27$ m

Other weight data of aircraft is not given, so the design will be based on the maximum take-off weight. The runway altitude is not given, so the most critical condition for the rudder is assumed to be at sea level.

Step 3:
The maximum crosswind speed (V_W) is given by the problem statement as 40 knots.

Step 4:
The aircraft approach speed is selected to be 1.1 V_s.

$$U_1 = 1.1V_s = 1.1 \times 120 = 132 \ knot = 67.91 \ \frac{m}{sec}$$

Step 5:
Determine the aircraft total airspeed (V_T).

$$V_T = \sqrt{U_1^2 + V_W^2} = \sqrt{(132)^2 + (40)^2} = 137.93 \ knot = 70.95 \ \frac{m}{sec} \tag{12.106}$$

Step 6:
Calculate the projected side area of the aircraft.
From side-view of the aircraft, it is seen that the wing and engines are projected into the fuselage. Both the fuselage and vertical tail have a rectangular shape, so the aircraft projected side area is mainly the fuselage projected side area plus vertical tail planform area. The geometry of the landing gear is not given, so a two percent is added to the sum of the fuselage and vertical tail projected side area to account for the landing gear:

$$S_S = 1.02[S_f + S_V] = 1.02[L_f D_f + S_V] = 1.02 \times (63 \times 5.5 + 50) = 404.4 \ m^2$$

Step 7:
Determine the center of the projected side area of the aircraft (S_S) and its distance to the aircraft cg (d_c). Both the fuselage and vertical tail have a rectangular shape, so the center of fuselage is at midpoint of the fuselage length from the nose (63/2 = 31.5 m). The vertical tail chord is:

$$S_V = b_V \cdot \overline{C_V} \Rightarrow C_V = \frac{S_V}{b_V} = \frac{50}{8} = 6.25 \ m \tag{6.80}$$

The center of the vertical tail area is at midpoint of the vertical tail chord from its leading edge (6.25/2= 3.125 m). The overall center of the projected side area of the aircraft from the fuselage nose is determined as follows:

$$x_{ca} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} = \frac{(L_f D_f)x_f + S_V x_V}{L_f D_f + S_V} = \frac{(63 \times 5.5) \times \frac{63}{2} + 50 \times \left[(63 - 6.25) + \frac{6.25}{2}\right]}{(63 \times 5.5) + 50} \tag{12.119}$$

or
x_{ca} = 35.078 m

35

The vertical tail aerodynamic center is assumed to at its quarter chord, so the distance between the center of gravity of the aircraft to the fuselage nose is (See Figure 12.52):

$$x_{cg} = L_f - l_V - 0.75C_V = 63 - 27 - 0.75 \times 6.25 = 31.313 \ m$$

Thus, the distance between the center of the projected side area of the aircraft and aircraft cg (See Figure 12.51) is:

$$d_c = x_{ca} - x_{cg} = 35.078 - 31.313 = 3.766 \ m$$

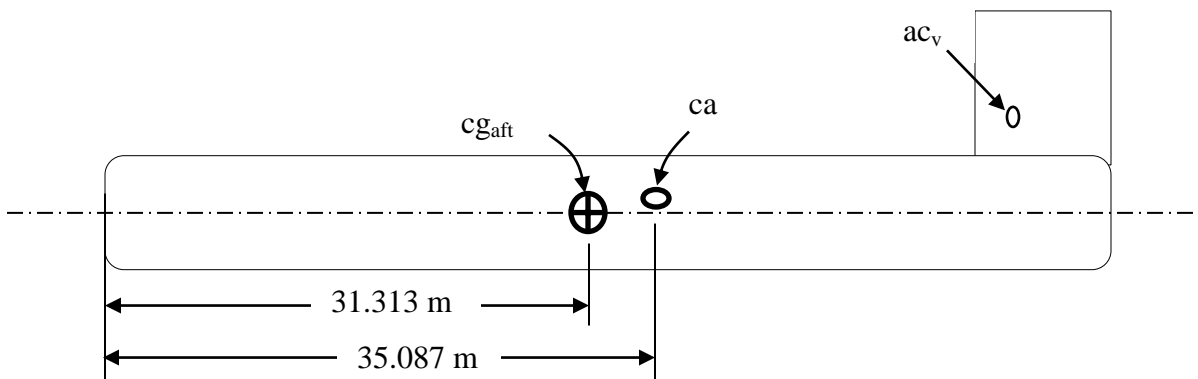Therefore the center of the projected side area of the aircraft is 3.766 m behind the aircraft cg.

Figure 12.12. Center of projected side area

Step 8:
Determine the aircraft side force produced by the crosswind (F_w). Due to simplicity, a crosswind from the right is assumed; which is generating a positive sideslip angle. Based on the side view of the aircraft (fuselage has a cylindrical shape); a side drag coefficient of 0.6 is selected (Section 12.6.2.3). The sea level air density is 1.225 kg/m^3.

$$F_w = \frac{1}{2}\rho V_W^2 S_S C_{D_y} = \frac{1}{2} \times 1.225 \times (40 \times 0.514)^2 \times 404.4 \times 0.6 = 62,936 \ N \qquad (12.113)$$

Step 9:
The rudder-span-to-vertical-tail-span ratio; b_R/b_V (from Table 12.3) is tentatively selected to be 1.

Step 10:
The rudder-to-vertical-tail-chord ratio (C_R/C_V) is tentatively selected to be 0.3 (Table 12.3).

Step 11:
Determine the aircraft side slip angle (β).

$$\beta = \tan^{-1}\left(\frac{V_W}{U_1}\right) = \tan^{-1}\left(\frac{40}{132}\right) = 0.294 \ rad = 16.86 \ \text{deg} \tag{12.105}$$

Step 12:
Calculate aircraft sideslip derivatives C_{n_β} and C_{y_β}.

From Chapter 6, Section 6.8.1, the parameter K_{f1} is selected to be 0.75; and from Section 12.6.2.3, the parameter K_{f2} is selected to be 1.35. Thus:

$$C_{n_\beta} = K_{f1}C_{L_{\alpha V}}\left(1 - \frac{d\sigma}{d\beta}\right)\eta_V \frac{l_{Vt} S_V}{bS} = 0.75 \times 4.5 \times (1-0) \times 0.96 \times \frac{27 \times 50}{60 \times 365} \tag{12.116}$$

or

$$C_{n_\beta} = 0.2 \ \frac{1}{rad}$$

$$C_{y_\beta} = -K_{f2}C_{L_{\alpha V}}\left(1 - \frac{d\sigma}{d\beta}\right)\eta_V \frac{S_V}{S} = -1.35 \times 4.5 \times (1-0) \times 0.96 \times \frac{50}{365} \tag{12.117}$$

or

$$C_{y_\beta} = -0.8 \ \frac{1}{rad}$$

Step 13:
A value of 0.51 for the rudder angle of attack effectiveness (τ_r) is extracted from Figure 12.12 for a control-surface-chord/lifting-surface-chord of 0.3.

Step 14:
Calculate aircraft control derivative $C_{y_{\delta_R}}$ and $C_{n_{\delta_R}}$.

$$C_{y_{\delta_R}} = C_{L_{\alpha V}}\eta_V \tau_r \frac{b_R}{b_V}\frac{S_V}{S} = 4.5 \times 0.96 \times 0.51 \times 1 \times \frac{50}{365} = 0.302 \ \frac{1}{rad} \tag{12.118}$$

$$C_{n_{\delta_R}} = -C_{L_{\alpha V}}V_V\eta_V \tau_r \frac{b_R}{b_V} = -4.5 \times 0.062 \times 0.96 \times 0.51 \times 1 = -0.136 \ \frac{1}{rad} \tag{12.100}$$

Step 15:
Compute the rudder deflection.
By simultaneous solution of equations 12.114 and 12.115, two unknowns of rudder deflection and side-wash angle is calculated.

$$\frac{1}{2}\rho V_T^2 Sb\left(C_{n_o} + C_{n_\beta}(\beta - \sigma) + C_{n_{\delta_R}}\delta_R\right) + F_w \cdot d_c \cos\sigma = 0 \tag{12.114}$$

$$\frac{1}{2}\rho V_W^2 S_S C_{D_y} = \frac{1}{2}\rho V_T^2 S\left(C_{y_o} + C_{y_\beta}(\beta - \sigma) + C_{y_{\delta_R}}\delta_R\right) \tag{12.115}$$

37

or

$$\frac{1}{2} \times 1.225 \times (70.95)^2 \times 365 \times 60 \times [0.2 \times (0.294 - \sigma) - 0.136\delta_R] + 62936 \times 3.766 \cos \sigma = 0$$

$$62936 = \frac{1}{2} \times 1.225 \times (70.95)^2 \times 365(-0.799 \times (0.294 - \sigma) + 0.302\delta_R)$$

Simultaneous solution of these equations results in:

δ_R = 0.458 rad = 26.2 degrees
σ = 0.191 rad = 10.95 degrees

The required rudder deflection (26.2 deg) is less than the maximum allowable deflection (30 deg). Thus, the aircraft is able to handle a crosswind of 40 knots and land safely.

Steps 16 and 17: Checked; OK:
Step 18:

Now that the vertical is designed based on the crosswind landing requirements, it is the time to evaluate the rudder design to ensure that other rudder design requirements (e.g. asymmetric thrust and spin recovery) are met. Since the aircraft is not spinnable, the only major requirement is asymmetric thrust directional control. Since the rudder geometry and derivatives are known, we start from step 5 of the procedure in Section 12.6.3.1.

Step: 18-5:
Determine/Select the aircraft minimum controllable speed (V_{MC}).
To consider a safety factor, it is decided to select a value equivalent to 80 percent of the stall speed.

$$V_{MC} = 0.8 \cdot V_S = 0.8 \times 120 = 96 \ knot = 49.39 \ \frac{m}{sec}$$

Step: 18-6:
Determine the required maximum yawing moment to directionally control/trim the aircraft in the most critical condition. It is desired to directionally trim the aircraft in an asymmetric thrust flight condition when the aircraft is flying with minimum controllable speed.

The aircraft has four engines, so the most crucial flight condition is when two engines (n = 2) in one side suddenly lose power and become inoperative. The distance between two internal engines is 20 m, and between two external engines is 40 m. Each engine is generating 140 kN of thrust. Hence, the rudder deflection to balance the asymmetric thrust at sea level is:

$$\delta_R = \frac{\sum\limits_{i=1}^{n2} T_{L_i} y_{T_i}}{-\bar{q}SbC_{n_{\delta_R}}} = \frac{\left(140{,}000 \times \dfrac{10}{2}\right) + \left(140{,}000 \times \dfrac{20}{2}\right)}{-\dfrac{1}{2} \times 1.225 \times (49.39)^2 \times 365 \times 60 \times (-0.136)} \tag{12.104b}$$

or

$$\delta_R = 0.945 \ rad = 54.16 \ \deg$$

The required rudder deflection exceeds the maximum allowable rudder deflection (i.e. 54.16 > 30). Therefore, this rudder geometry is not acceptable to satisfy the asymmetric thrust balance requirement when the aircraft is flying with a speed equal to 80 percent of the stall speed.

At this point, there are two options to the designer: 1. Redesign the rudder, 2. Redefine the minimum controllable speed. Due to the fact that, the aircraft may have a minimum controllable speed slightly greater than the stall speed, the second option is chosen. The maximum allowable rudder deflection of 30 degrees is employed. Thus, the new minimum controllable speed is calculated as follows:

$$V_{MC} = \sqrt{\frac{\sum\limits_{i=1}^{n2} T_{L_i} y_{T_i}}{-\dfrac{1}{2}\rho SbC_{n_{\delta_R}}\delta_R}} = \sqrt{\frac{\left(140{,}000 \times \dfrac{20}{2}\right) + \left(140{,}000 \times \dfrac{40}{2}\right)}{-\dfrac{1}{2} \times 1.225 \times 365 \times 60 \times (-0.136) \times \dfrac{30}{57.3}}} \tag{12.104b}$$

or

$$V_{MC} = 66.356 \ \frac{m}{\sec} = 129 \ knot$$

This minimum controllable speed is about 7.5% greater than the aircraft stall speed.

$$\frac{V_{MC}}{V_s} = \frac{129}{120} = 1.075$$

Steps 19 through 23:

The rest of the example is left to the interested reader to continue.

Step 24:

Calculate the rudder span, rudder chord, and rudder area.

$$\frac{C_R}{C_V} = 0.3 \Rightarrow C_R = 0.3C_V = 0.3 \times 6.25 = 1.875 \ m$$

$$\frac{b_R}{b_V} = 1 \Rightarrow b_R = b_V = 8 \ m$$

$$S_R = b_R C_R = 8 \times 1.875 = 15 \ m^2$$

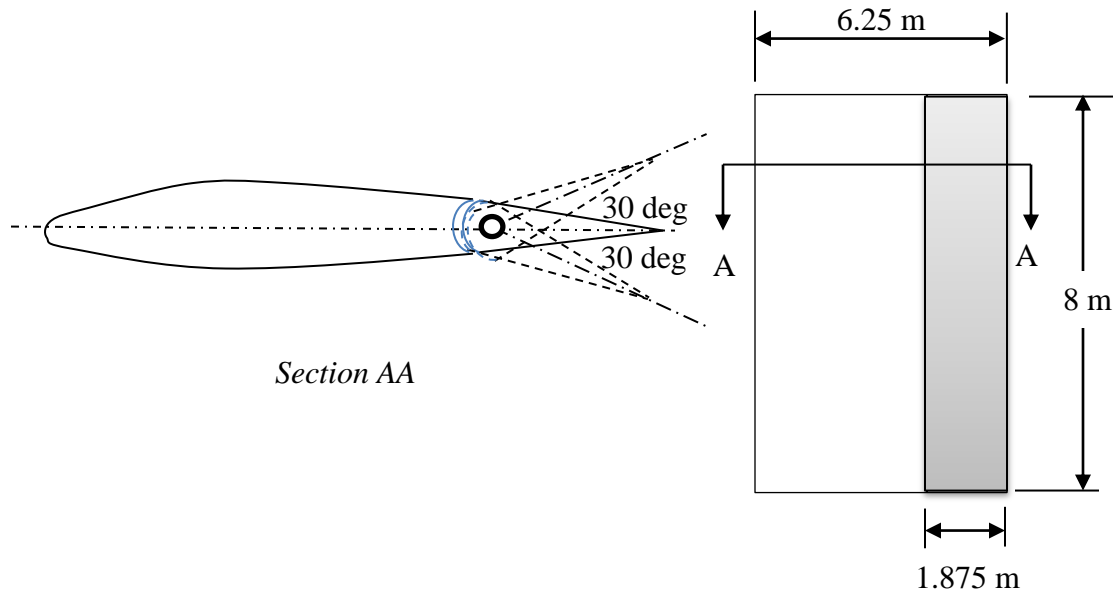The geometry of the rudder is depicted in Figure 12.53.

Section AA

Figure 12.13. Geometry for the rudder of Example 12.6